Stuff you need to know by heart

Types of Numbers

Real numbers

$\mathbb{R}\begin{cases}\mathbb{Z} \text{ integers } = -3, -1, 0, 1, 2, 3 \\ \mathbb{N} \text{ atural } = 0, 1, 2, 3 \\ \mathbb{Q} \text{ Rational } = 3/2, 3.5, 6.\overline{8} \\ \mathbb{Q}' \text{ Irrational } = \pi, 1.683542 \\ \mathbb{W} \text{ hole } = 1, 2, 3\end{cases}$

Fractions

$\frac{1}{2_{\times 9}} + \frac{3}{9_{\times 2}} = \frac{15}{18} = \frac{5}{6}$

$\frac{1}{2_{\times 3}} - \frac{3}{6} = 0$

$4\,\frac{\overset{\div 4}{\cancel{16}}}{\underset{1}{\cancel{25}}} \times \frac{\overset{4}{\cancel{100}}}{\underset{9}{\cancel{36}}\div 4} = \frac{16}{9}$

$\frac{15}{27} \div \frac{25}{9} = \frac{15}{27} \cdot \frac{25}{9} = \frac{3}{15} = \frac{1}{5}$

$\frac{3}{1}(-\frac{6}{7}x) = -\frac{18}{7}x$

$\frac{4}{5}\left(\frac{3}{2}x\right) = \frac{12}{10}x = \frac{6}{5}x$

$\frac{3/2}{2/9} = \frac{3}{2} \div \frac{2}{9} = \frac{3}{2} \cdot \frac{9}{2} = \frac{27}{4}$

Order of Operations with Fractions

11. $(-\frac{4}{7}) \div (-\frac{12}{7}) - \frac{1}{3} =$

$(-\frac{4}{\cancel{7}_1}) \cdot (-\frac{\cancel{7}^1}{12_3}) - \frac{1}{3}$

$\frac{1}{3} - \frac{1}{3}$

0

12. $4\frac{1}{2} \times (-\frac{2}{3}) \div \frac{7}{8} =$

$\frac{9}{2} \cdot \left(-\frac{2}{3}\right) \cdot \frac{8}{7}$

$= \frac{9^3}{2_1} \cdot \frac{-16^8}{27_9}$

$\frac{-24}{9} = -\frac{8}{3}$

13. $\frac{4}{9} \div \frac{-3}{4} \times \frac{2}{9} \div \frac{3}{5} =$

$\frac{4}{9} \cdot \frac{4}{-3} \times \frac{2}{9} \cdot \frac{5}{3}$

$-\frac{160}{729}$

14. $\frac{1}{3} \times \frac{5}{8} + \frac{1}{4} \times (\frac{-1}{2})$

$\frac{5}{24} + \frac{1}{4} \cdot \left(\frac{-1}{2}\right)$

$\frac{5}{24} + \frac{-1^{\times 3}}{8_{\times 3}}$

$\frac{5}{24} + -\frac{3}{24}$

$\frac{2}{24} = \frac{1}{12}$

Solve

$\frac{2}{3}x + 7 = 4 \quad -7$

$\frac{2}{3}x = \frac{-3}{1}$

$x = -\frac{9}{2}$

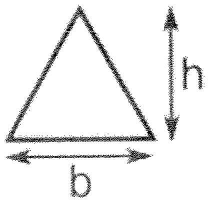
$x = -4.5$

$\frac{3}{4}x = \frac{6}{11}$

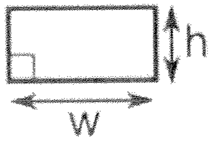
$x = \frac{24}{33}$

$x = \frac{8}{11}$

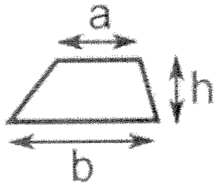
Area of 2D Shapes



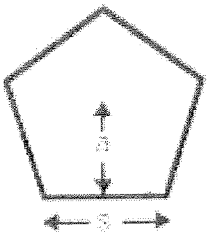
$\frac{b \cdot h}{2}$



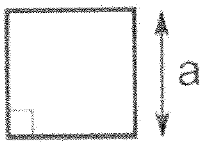
$h \cdot w$



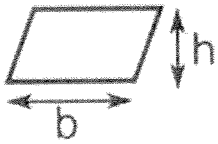
$\frac{(b+a)h}{2}$



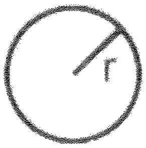
$\frac{a \cdot s \cdot s}{2}$



a^2



$b \cdot h$



r^2

Long Division

16
32)528
32 ↓
208
192
16

1 = Divide
2 = Multiply
3 = Subtract
4 = Bring down
5 = Repeat

Solve equations

Case 1: $2x - 8 = 0$ $+8$

$2x = \frac{8}{2}$

$x = 4$

Case 2: $2x + 3 = -3x + 5x - 6$

$2x = 2x - 6 - 3$

$0x = -9$

$\varnothing$ No solution

Case 3: $3x - 8 = 2x + 1x - 5 - 3$

$3x - 8 = 3x - 8$

$0\cancel{x} = 8 + 8$

$0x = 0$

$x \in \mathbb{R}$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{a \pm b} \neq \sqrt{a} \pm \sqrt{b}$$

Simplify

$\sqrt{50x^3}$ $5x\sqrt{2 \cdot \cancel{25} \cdot \cancel{x^2} \cdot x}$

$5x\sqrt{2x}$

50: 2 25 x^3: x^2 x

$\sqrt{125x^2}$ $\sqrt{5 \cdot 25 \cdot x^2}$

$5x\sqrt{5}$

$\sqrt{200}$

$\sqrt{2 \cdot 100}$

$10\sqrt{2}$

Rationalize the denominator

$\frac{3}{\sqrt{5}} =$

$\frac{6}{\sqrt{3}}$

Properties of a function

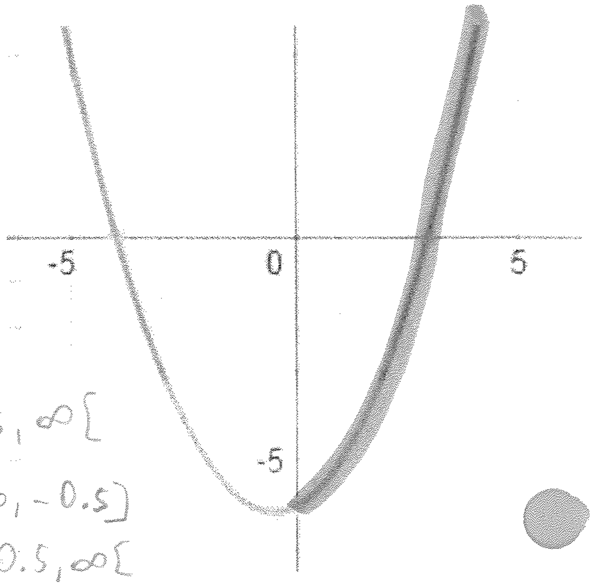
Positive/Negative: where the curve is above x-axis / where the curve is below x-axis

Strictly Positive/Negative: same as above but not including on axis

Increasing/Decreasing: the values of x where the line goes up / values of x where the line goes down

Strictly Increasing/Decreasing: Same as above, but don't include the axis point

erything always i the -axis



⊕ $]-\infty, -4] \cup [3, \infty[$

⊖ $[-4, 3]$

St ⊕ $]-\infty, -4[\cup]3, -\infty[$

St ⊖ $]-4, 3[$

(↑) $[-0.5, \infty[$

(↓) $]-\infty, -0.5]$

(St↑) $]-0.5, \infty[$

(St↓) $]-\infty, -0.5[$

Notation $f(x)$ same as $s(x)$ same as $h(t)$

$y = ax + b$	$f(x) = ax + b$
$y = 2x - 3$	$f(x) = 2x - 3$
$y = 2(5) - 3$	$f(5) = 2(5) - 3$

The height (m) of a ball as a function of time (sec) follows the equation ~~$h(t) = 2x^2 + 1$~~.

$h(t) = 2t^2 + 1$

Find the time it takes to reach 19 m → $h(t) = 19$.

$h(t) = 2t^2 + 1$

$19 - 1 = 2t^2 \boxed{+1}$

$\frac{18}{2} = 2t^2$

$\sqrt{9} = t^2$

$3 = t$

$a > 0$ means a is positive

Chapter 1 –

1.1 Polynomials pg. 5 – 8 omit 10, 11

Polynomial: An algebraic expression involving numbers and variables.

Monomial: A single expression, one term.

→ can't have exponent as fraction

exponents → must be positive

ax^ny^m

• allowed to be a fraction

coefficient → could be negative

ex: Are the following monomials?

a. 7, 16x, x^3, $2x^5y$, -3xyz

Yes

b. $\frac{2}{x} = 2 \cdot \frac{1}{x} = 2x^{-1}$

No

c. $0.5\sqrt{x} = 0.5\ x^{1/2}$

No

Notes :

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$x^{-2} = \frac{1}{x^2}$$

$$x^{-1} = \frac{1}{x}$$

$$x^{1/2} = \sqrt{x}$$

$$x^{3/2} = \sqrt[2]{x^3} = \sqrt{x^3}$$

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

Degree: the value of the exponent of the variable (add if multiple exponents)

Find the degree.

$-4 \; x^0 \rightarrow 0$

$4x^2 \rightarrow 2$

$-5x^4y^5 \rightarrow 9$

$\frac{5}{2}x^0yz^2 \rightarrow 3$

- In a polynomial, you take the highest number for the degree
  ex: $3x^{\textcircled{2}}+x = 2$

Binomial	→	2 terms	$2x+3$	Deg = 1
Trinomial	→	3 terms	$3x^2 + 2x + 6$	Deg = 2

Two monomials are called like terms if the exponents and variables on one monomial corresponds to the exponents and variables on the 2nd monomial.

Ex: Are they like terms?

1) $-2x$ and $5.2x$ Yes
2) $\frac{5}{2}x^2y$ and $-3yx^2$ Yes
3) $\sqrt{2}yz$ and $-5x^0yz$ Yes
4) $-2x^3y^2z$ and $5x^2y^3z$ No

$2^0 =$ 1

$(-5)^0 =$ 1

Operations on Polynomials

Addition

1) $x + x =$ $2x$
2) $(-2x^2 - 3x + 5) + (-5x^2 - 6x - 2)$ $-7x^2 - 9x - 3$
3) $2x^2y - 3xy^2 + 5xy - 6x^2y + 5xy - 5x + 2y$ $-4x^2y - 3xy^2 + 10xy - 5x + 2y$
4) $2y^2 \quad - 6y \quad + 5$

$\underline{+ \ 10y^2 - 4y - 20}$

$12y^2 - 10y - 15$

Subtraction

1) $(-2x^2 - 3x + 5) - (-5x^2 - 6x - 2)$ $3x^2 + 3x + 7$
2) $2y^2 \quad - 6y \quad + 5$

$-\underline{(10y^2 \quad - 4y \quad - 20)}$

$-8y^2 - 2y + 25$

Multiplication

1) $-2(x^2 - 3x + 5)$ $-2x^2 + 6x - 10$
2) $(2x - 3)(4x - 8)$ $8x^2 - 16x - 12x + 24 =$ $8x^2 - 28x + 24$
3) $2xy(3x^2y - 5xy + 2)$ $6x^3y^2 - 10x^2y^2 + 4xy$
4) $(xy - y)(x^2y + 3xy - 5)$

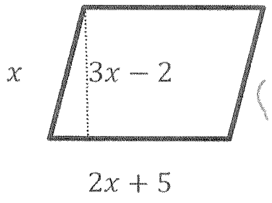
$x^3y^2 + 3x^2y^2 - 5xy - x^2y^2 - 3xy^2 + 5y =$ $x^3y^2 + 2x^2y^2 - 5xy - 3xy^2 + 5y$

Division (a polynomial by monomial)

1) $(6x^2 - 8x + 2) \div (-2)$ $-3x^2 + 4x - 1$
2) $\frac{8x^3 - 10x^2 + 12x}{2x}$ $4x^2 - 5x + 6$
3) $\frac{6x^2y^4 - 18x^2y^3 + 21xy}{3xy}$ $2xy^3 - 6xy^2 + 7$

- To divide a polynomial by a monomial, we utilize the same procedure

F[illegible] the area and perimeter



Area

$(2x+5)(3x-2)$

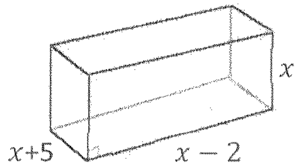
$6x^2 - 4x + 15x - 10$

$6x^2 + 11x - 10$

Perimeter

$(x) + (x) + (2x+5) + (2x+5)$

$6x + 10$



Area

$(x-2)(x)$
$2(x^2 - 2x)$
$2x^2 - 4x$

$(x+5)(x)$
$2(x^2 + 5)$
$2x^2 + 10$

$(x+5)(x-2)$
$x^2 - 2x + 5x - 10$
$2(x^2 + 3x - 10)$
$2x^2 + 6x - 20$

Total Area

$\boxed{2x^2}\underline{-4x}\boxed{+2x^2}+10\boxed{+2x^2}\underline{+6x}-20$

$6x^2 + 2x - 10$

Evaluate

To find the numerical value, plug in the number that corresponds to the variables in the expression.

1) $P(x) = x^2 - 3x + 1$
   P(2) = Replace x by 2
   $P(2) = (2)^2 - 3(2) + 1 = -1$
2) $P(x,y) = -x^2y + xy - 3$
   $P(-1,2) = -(-1)^2(2) + (-1)(2) - 3 = -7$

3) $2A - C + B$ where $A = 2x^2 + 3x - 5$, $B = x^2 + 6x - 2$, $C = -7x^2 + 10$

$2(2x^2 + 3x - 5) - (-7x^2 + 10) + (x^2 + 6x - 2)$

$4x^2 + 6x - 10 + 7x^2 - 10 + x^2 + 6x - 2$

$\boxed{12x^2 + 12x - 22}$

1.2 Remarkable Identities pg. 10-11

Identity # 1 : $(a + b)^2 = a^2 + 2ab + b^2$ ← Perfect Square Trinomial

$$(a + b)^2 = (a + b)(a + b)$$

$$=$$

1) $(x+2)^2 \neq x^2 + 4$

$(x+2)^2 = x^2 + 4x + 4$

2) $(2x+5)^2 = 4x^2 + 20x + 25$

3) $(-3x+2y)^2 = -9x^2 - 12xy + 4y^2$

4) $\left(\frac{1}{2}x + \frac{3}{4}y\right)^2 = (1/2\,x)^2 + 2(1/2\,x)(3/4\,y) + (3/4\,y)^2 = \boxed{\frac{1}{4}x^2 + \frac{3}{4}xy + \frac{9}{16}y^2}$

$a^2 + 2\,a\,b + b^2$

5) $\left(\frac{2}{3}y^2 + \frac{1}{2}x\right)^2 = \left(\frac{2}{3}y^2\right)^2 + 2\left(\frac{2}{3}y^2\right)\left(\frac{1}{2}x\right) + \left(\frac{1}{2}x\right)^2 = \boxed{\frac{4}{9}y^4 + \frac{2}{3}xy^2 + \frac{1}{4}x^2}$

Identity #2: $(a - b)^2 = a^2 - 2ab + b^2$

$(x-5)^2 = x^2 - 10x + 25$

$(y^2-3)^2 = y^4 - 6y^2 + 9$

$\left(\frac{3}{4}x - \frac{1}{2}y^2\right)^2 = \boxed{\frac{9}{16}x^2 - \frac{3}{4}xy^2 + \frac{1}{4}y^4}$

$(a-b)(a-b)$

$a^2 - ab - ab + b^2$

$a^2 - 2ab + b^2$

Identity 3: $(a-b)(a+b) = a^2 - b^2$ ←Difference of two squares

1) $(\overset{a}{3x} + \overset{b}{2})(3x - 2) = a^2 - b^2 = 9x^2 - 4$

2) $(2y^2 + x)(2y^2 - x) = 4y^4 - x^2$

3) $(3x^2yz - 2xy)(3x^2yz + 2xy) = 9x^4y^2z^2 - 4x^2y^2$

work backwards

4) $\underset{a^2 - b^2}{x^2 - 4} = (x)^2 - (2)^2 = (x-2)(x+2)$

5) $\sqrt{4x^2y^4} - \sqrt{9x^2} = (2xy^2)^2 - (3x) = (2xy^2 - 3x)(2xy^2 + 3x)$

6) $x^4 - 1 = (x^2)^2 - (1)^2$ $(x-1)(x+1)(x^2+1)$

SQUARE

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

$11^2 = 121$

$12^2 = 144$

$13^2 = 169$

Find the missing term

7) $x^2 + 4xy + 4y^2$

8) $9x^2 + 12x + 4$

9) $\frac{4}{9}x^4 - x^2y + \frac{9}{16}y^2$

10) $\frac{9}{16}x^2 - \frac{5}{4}x + \frac{25}{36} =$

Identity 4: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^3 = (a+b)^2(a+b)$

Identity 5: $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a-b)^3 = (a-b)^2(a-b)$

Identity 6: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Identity 7: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

7) $a^2 = x^2$ $b^2 = 4y^2$ $2ab$
$a = x$ $b = 2y$ $2(x)(2y)$

8) $b^2 = 4$ $12x = 2(a)(b)$
$b = 2$ $12x = 2(a)(2)$
$12x = 4a$
$3x$

9) $a = \frac{2}{3}x^2$
$b = \frac{3}{4}y$
$2ab = 2(\frac{2}{3}x^2)(\frac{3}{4}y)$

10) $a = \frac{3}{4}x$
$2ab = 2(\frac{3}{4}x)(b)$
$\frac{3}{2}x(b)$
$5/4 \div \frac{3}{2}$
$b = \frac{5}{6}$

1.3 Polynomial Division pg.12-14 (a polynomial by a polynomial)

Quotient
$Q(x)$
$B(x) \overline{)\, A(x)}$ ← dividend
Divisor
$\overline{R(x)}$
Remainder

*need to always write the terms that are missing

1) $\frac{x^2+7x+12}{x+3}$

x+4

1) $x+4 \quad R=0$
$x+3 \overline{)\, x^2+7x+12}$
$-\ x^2+3x \downarrow$
$4x+12$
$-\ 4x+12$
0

2) $\frac{x^2+3x+3}{x+1}$

$x+2+\left(\frac{1}{x+1}\right)$

2) $x+2 \quad R:1$
$x+1 \overline{)\, x^2+3x+3}$
$-\ x^2+1x \downarrow$
$2x+3$
$2x+2$
1

3) Find the length of a rectangle whose area is $(16y^2-1)$ and the width is (4y + 1).

$L = 4y-1$

3) $A = L \cdot W$
$16y^2-1 = L(4y+1)$
$4y-1$
$4y+1 \overline{)\, 16y^2+0y-1}$
$-\ 16y^2+4y \downarrow$
$-4y-1$
$-\ 4y-1$
0

4) $(x^3+27) \div (x^2+1)$

$x+\left(\frac{-x+27}{x^2+1}\right)$

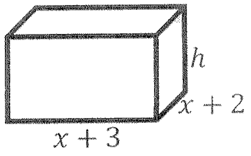
4) x
$x^2+1 \overline{)\, x^3+0x^2+0x+27}$
$-\ x^3 \quad +x \downarrow$
$-x+27$

5) $(x^4-1) \div (x^2+1)$

x^2-1

5) $x^2 \quad -1$
$x^2+1 \overline{)\, x^4+0x^3+0x^2+0x-1}$
$-\ x^4 \quad +x^2$
$-x^2+0x-1$
$-x^2 \quad -1$
0

6) Find the height of a rectangular prism if $V = x^3+3x^2-4x-12$



$V = Ab \cdot h$
$\frac{x^3+3x^2-4x-12}{x^2+5x+6}$

$Ab = (x+3)(x+2)$
$= x^2+2x+3x+6$
$= x^2+5x+6$

$x-2$
$x^2+5x+6 \overline{)\, x^3+3x^2-4x-12}$
$x^3+5x^2+6x \downarrow$
$-2x^2-10x-12$
$-2x^2-10x-12$
0

$h = x-2$

7) $(4x^2 + 20xy + 25y^2) \div (2x + 5y)$

$$\begin{array}{r} 2x+5y \\ 2x+5y\,\overline{)\,4x^2+0x+20xy+25y^2+0y} \\ -\,4x^2 \quad +10xy \\ \hline 10xy+25y^2 \\ -\,10xy+25y^2 \\ \hline 0 \end{array}$$

$2x+5y$

8) **Find the height of a right triangle if the A =** $2x^4 - 2x^3 + 6x^2 - 6x$ **and base =** $2x^3 + 6x$

$$\text{Triangle} = \frac{B \cdot H}{2}$$

$$2x^4 - 2x^3 + 6x^2 - 6x = \frac{(2x^3+6x)H}{2}$$

$$\frac{2x^4 - 2x^3 + 6x^2 - 6}{x^3+3x} = (x^3 + \overset{2}{3}x)h$$

$$\begin{array}{r} 2x-2 \\ x^3+3x\,\overline{)\,2x^4-2x^3+6x^2-6x} \\ 2x^4 \;\downarrow\; +6x^2 \;\downarrow \\ \hline -2x^3 \quad -6x \\ -\quad -2x^3 \quad -6x \\ \hline 0 \end{array}$$

Height = $2x-2$

✓

1.4 Factoring a polynomial pg. 15 -16

Factoring an integer n involves finding integers whose product is n.
These 2 numbers are called factors.
What are the common factors of 12 and 24?

12: (1, 2, 3, 4, 6, 12)
24: (1, 2, 3, 4, 6, 8, 12, 24)

Removing the common factor $ab + ac = a(b + c)$

1) $5x + 15 = 5(x+3)$
2) $2x^2 + 2x = 2x(x+1)$
3) $4ab + 6b^2 \quad 2b(2a+3b)$
4) $4a^2 - 8ab + 12 \quad 4(a^2-2ab+3)$
5) $2ac + 6bc + 12c^2 \quad 2c(a+3b+6c)$
6) $b(5 - 2b) - (5 - 2b) = (5-2b)(b-1)$

7) $(a + 2)(a + 1) + (a - 3)(a + 1)$

$$\boxed{(a+1)\left[(a+2)+(a-3)\right] = (a+1)(2a-1)}$$

8) $2a(2ax - 1) - (-1 + 2ax)$

$$\boxed{(2ax-1)(2a-1)}$$

9) $(x + 1)\overbrace{(2x + 6)}^{\text{takeout common factors}} - (x - 2)\overbrace{(3x + 9)}^{\text{take out common factors}}$

$$(x+1)2(x+3)-(x-2)3(x+3)$$

$$2(x+1)(x+3) - 3(x-2)(x+3)$$

$$(x+3)\underbrace{\left[2(x+1) - 3(x-2)\right]}_{\text{Simplify}}$$

$$(x+3)(2x+2-3x+6)$$

$$(x+3)(-x+8)$$

Fa[illegible]or by grouping (4 terms) pg. 17 $ac + ad +$ [illegible] $+ bd =$

bc

$a(c+d)+b(c+d) = (c+d)(a+b)$

1) $6ab + 3b - 4a - 2$ **←Use first 2 terms**

$3b(2a+1)-2(2a+1)$

$\boxed{(2a+1)(3b-2)}$

2) $6ab + 3b - 4a - 2$ **←Use first and 3rd term**

$2a(3b-2)+1(3b-2)$

$\boxed{(3b-2)(2a+1)}$

if there is a ⊖ in the 1st term, take it out as common Factor

3) $xy + 3x + 2y + 6$

$x(y+3)+2(y+3)$

$\boxed{(y+3)(x+2)}$

4) $x^2 + 4y + 2x + 2xy$

$x(x+2)+2y(2+x)$

$\boxed{(x+2)+(x+2y)}$

5) $4xy - 12x - 10y + 30$

$4x(y-3)-10(y-3)$

$(y-3)(4x-10)$ ← ÷2 there is still a GCF

$\boxed{2(2x-5)(y-3)}$

Di ce of Squares pg. 18 #9-10 $(a+b)(a \quad = a^2 - b^2$

1) $x^2 - 4$

$(x+2)(x-2)$

2) $25y^2 - x^4$

$(5y+x^2)(5y-x^2)$

3) $2x^2 - 18$ $2(x^2-9)$

$\boxed{2(x+3)(x-3)}$

irst think GCF

4) $-\frac{x^2}{4} + 81$

$-\left(\frac{x^2}{4} - 81\right)$

$-\left(\frac{x}{2}+9\right)\left(\frac{x}{2}-9\right)$

Note: A sum of squares cannot be factored!

5) $4a^6 - 64$ GCF

$4(a^6-16)$

$\boxed{4(a^3+4)(a^3-4)}$

- $\sqrt{}$Square 1st term
- $\sqrt{}$Square 2nd term
- put + and −

6) $x^4 - 1$

$(x^2+1)\underbrace{(x^2-1)}_{\text{difference of squares}}$

$\downarrow$

$\boxed{(x^2+1)(x-1)(x+1)}$

$\sqrt{\frac{1}{4}x^2} = \frac{1}{2}x$

7) $4(x+3y)^2 - (-x+y)^2$

$[2(x+3y)+(-x+y)][2(x+3y)-(-x+y)]$

$(2x+6y-x+y)(2x+6y+x-y)$

$\boxed{(x+7y)(3x+5y)}$

$\sqrt{x^4} = x^2$

8) $25(x-3)^2 - 9(x+1)^2$

$[5(x-3)-3(x+1)][5(x-3)+3(x+1)]$

$(\boxed{5x}-15\boxed{-3x}-3)(\boxed{5x}-15\boxed{+3x}+3)$

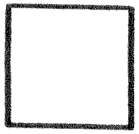
$(2x-18)(8x-12) \longrightarrow$ GCF

$2(x-9) \quad 4(2x-3)$

multiply

$\boxed{8(x-9)(2x-3)}$

Factoring a Perfect Square Trinomial pg 20



They are squares

$a^2 + 2ab + b^2 = (a+b)^2$ Identity 1

$a^2 - 2ab + b^2 = (a-b)^2$ Identity 2

1) $x^2 + 6x + 9$

$a = x$
$b = 3$
$2ab = 2(x)(3)$

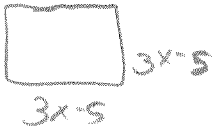
$\boxed{(x+3)^2}$

2) $\sqrt{4x^2} + 20x \sqrt{+ 25}$

$\boxed{(2x+5)^2}$

3) **The area of a square is $A = 9x^4 - 30x^2 + 25$. Find the perimeter.**

$A = (3x^2 - 5)^2$



$P = 4(s)$
$P = 4(3x^2 - 5)$
$\boxed{P = 12x^2 - 20}$

4) **The area of a square is $A = \frac{4}{9}x^2 - y^4$. Find the perimeter.**

$A = \left(\frac{2}{3}x + y^2\right)\left(\frac{2}{3}x - y^2\right)$

$P = 2(s) + 2(s)$
$= 2\left(\frac{2}{3}x + y^2\right) + 2\left(\frac{2}{3}x - y^2\right)$
$= \frac{4}{3}x + \cancel{2y}^2 + \frac{4}{3}x - \cancel{2y}^2$

$\boxed{P = \frac{8}{3}x}$

5) **Is the polynomial $4x^2 + 6x + 9$ a perfect square trinomial?**

$a = 2x$
$b = 3$
$2ab = 12x$

No, it is not because the middle term should be 12x.

Fa ng Trinomials, in the form $1x^2 + bx + c$ pg 3

1) $x^2 - 6x + 8$ a=1

Product='c' = 8 } (-2)
Sum ='b' = -6 } (-4)

$\boxed{(x-2)(x-4)}$

2) $x^2 - 3x - 4$

Product = -4
Sum = -3

$\boxed{\underbrace{(x+1)(x-4)}_{\text{check middle term}}}$

3) $4y^2 + 16y - 48$

GCF → $4(y^2+4y-12)$ product = -12 } 6·-2
Sum = 4

$\boxed{4(y+6)(y-2)}$

4) $x^2 + 6x + 5$

product = 5 } 5·1
Sum = 6

$\boxed{(x+5)(x+1)}$

5) $x^4 - 6x^2 + 8$

product = 8 } -4·-2
Sum = -6

$(x^2-2)\underbrace{(x^2-4)}_{\text{difference of squares}}$

$\boxed{(x^2-2)(x-2)(x+2)}$

{ Product = 'c'
{ Sum = 'B'

Checking

$(x-2)(x-4)$

$x^2 - 4x - 2x + 8$

$x^2 - 6x + 8$

Find two numbers that multiply to 'c' and add to 'B'

Factoring trinomials in the form $ax^2 + bx + c, where\ a \neq 1$

product = (a • c)
Sum = b

* The middle term splits into 2 terms *

Last Step = solve by grouping

1) $\underset{a}{2x^2} \underset{b}{\boxed{-11x}} + \underset{c}{5}$

product = 10 $<$ −10, −1
Sum = −11

$\underline{2x^2 - 10x} \underline{- 1x + 5}$

$2x(x-5) - 1(x-5)$

$(2x-1)(x-5)$

2) $3a^2 + 10ab + 7b^2$

P = 21 $<$ 7, 3
S = 10

$3a^2 + 7ab + 3ab + 7b^2$

$3a(a+b) + 7b(a+b)$

$(3a+7b)(a+b)$

3) $6x^2 + 22x + 12$

GCF

P = 18 $<$ 2, 9
S = 11

$2(3x^2 + 11x + 6)$

$2(3x^2 + 2x + 9x + 6)$

$2[3x(x+3) + 2(x+3)]$

$2(3x+2)(x+3)$

4) $6x^2 - x - 40$

P = −240 $<$ 15, −16
S = −1

$6x^2 + 15x - 16x - 40$

$3x(2x+5) + 8(2x-5)$

$(2x+5)(3x+8)$

Mu[illegible]ep Factoring

1) $4x(2x+3)+4x^2-9$

2) x^3-16x

3) $(x^2-1)^2+(x-1)^2$

4) Find the dimensions of a rectangular prism whose $V=2x^3+6x^2+4x$

5) Find the dimensions of a rectangular prism whose $V=x^3+4x^2+x-6$ and height is $(x+3)$.

Factorin eview

When factoring, you will have to analyze and decide which method to use. You will not be told the method.

Here's are a few directions:

3) Now factor the polynomial that remains:

a. If there are 4 terms, try to factor them by grouping.

b. If there are three terms (trinomial), try to use product/sum method. You may also watch for the perfect square trinomial pattern.

$a^2 + 2ab + b^2 = (a+b)^2$

c. If there are two terms (binomial), then there is no middle term, try to factor by difference of squares.

$\boxed{a^2 - b^2}$ ✓

$\underbrace{a^2+b^2}$

can't be done

1) Write the polynomial in descending order of degree. $2xy + x^2 + y^2 = \boxed{x^2 + 2xy + y^2}$

2) Factor out the GCF, it there is one. Remember, if the leading term is negative, factor out the negative GCF

4) Always check to see if anything factors further!!

5) If we can't factor anything, not even a GCF, then we call the polynomial prime.

3) $\overbrace{(x^2-4)}' + (x-2)^2$

$\underbrace{(x-2)}(x+2)+(x-2)\underbrace{(x-2)}$

$(x-2)[(x+2)+(x-2)]$

$(x-2)(2x)$

$2x(x-2)$

2) x^3-25x

$x(x^2-25)$

$x(x-5)(x+5)$

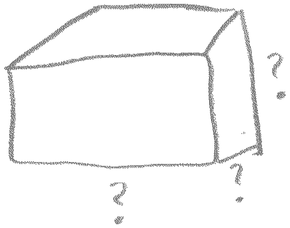
1) $6x(3x+4)+\underbrace{9x^2-16}$ ← difference of Squares

$6x\cancel{(3x+4)}+(3x-4)\cancel{(3x+4)}$

$(3x+4)[6x+(3x-4)]$

$(3x+4)(9x-4)$

4) Find the dimensions of a rectangular prism whose $V = 3x^3+9x^2-12x$



Factor

$3x(x^2+3x-4)$

product $= -4$ < 4
Sum $= 3$ < -1

$3x(x+4)(x-1)$

The dimensions are $3x$, $(x+4)$ and $(x-1)$.

5) Find the dimensions of a rectangular prism whose $V = x^3+4x^2+x-6$ and height is $(x+3)$.

$V = Ab \cdot h$

Division

~~Ab~~ $= x^2+x-2$

$x+3 \overline{)\, x^3+4x^2+x-6}$

$\underline{x^3+3x^2}$ ↓

x^2+x ↓

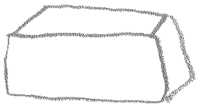
$-\underline{x^2+3x}$

$-2x-6$

$\underline{-2x-6}$

0

does
work



Product/Sum

x^2+x-2 $P=-2$, $S=1$ < $2, -1$

$(x+2)(x-1)$

The dimensions are $(x+2)$ and $(x-1)$

1.5 Rational Expressions pg.25-27

Def: An expression in the form $\frac{P(x)}{Q(x)}$ where P(x) and Q(x) are polynomials and Q(x)≠0 is a rational expression.

For what values is the denominator zero?

1) $\frac{5x^2+60}{x-3}$ 2) $\frac{3x^2+4}{2x}$ 3) $\frac{7}{3x+10}$

Factoring Summary

Before factoring any polynomial, write the polynomial in **descending order** of one of the variables.

1. Factor out the Greatest Common Factor (GCF). Look for this in **every** problem. This includes factoring out a -1 if it precedes the leading term.

   Example: $-3x^2+12x-18=-3(x^2-4x+6)$

2. If there are **FOUR TERMS**, try to factor by grouping (GR).

   Example: $x^3+6x^2-2x-12$

   $\underline{x^3+6x^2}\ \ \underline{-2x-12}=$ *group the first two terms, last two terms*

   $x^2(x+6)-2(x+6)=$ *factor out GCF from each grouping*

   $(x+6)(x^2-2)$ *factor out the common grouping*

3. If there are **TWO TERMS**, look for these patterns:

   a. The difference of squares (DOS) factors into conjugate binomials:

   $a^2-b^2=(a-b)(a+b)$

   Example: $9x^4-64y^2=(3x^2-8y)(3x^2+8y)$

   ***Note:* a variable is a perfect square if the exponent is even**

   b. The sum of squares does not factor:

   a^2+b^2 is prime

   Example: $9x^4+64y^2$ is PRIME

   c. The sum of cubes (SOC) or difference of cubes (DOC) factors by these patterns: each type contains a binomial (small bubble) times a trinomial (large bubble). Only the sign patterns differ between sum of cubes and difference of cubes.

   $a^3+b^3=(a+b)(a^2-ab+b^2)$

   Example : $(8x^3+27)=(2x+3)(4x^2-6x+9)$

   $a^3-b^3=(a-b)(a^2+ab+b^2)$

   Example : $(64x^6-125y^3)=(4x^2-5y)(16x^4+20x^2y+25y^2)$

   ***Note:* a variable is a perfect cube if the exponent is a multiple of three**

x	x^2	x^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	
10	100	
11	121	
12	144	
13	169	
14	196	
15	225	

4. If there are **THREE TERMS**, look for these patterns:

a. Quadratic trinomials of the form $ax^2 + bx + c \;\; where \;\; a = 1$ ($QT \;\; a = 1$))factor into the product of two binomials (double bubble) where the factors of c must add to b.

Example: $x^2 - 4x - 12 = (x-6)(x+2)$

b. Quadratic trinomials of the form $ax^2 + bx + c \;\; where \;\; a \neq 1$ ($QT \;\; a \neq 1$) eventually factor into the product of two binomials (double bubble), but you must first find the factors of *ac* that add to *b*, rewrite the original replacing *b* with these factors of *ac*, then factor by grouping to finally get to the double bubble.

Example:

$9x^2 + 15x + 4 \qquad ac = (9)(4) = 36$

$factors \;\; of \;\; 36 \;\; that \;\; add \;\; to \;\; 15: \;\; 12 \;\; and \;\; 3$

$9x^2 + 12x + 3x + 4 =$

$3x(3x+4) + 1(3x+4) =$

$(3x+4)(3x+1)$

c. Quadratic square trinomials (*QST*) of the form $ax^2 + bx + c$ may factor into the square of a binomial. Look for the pattern where two of the terms are perfect squares, and the remaining term is twice the product of the square root of the squares:

$a^2 \pm 2ab \pm b^2 = (a \pm b)^2$

Example: $16x^2 - 40x + 25 = (4x-5)^2$

5. Factor all expressions completely. Sometimes, you will need to use two or three types of factoring in a single problem.

Example:

$-2x^4 + 32 =$ $\quad factor \;\; out \;\; the \;\; GCF \;\; of \;\; -2$

$-2(x^4 - 16) =$ $\quad factor \;\; the \;\; difference \;\; of \;\; squares$

$-2(x^2 - 4)(x^2 + 4) =$ $\quad factor \;\; the \;\; remaining \;\; difference \;\; of \;\; squares$

$-2(x-2)(x+2)(x^2+4)$ $\quad (remember \;\; that \;\; the \;\; sum \;\; of \;\; squares \;\; is \;\; prime)$

Rational Expressions

- An expression in the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$ is a rational expression.

$\frac{2}{0}$ = ERROR $\frac{1}{0}$ = BIG PROBLEM

For what values is denominator 0?

1) $\frac{5x^2+60}{x-3}$ $x-3=0$ $x \neq 3$ cannot equal

2) $\frac{3x^2+4}{2x}$ $\frac{2x}{2}=\frac{0}{2}$ $x \neq 0$

3) $\frac{7}{3x+10}$ $3x+10=0$ $x \neq -\frac{10}{3}$ $3\left(-\frac{10}{3}\right)+10=0$

$3x = \frac{-10}{3}$

- A rational expression is undefined for the values where its denominator is equal to 0. When simplifying rational expressions, you must specify the restrictions. (make denominator = 0 and solve)

If $a \cdot b = 0$ then $a=0$ or $b=0$ or both $=0$

Examples: Determine the restrictions.

1) $\frac{x^2-3x}{9x^2+24x+16}$

PST

$9x^2+24x+16=0$

$(3x+4)^2=0$

$\underset{a}{(3x+4)}\underset{b}{(3x+4)}=0$

$3x+4=0 \quad 3x+4=0$

$a = 3x+4=0$

↑ also B they're the same

$3x = \frac{-4}{3}$

$x \neq \frac{-4}{3}$

2) $\frac{3x^2+15x-9}{x^2+2x-15}$

$x^2+2x-15=0$

$p=-15$ < 5

$s=2$ < -3

$(x+5)(x-3)=0$

$x+5=0 \quad x-3=0$

a: $x+5=0$

$x \neq -5$

b: $x-3=0$

$x \neq 3$

$x \neq -5, 3$

3) $\frac{5x^2-4x}{16x^2-25}$

$16x^2-25=0$

$(4x-5)(4x+5)=0$

$4x-5=0 \quad 4x+5=0$

$4x-5=0$

$4x = \frac{5}{4}$

$4x+5=0$

$4x = \frac{-5}{4}$

$x \neq \frac{5}{4}, -\frac{5}{4}$

Simplifying Rational Expressions

$\frac{20}{70} = \frac{\cancel{5}\cdot 2\cdot \cancel{2}}{7\cdot \cancel{5}\cdot \cancel{2}} = \frac{2}{7}$ ← Factor top and factor bottom and cancel

* only allowed to cancel out factors of the numerator and denominator

Examples: Simplify and state restrictions.

1) $\frac{12x^2-4x}{5x^2+15x}$

$\frac{4\cancel{x}(3x-1)}{5\cancel{x}(x+3)}$

$\frac{4(3x-1)}{5(x+3)}$

Restriction

$5x^2+15x=0$

$5x(x+3)=0$

$5x=0 \quad (x+3)=0$

$x \neq 0 \quad x \neq -3$

2) $\frac{(x+2)^2-16}{x^2-25}$

$\frac{[(x+2)-4][(x+2)+4]}{(x-5)(x+5)}$ = Nothing Cancels

$\frac{(x-2)(x+6)}{(x-5)(x+5)}$

Mutiply Rational Expressions

$$\frac{1}{2} \times \frac{12}{15} \times \frac{10}{30} \times \frac{15}{30} = \frac{1 \cdot 2 \cdot 2 \cdot 3 \cdot 2 \cdot 5 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 5 \cdot 2 \cdot 3 \cdot 5 \cdot 2 \cdot 3 \cdot 5} = \frac{1}{3 \cdot 5} = \frac{1}{15}$$

Simplify and state restrictions

1. $$\frac{x^2-9}{x^2-2x+1} \times \frac{x^2-3x+2}{x+3}$$

$$\frac{(x-3)\cancel{(x+3)}}{(x-1)\cancel{(x-1)}} \cdot \frac{(x-2)\cancel{(x-1)}}{\cancel{x+3}}$$

$$\frac{(x-3)(x-2)}{(x-1)}$$

$x \neq 1 \qquad x \neq -3$

2. $$\frac{y^2+6y+5}{7y^2-63} \cdot \frac{7y+21}{25+10y+y^2}$$

$$\frac{(y+5)(y+1)}{7(y^2-9)} \cdot \frac{7(y+3)}{(y+5)^2}$$

$$\frac{\cancel{(y+5)}(y+1)}{\cancel{7}(y-3)\cancel{(y+3)}} \cdot \frac{\cancel{7}\cancel{(y+3)}}{\cancel{(y+5)}(y+5)}$$

$$\frac{(y+1)}{(y-3)(y+5)}$$

$y \neq 3 \qquad y \neq -5 \qquad y \neq -3$

3. $$\frac{c^2+16c+64}{2c^2-128} \cdot \frac{c^2-6c-16}{3c^2+30c+48}$$

$$\frac{(c+8)(c+8)}{2(c^2-64)} \cdot \frac{(c-8)(c+2)}{3(c^2+10c+16)}$$

$$\frac{(c+8)\cancel{(c+8)}}{②(c-8)(c+8)} \cdot \frac{(c-8)\cancel{(c+2)}}{③\cancel{(c+8)}\cancel{(c+2)}}$$

$$2 \cdot 3 = \frac{1}{6}$$

$c \neq 8, -8, -2$

4. $$\frac{2x^2+2x}{x+5} \cdot \frac{x^2+10x+25}{2x^3-2x}$$

$$\frac{2x(x+1)}{x+5} \cdot \frac{(x+5)(x+5)}{2x(x^2-1)}$$

$$\frac{\cancel{2x}\cancel{(x+1)}}{\cancel{x+5}} \cdot \frac{\cancel{(x+5)}(x+5)}{\cancel{2x}(x-1)\cancel{(x+1)}}$$

$$\frac{x+5}{x-1}$$

$x \neq -5, 0, 1, -1$

Division of Rational Expressions

✱ Perform the reciprocal

$$\frac{9}{16} \div \frac{24}{18} = \frac{9}{16} \cdot \frac{18}{24} = \frac{3\cdot 3\cdot \cancel{2}\cdot 3\cdot 3}{\cancel{2}\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot \cancel{3}} = \frac{27}{64}$$

ill 3
re restriction

1. $$\frac{3x}{\boxed{x+2}} \div \frac{\boxed{5x}}{\boxed{x+2}}$$

$$\frac{3\cancel{x}}{\cancel{x+2}} \cdot \frac{\cancel{x+2}}{5\cancel{x}}$$

$$\frac{3}{5} \qquad x \neq -2, 0$$

2. $$\frac{2x^2+2y^2}{x^2-4y^2} \div \frac{x^2+y^2}{x+2y}$$

$$\frac{2\cancel{(x^2+y^2)}}{(x-2y)\cancel{(x+2y)}} \cdot \frac{\cancel{x+2y}}{\cancel{x^2+y^2}}$$

$$\frac{2}{x-2y} \qquad x \neq 2y, -2y$$

3) $\frac{x^2+2x-15}{x^2-4x-45} \div \frac{x^2+x-12}{x^2-5x-36}$

$$\frac{\cancel{(x+5)}\cancel{(x-3)}}{\cancel{(x-9)}\cancel{(x+5)}} \cdot \frac{\cancel{(x-9)}\cancel{(x+4)}}{\cancel{(x+4)}\cancel{(x-3)}}$$

$$1$$

$x \neq 9, -5, -4, 3$

Adding and Subtracting Rational Expressions
pg 28 #5-8

FACTOR ALL TERMS

$$\frac{4^{\times 2}}{5_{\times 2}} + \frac{1^{\times 5}}{2_{\times 5}} = \frac{8}{5 \cdot 2} + \frac{5}{5 \cdot 2} = \frac{13}{10}$$

*Add numerator, keep the same denominator

$$\frac{4^{\times 4}}{\underset{3 \cdot 3^{\times 4}}{9}} + \frac{1^{\times 3}}{\underset{4 \cdot 3^{\times 3}}{12}} = \frac{16}{3 \cdot 3 \cdot 4} + \frac{3}{3 \cdot 3 \cdot 4} = \frac{19}{36}$$

* After finding common denominator, do not cancel. Only last steps

$$\frac{1^{\times 3}}{\underset{5 \cdot 4}{20}} + \frac{2^{\times 4}}{\underset{5 \cdot 3}{15}} = \frac{3}{5 \cdot 4 \cdot 3} + \frac{8}{5 \cdot 3 \cdot 4} = \frac{11}{60}$$

Examples: Simplify and state restrictions.

1) $$\left(\frac{x-2}{x+1}\right)\frac{x}{x} + \frac{3}{x}\frac{(x+1)}{(x+1)}$$

$$\frac{x(x-2)}{(x+1)(x)} + \frac{3(x+1)}{(x+1)(x)}$$

$$\frac{x^2-2x \qquad 3x+3}{x(x+1)}$$

$$\frac{x^2+x+3}{x(x+1)}$$

$x \neq -1, 0$

Note

Whatever is in the 1st denominator, you put in the new denominator as well as, extra Factors from the 2nd denominator

Add numerator and leave the same denominator

✱ Ask yourself if numerator can be Factored more

2) $$\frac{3}{x^2-16} + \frac{x}{x+4}$$

$$\frac{3}{(x-4)(x+4)} + \frac{x}{x+4}(x-4)$$

$$\frac{3}{(x-4)(x+4)} + \frac{x(x-4)}{(x-4)(x+4)}$$

$$\frac{3+x(x-4)}{(x-4)(x+4)}$$

$P = 3$
$S = -4$
$(x-3)(x-1)$

$$\frac{3+x^2-4x}{(x-4)(x+4)} = \frac{x^2-4x+3}{(x-4)(x+4)} = \frac{(x-3)(x-1)}{(x-4)(x+4)}$$

$x \neq -4, 4$

3) $\frac{2^{(x+3)}}{x-3} - \frac{5^{(x-3)}}{x+3}$

$$\frac{2(x+3)-5(x-3)}{(x-3)(x+3)}$$

$x \neq 3, -3$

$$\frac{2x+6-5x+15}{(x-3)(x+3)}$$

$$\frac{-3x+21}{(x+3)(x-3)}$$

$$\frac{-3(x-7)}{(x-3)(x+3)}$$

4) $\frac{a-2^{(2)}}{\underset{4a\,(2)}{4a}} - \frac{4-6a}{\underset{4a\cdot 2}{8a}}$

$$\frac{2(a-2)-(4-6a)}{4a\cdot 2}$$

$x \neq 0$

$$\frac{2a-4-4+6a}{8a}$$

$$\frac{8a-8}{8a}$$

$$\frac{\cancel{8}(a-1)}{\cancel{8}a}$$

$$\frac{a-1}{a}$$

5) $\frac{x+2}{x^2+4x+4}-\frac{x-4}{x^2-2x-8}$

$p=-8$
$s=-2$ $<$ -4, $+2$

$$\frac{x+2^{(x-4)}}{(x+2)^2}-\frac{x-4^{(x+2)}}{(x-4)(x+2)}$$

$$\frac{(x+2)(x-4)-(x-4)(x+2)}{(x+2)^2(x-4)}$$

$$\frac{(\cancel{x^2}-4x+2x-8)\cancel{-}\cancel{(x^2}+2x-4x-8)}{(x+2)^2(x-4)}$$

$$(-\cancel{2x}-8)-(-\cancel{2x}-8)$$

$$0$$

$$\frac{(\cancel{x+2})}{(x+2)(\cancel{x+2})}-\frac{(\cancel{x-4})}{(x+2)(\cancel{x-4})}$$

$$\frac{1}{(x+2)}-\frac{1}{(x+2)}$$

$$0$$

* you can't cancel diagonally when subtract/add. You can cancel only if division/multiply

6) $\frac{5x+1}{x^2-2x-3}-\frac{5x-3}{x^2-x-6}$

$$\frac{5x+1}{(x-3)(x+1)}-\frac{5x-3}{(x-3)(x+2)}$$

$$\frac{(5x+1)(x+2)-(5x-3)(x+1)}{(x-3)(x+1)(x+2)}$$

$$\frac{(5x^2+10x+x+2)-(5x^2+5x-3x-3)}{(x-3)(x+1)(x+2)}$$

$$\frac{(\cancel{5x^2}+11x+2)-(\cancel{5x^2}+2x-3)}{(x-3)(x+1)(x+2)}$$

$$\frac{9x+5}{(x+3)(x+1)(x+2)}$$

Steps

1. Factor
2. Simplify by cancelling
3. Common denominato
4. Combine terms in numerator
5. Factor
6. Cancel (if possible)

1. Solving a second degree equation by factoring pg. 29-32 #1-8

FIRST DEGREE EQUATIONS

Let's start by reviewing the 3 cases for first degree equations with form: $bx + c = 0$.

Case 1: 1 solution → Ex: $3x - 9 = 0$

$3x = \frac{9}{3}$

$\boxed{x = 3}$

Case 2: No solution → Ex: $4x - 6 = 9x - 5x + 20$

$4x - 6 = 4x + 20$

$4x - 4x = 20 + 6$

$\boxed{\emptyset}$

Case 3: Many Solutions →

Ex: $5x - 1 = 4x + 1x + 9 - 10$

$5x - 1 = 5x - 1$

$5x - 5x = -1 + 1$

$\boxed{S = \mathbb{R}}$

A second degree equation/quadratic equation is any equation written in the form:

$$ax^2 + bx + c = 0 \text{ where } a \neq 0.$$

There are 4 ways of solving quadratic equations.

✳ Put all numbers on the left

Case 1: Solving quadratic equations of the form $ax^2 + bx + c = 0$ (Can Be Factored) pg 29#1-3

Ex 1: Solve

$x^2 - 7x = -10$

Put your answers in solution set notation, meaning

S = {....}

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x - 5 = 0$ $\quad$ $x - 2 = 0$

$x = 5$ $\quad$ $x = 2$

Verify answer

Ex 2: Solve $x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

$\downarrow$

$x - 3 = 0$

$x = 3$

Ex 3: Solve $x^2 - 7x = -7x - 25$

$x^2 \cancel{-7x+7x} + 25 = 0$

$(x^2+25) = 0$

CANNOT BE FACTORED

$S = \emptyset$

Ex: 4: Solve $16x^2 = 25$ by factoring

$16x^2 - 25 = 0$

$(4x-5)(4x+5) = 0$

$4x-5=0$ $\quad$ $4x+5=0$

$4x = \frac{5}{4}$ $\quad$ $4x = \frac{-5}{4}$

$\boxed{x = \frac{5}{4}}$ $\quad$ $\boxed{x = \frac{-5}{4}}$

Ex 5: Solve $x^2 - 25x = 0$

$x(x-25) = 0$

$x=0$ $\quad$ $x=25$

Ex 6: Solve $2x^2 - 13x - 15 = 0$

$P = -30$, $S = -13$: -15, $+2$

$2x^2 - 13x - 15 = 0$

$2x^2 + 2x - 15x - 15 = 0$

$2x(x+1) - 15(x+1) = 0$

$(x+1)(2x-15) = 0$

$x = -1$ $\quad$ $2x - 15 = 0$

$\boxed{x = \frac{15}{2}}$

Case 2: Solving quadratic equations of the form: $ax^2 = k$

$\{x^2 = k\}$

If $k < 0$ then $S = \emptyset$
If $k = 0$ then $S = 0$
If $k > 0$ then $S =$ 2 answers → +, −

* When square rooting a number you get two results.

Ex 1: Solve $x^2 + \cancel{3x} = \cancel{+3x} + 100$

$\sqrt{x^2} = \sqrt{100}$

$x = 10, -10$

Ex 2: Solve $\frac{4x^2}{4} = \frac{49}{4}$

$\sqrt{x^2} = \sqrt{\frac{49}{4}}$

$x = 7/2, -\frac{7}{2}$

Ex 3: Solve $x^2 - 13 = 0$

$\sqrt{x^2} = \sqrt{13}$

$x = +\sqrt{13}, -\sqrt{13}$

Ex 4: Solve $x^2 + 9 = 0$

$\sqrt{x^2}\ \sqrt{-9}$

$\boxed{S = \{\emptyset\}}$

Ex 5: Solve $\frac{4x^2}{4} = \frac{64}{4}$

$\sqrt{x^2} = \sqrt{16}$

$\boxed{S = \{4, -4\}}$

Case 3: Solving quadratic equations of the form $a(x - h)^2 + k = 0$

Ex 1: Solve $\frac{3(x - 4)^2}{3} = \frac{0}{3}$

$\sqrt{(x-4)^2} = \sqrt{0}$ $\quad x = 4$

$x - 4 = 0$

Ex 2: Solve $\frac{-0.4(x - 4)^2}{-0.4} = \frac{-40}{-0.4}$

$\sqrt{(x-4)^2} = \sqrt{100}$

$x - 4 = 10 \qquad x - 4 = -10$

$x = 14 \qquad x = -6$

$\boxed{S = \{-6, 14\}}$

Case 4: Solving quadratic equations of the form $ax^2 + bx + c = 0$ (Can Not Be Factored)

To solve for the solutions to x,

➔ Use Quadratic Formula

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

← Delta

Where the discriminant $\Delta = b^2 - 4ac$

✳ usually decimal answers

The formula tells us the number of solutions that an equation has.

Sign of Δ	# of solutions
$\Delta > 0$	2
$\Delta = 0$	1
$\Delta < 0$	0

The quadratic formula can be written as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a=1 \quad b=-1 \quad c=-20$

Ex 1: Solve $x^2 - x - 20 = 0$

By Factoring

$(x-5)(x+4)=0$

$x-5=0 \qquad x+4=0$

$x=5 \qquad x=-4$

$$\boxed{S=\{-4,5\}}$$

By Quadratic Formula

Step 1

$\Delta = b^2 - 4ac$

$\Delta = (-1)^2 - 4(1)(-20)$

$\Delta = 1 - 4(-20)$

$\Delta = 1 + 80$

$\Delta = 81 \rightarrow$ 2 solutions

Step 2

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{81}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{81}}{2}$$

$\frac{1+9}{2} \qquad \frac{1-9}{2}$

$5 \qquad -4$

$$\boxed{S=\{-4,5\}}$$

Ex 2: Solve $-3(x+1)^2+9=0$

By isolating x

$$\frac{-3(x+1)^2}{-3}=\frac{-9}{-3}$$

$$\sqrt{(x+1)^2}=\sqrt{3}$$

$$x+1=\pm\sqrt{3}$$

$-1+\sqrt{3}$ $\quad$ $-1-\sqrt{3}$

$=0.73$ $\quad$ $x=-2.73$

By Quadratic Formula

$$-3(x^2+2x+1)+9=0$$

$$-3x^2-6x-3+9=0$$

$$-3x^2-6x+6=0$$

$$\Delta=b^2-4ac$$

$$\Delta=(-6)^2-4(-3)(6)$$

$$=36+72$$

$\Delta=108 \to 2$ solutions

$$x=\frac{-b\pm\sqrt{\Delta}}{2a}$$

$$x=\frac{-(-6)\pm\sqrt{108}}{2(-3)}$$

$\frac{6+10.4}{-6}$ $\quad$ $\frac{6-10.4}{-6}$

$x=-2.73$ $\quad$ $x=0.73$

$$\boxed{S=\{-2.73, 0.73\}}$$

Ex 3: Solve $\overset{a}{3}x^2-\overset{b}{5}x-\overset{c}{8}=0$

$$\Delta=b^2-4ac$$

$$=(-5)^2-4(3)(-8)$$

$$=25+96$$

$$\Delta=121$$

$$x=\frac{-b\pm\sqrt{\Delta}}{2a}$$

$$x=\frac{-(-5)\pm\sqrt{121}}{2(3)}$$

$$x=\frac{5\pm 11}{6}$$

$x=\frac{5+11}{6}$ $\quad$ $x=\frac{5-11}{6}$

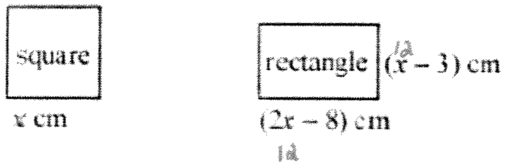
$x=2.6$ $\quad$ $x=-1$

$$\boxed{S=\{-1, 2.6\}}$$

Quadratic Word Problems

pg 34 #14-22

1. Find the perimeter of a rectangle if the two shapes are equal in area.



✱ Put things were x^2 remains ⊕

✱ If there's 2 answers, check your result

$A_{square} = A_{rectangle}$

$s^2 = L \cdot W$

$(x)^2 = (2x-8)(x-3)$

$x^2 = 2x^2 - 6x - 8x + 24$

$\boxed{x^2} = 2x^2 - 14x + 24$

$0 = 2x^2 - x^2 - 14x + 24$

$0 = x^2 - 14x + 24$

$0 = (x-12)(x-2)$

$x = 12 \quad \cancel{x = 2}$

Can't work when you plug it in

$P_{rectangle}$

$L = 2(12) - 8 = 16$

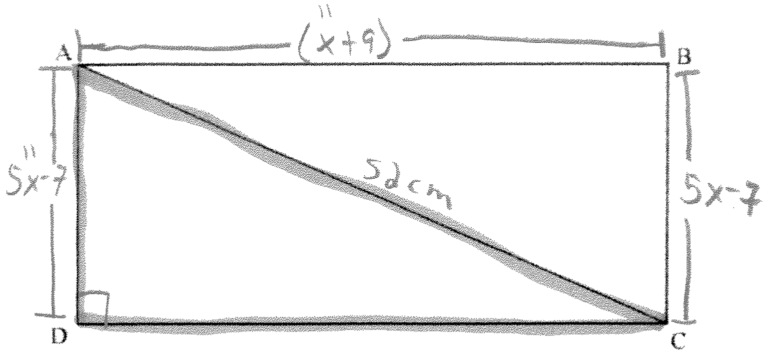
$W = 12 - 3 = 9$

$2(9) + 2(16)$

$P = 50cm$

2.

The length of the sides of rectangle ABCD below can be represented by binomials. The area of this rectangle is then represented by the trinomial $5x^2 + 38x - 63$. In addition, the length of diagonal AC of this rectangle is 52 cm. What is the numerical perimeter of rectangle ABCD in centimetres?



$P = -315$, $S = 38$ $<^{45}_{-7}$

① Arect $= 5x^2 + 38x - 63$

$A = (L)(W)$

$= 5x^2 - 7x + 45x - 63$

$= 5x(x+9) - 7(x+9)$

$A = (x+9)(5x-7)$

$L = 11 + 9 = 20$

$W = 5(11) - 7 = 48$

② Find Value of x

$a^2 + b^2 = c^2$

$(5x-7)^2 + (x+9)^2 = (52)^2$

$(25x^2 - 70x + 49) + (x^2 + 18x + 81) = 2704$

$26x^2 - 52x + 130 - 2704 = 0$

$26x^2 - 52x - 2574 = 0$

$26(x^2 - 2x - 99) = 0$

$26(x - 11)(x + 9) = 0$

$\boxed{x = 11}$ $\cancel{x = -9}$

$P = -99$, $S = -2$ $<^{-11}_{+9}$

③ Perimeter

$P = 2(L) + 2(W)$

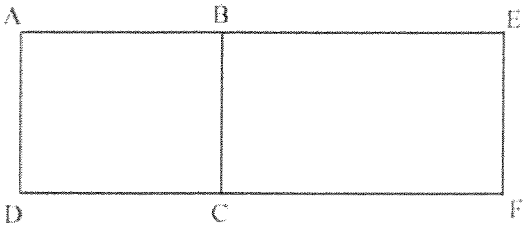
$= 2(20) + 2(48)$

$P = 136\text{cm}$

3. Sam is 10 years older than Frank. Ten years from now, the product of their ages will be 1200. What's Sam's present age?

4.

Rectangles ABCD and BEFC below have side BC in common.
The lengths of their bases and their heights can be represented by binomials. This therefore means that the area of each rectangle is represented by a trinomial as follows:



- The area of rectangle ABCD is represented by the trinomial $6x^2 + 17x + 5$.
- The area of rectangle BEFC is represented by the trinomial $8x^2 + 14x - 15$.
  What binomial represents the length of side BC?

Solving Rational Equations
Not in Workbook – (Handout Given)

$$\frac{2x}{3}+\frac{x}{5}=13$$

The usual technique is to put all terms on the lowest common denominator (LCD) and then multiply the entire equation by the LCD, meaning both sides of the equation. The resulting equation will be cleared of fractions, and we can then proceed to solve for the variable.

Clearing the Fractions in an equation

Example 1:

NO RESTRICTIONS

Solve $$15\left(\frac{2x}{3}+\frac{x}{5}\right)=(13)(15)$$

$$\overset{5}{\cancel{15}}\left(\frac{2x}{\div\cancel{3}}\right)+\overset{3}{\cancel{15}}\left(\frac{x}{\div\cancel{5}}\right)=(13)(15)$$

NOTE The LCM for 3 and 5 is 15.

The LCD for $\frac{2x}{3}$ and $\frac{x}{5}$ is 15.

$$10x+3x=195$$

$$\frac{13x}{13}=\frac{195}{13}$$

$$x=15$$

$$\frac{2(15)}{3}+\frac{(15)}{5}=13$$

Verify Answer: To check, substitute result in the original equation.

Be Careful! A common mistake is to confuse an *equation* such as

$$\frac{2x}{3} + \frac{x}{5} = 13$$

and an *expression* such as

$$\frac{2x}{3} + \frac{x}{5}$$

Let's compare.

Equation: $\frac{2x}{3} + \frac{x}{5} = 13$

Here we want to *solve the equation for x,* as in Example 1. We multiply both sides by the LCD to clear fractions and proceed as before.

Expression: $\frac{2x}{3} + \frac{x}{5}$

Here we want to find *a third fraction* that is equivalent to the given expression. We write each fraction as an equivalent fraction with the LCD as a common denominator.

$$\begin{aligned} \frac{2x}{3} + \frac{x}{5} &= \frac{2x \cdot 5}{3 \cdot 5} + \frac{x \cdot 3}{5 \cdot 3} \\ &= \frac{10x}{15} + \frac{3x}{15} = \frac{10x + 3x}{15} \\ &= \frac{13x}{15} \end{aligned}$$

Steps to Solve a Rational Equation

1. Factor the denominators of all rational expressions. Identify any restrictions on the variable.
2. Identify the LCD of all expressions in the equation.
3. Multiply both sides of the equation by the LCD.
4. Solve the resulting equation.
5. Check each potential solution.

* First Reduce, then multiply

Solving an equation Involving Rational Expressions

Example 2

$\boxed{\text{Restriction} \neq 0}$

NOTE We assume that x cannot have the value 0. Do you see why?

Solve.

$$4x^2\left(\frac{7}{4x} - \frac{3}{x^2}\right) = \left(\frac{1}{2x^2}\right)(4x^2)$$

The LCM of $4x$, x^2, and $2x^2$ is $4x^2$. So, the LCD for the equation is $4x^2$.

$2 \cdot 2 \cdot x \quad x \cdot x \quad 2 \cdot x \cdot x = 2 \cdot 2 \cdot x \cdot x$

$$\overset{x}{\cancel{4x^2}}\left(\frac{7}{\cancel{4x}}\right) - \overset{4}{\cancel{4x^2}}\left(\frac{3}{\cancel{x^2}}\right) = \frac{\cancel{4x^2}\ 2}{\cancel{2x^2}}$$

$$7x - 12 = 2$$

$$7x = 14$$

$$x = 2$$

$$\frac{7}{4(2)} - \frac{3}{(2)^2} = \frac{1}{2(2)^2}$$

$$\frac{7}{8} - \frac{3}{4} = \frac{1}{8}$$

Be sure to return to the original equation and substitute the result for x.

Example 3: Solve

(Always mention restrictions.)

$$(10\cancel{+}m)(10-m)\left[\frac{24}{(10\cancel{+}m)}+1\right]=\left[\frac{24}{(10\cancel{-}m)}\right](10+m)(1\cancel{0}-m)$$

$LCD = (10+m)(10-m)$

Restrictions

$X \neq -10, 10$

$24(10-m) + (10+m)(10-m) = 24(10+m)$

$240 - 24m + (100 - m^2) = 240 + 24m$

$\boxed{-m^2 - 24m + 340} = 24m + 240$

$m^2 + 48m - 100$

$(m+50)(m-2)$

$m = -50 \quad m = 2$

- $\frac{24}{10+(2)} + 1 = \frac{24}{10-(2)}$

$\frac{24}{12} + 1 \overset{?}{=} \frac{24}{8}$

136

3

- $\frac{24}{10-50} + 1 = \frac{24}{10+50}$

Example 4: Solve

$$\frac{11}{x^2-4}+\frac{x+3}{2-x}=\frac{2x-3}{x+2}$$

$LCD = (x-2)(x+2)(2-x)$

$$\frac{11}{(x-2)(x+2)} + \frac{x+3}{2-x} = \frac{2x-3}{x+2}$$

Example 5: Solve and check

$$\frac{x}{x-2} - 7 = \frac{2}{x-2}$$

Bell Work Question

Solve

$$\frac{(x+2)}{x+3} - \frac{x^2}{x^2-9} = 1 - \frac{x-1}{3-x}$$

Bell work Question

Solve.

$$\frac{x}{x-4} = \frac{15}{x-3} - \frac{2x}{x^2-7x+12}$$

Bell Work Question

$$\frac{x+3}{x^2-x} - \frac{8}{x^2-1} = 0$$

1:7 Sec[illegible]d degree Inequalities pg. 38-42 #1-8

Greater than

We will study the symbols: $\boxed{\geq, >}$, $\boxed{\leq, <}$. less than
Each symbol is associated with an interval.

In this section, we create a number line in order to find the interval for the inequality.

≥ & ≤	Closed d●t
< & >	Open d○t

Sign of a 1st degree binomial

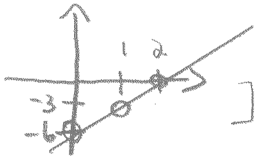
Find the interval for which $3x + 5 > \boxed{11}$.

Bring everything to the left and graph the line. Write the interval for which the LHS is greater than 0. (Sec 3 Method)

$3x + 5 - 11 > 0$

$3x - 6 > 0$

x	y
0	-6
1	-3
2	0



$]2, \infty[$

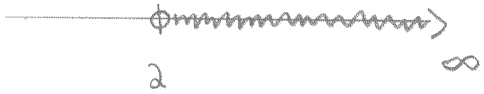
Find the interval for which $3x + 5 > 11$.

Isolate x and make a number line. Shade according to your inequality and write interval. (Sec 4 SN Method)

$3x > 11 - 5$

$\frac{3x}{3} > \frac{6}{3}$

$x > 2$



$]2, \infty[$

Solving a second degree inequality algebraically

The SIGN of the quadratic equation $f(x) = ax^2 + bx + c$ is:

$$\textbf{positive}\ when\ f(x) \geq 0$$
$$\textbf{strictly positive}\ when\ f(x) > 0$$
$$\textbf{negative}\ when\ f(x) \leq 0$$
$$\textbf{strictly negative}\ when\ f(x) < 0$$

Procedure to solve quadratic inequalities

1. Move all terms to one side.
2. Simplify and factor the quadratic expression.
3. Find the roots by sing the zero product principle.
4. Use the roots to divide the number line into regions.
5. Test each region using the inequality.

Example 1 Solve the inequality, $x^2 > x + 2$.

Solution

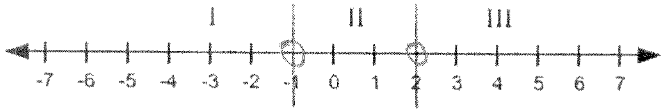
$$x^2 > x + 2$$
$$x^2 - x - 2 > 0$$
$$(x - 2)(x + 1) > 0$$

The corresponding equation is $(x - 2)(x + 1) = 0$ so…

Roots/zeros

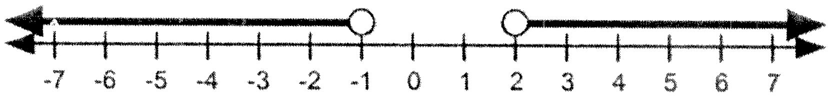
$x - 2 = 0$ or $x + 1 = 0$

$x = 2$ $\quad$ $x = -1$



Now we test one point in each region.

gion	Test Point	Inequality	Status
I	$x = -2$	$(x - 2)(x + 1) = (-2 - 2)(-2 + 1) = 4 > 0$	True
II	$x = 0$	$(x - 2)(x + 1) = (0 - 2)(0 + 1) = -2 > 0$	False
III	$x = 3$	$(x - 2)(x + 1) = (3 - 2)(3 + 1) = 4 > 0$	True



So the solution to this inequality is $x < -1$ or $x > 2$.

$]-\infty, -1[\cup]2, \infty[$

2. For $P(x) = x^2 - 5x + 4$,
find the values of x where

a. $P(x) \geq 0$ Answer: $x \in]-\infty, 1] \cup [4, \infty[$

b. $P(x) > 0$ Answer: $x \in]-\infty, 1[\cup]4, \infty[$

c. $P(x) \leq 0$ Answer: $x \in [1, 4]$

d. $P(x) < 0$ Answer: $x \in]1, 4[$

e. $P(x) = 0$ Answer: $x \in \{1, 4\}$

① Find roots by factoring

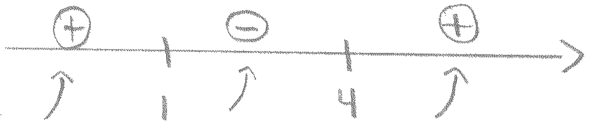
$p = 4$
$s = -5$

$X^2 - 5x + 4 = 0$

$(x-4)(x-1)$

$X = 4 \quad x = 1$

② Make number line



) Test
ch
ion

$P(-2) = 18 \qquad P(3) = -2 \qquad P(5) = 4$

$\uparrow \qquad \uparrow \qquad \uparrow$

$(-2)^2 - 5(-2) + 4 \qquad (3)^2 - 5(3) + 4 \qquad (5)^2 - 5(5) + 4$

3. Find the solution set for which
$f(x) = -x^2 + 6x + 3 \geq 0$.

$-x^2 + 6x + 3 = 0$

$-(x^2 - 6x - 3) = 0$

$\Delta = b^2 - 4ac$

$(-6)^2 - 4(1)(-3)$

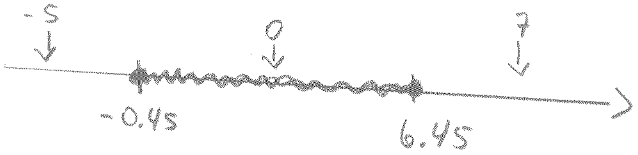
$36 + 12$

48

①
$$X = \frac{-6 \pm \sqrt{48}}{2(-1)}$$

$$X = \frac{-6 + \sqrt{48}}{-2} \qquad X = \frac{-6 - \sqrt{48}}{-2}$$

$X = -0.45 \qquad X = 6.45$



② $f(-5) = -52$

$f(0) = 3$

$f(7) = -4$

③ $[-0.45, 6.45]$

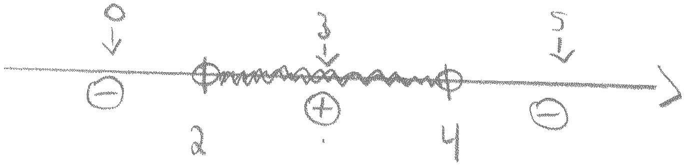
4. baseball player hits a ball which follows the trajectory $h(t) = -t^2 + 6t + 5$. When is the ball higher than 13m?

$-t^2 + 6t + 5 = \boxed{13}$

$-t^2 + 6t - 8 = 0$

$-(t-4)(t-2)$

$t = 4 \quad t = 2$



$h(0) = -8$

$h(3) = 14$

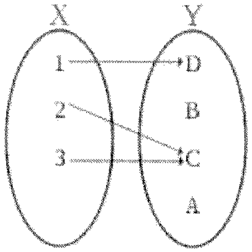
$h(5) = -3$

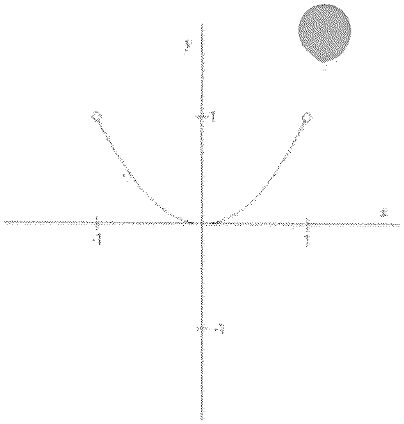
$\boxed{]2,4[}$

PROPERTIES OF A FUNCTION PG. 49-50

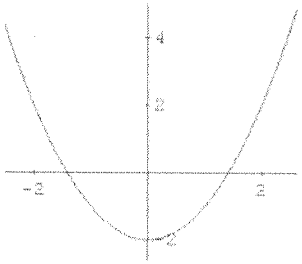
Domain (*dom f*): the values of x that the function covers (from left to right if on a graph)

Range (*ran f*): the values of y that the function covers (from bottom to top if on a graph)





$R = \{(2,3),(4,5),(6,3)\}$



INTERCEPTS PG. 52

Zeros (x-intercepts): where the function crosses the x axis (let f(x)=0)

Y-intercept (initial value): where the function crosses the y-axis (find f(0))

Find the x and y intercepts

1. Linear:

$$f(x) = \frac{2}{5}x - 6$$

2. 2nd degree:

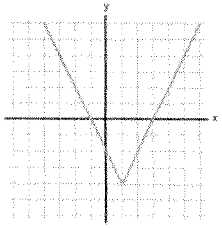
$$f(x) = x^2 - 1x - 30$$

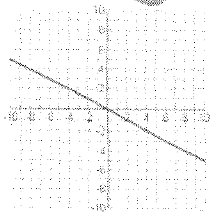
Sig pg. 53 #13-15

A function is POSITIVE for all values of x from left to right for which the graph is ABOVE the x-axis.

A function is NEGATIVE for all values of x from left to right for which the graph is BELOW the x-axis

STRICTLY POSITIVE OR NEGATIVE means that we do NOT include the zeros.





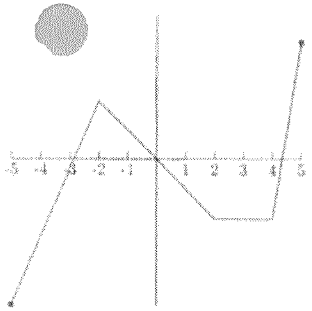
Variation of a function pg 55 #16-19

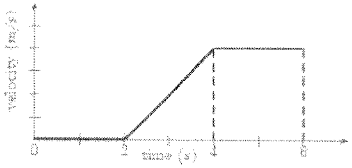
A function is INCREASING for all values of x from left to right for which the line goes up (the y values are increasing) or remains constant.

A function is DECREASING for all values of x from left to right for which the line goes down (the y values are decreasing) or remains constant.

A constant function is a horizontal line. Constant functions are considered both increasing and decreasing.

When the term <strictly> is used, do not include the values where the function is constant.





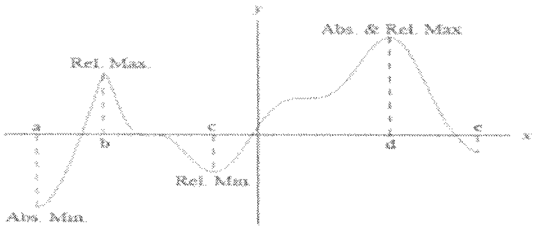
Extema of a function pg 58 #20-22, p70#1-4 , pg 45 #1-2

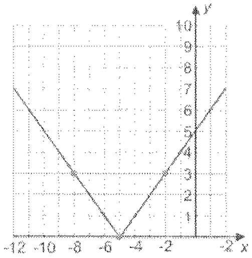
Absolute Maximum: is the highest y value, if it exists

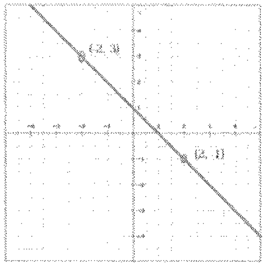
Relative Maximum: is the second highest y value, if it exists

Absolute Minimum: is the lowest y value, if it exists

Relative Minimum is the second lowest y value, if it exists


y
Abs. & Rel. Max
Rel. Max.
a
b
c
d
e
x
Rel. Min.
Abs. Min.


y
10
9
8
7
6
5
4
3
2
1
-12
-10
-8
-6
-4
-2
-2
x


(-2, 3)
(2, -1)

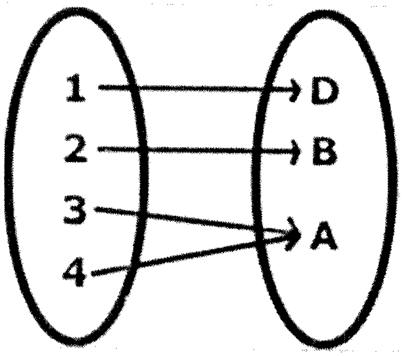
Chap 2: Functions

2.1 A function vs a relation pg. 47-48

DEFINITION OF RELATION

A **relation** is a relationship/link between sets of information (x and y value). For example, think of all the people in one of your classes, and think of their heights. The pairing of names and heights is a relation.

A relation is a mapping, or pairing of input values with output values.



LABEL

OUTPUT
INPUT
DOMAIN
RANGE
SOURCE
TARGET

Relations can be represented by a list of ordered pairs, by a correspondence between the domain and range, by a graph or by an equation.

- A relation may be defined as a set of ordered pairs.

$$\{(1, 2), (-3, 4), (1, -4), (3, 4)\}$$

- A relation may be defined by a correspondence (Figure 4-1). The corresponding ordered pairs are $\{(1, 2), (1, -4), (-3, 4), (3, 4)\}$.

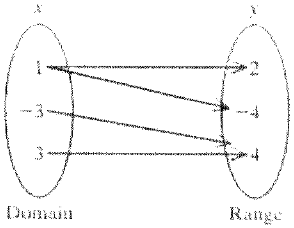


Figure 4-1

- A relation may be defined by a graph (Figure 4-2). The corresponding ordered pairs are $\{(1, 2), (-3, 4), (1, -4), (3, 4)\}$.

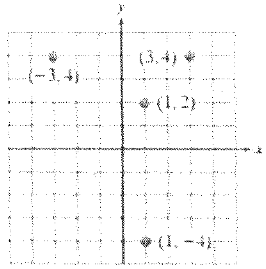


Figure 4-2

- A relation may be expressed by an equation such as $x = y^2$. The solutions to this equation define an infinite set of ordered pairs of the form $\{(x, y) | x = y^2\}$. The solutions can also be represented by a graph in a rectangular coordinate system (Figure 4-3).

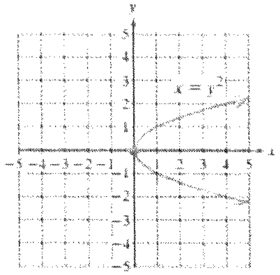


Figure 4-3

Representing Relations

A relation can be represented in the following ways.

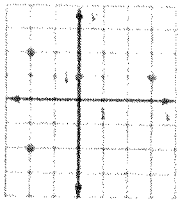
Ordered Pairs

(−2, 2)
(−2, −2)
(0, 1)
(3, 1)

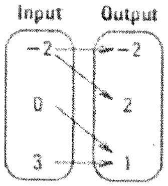
Table

x	y
−2	2
−2	−2
0	1
3	1

Graph



Mapping Diagram

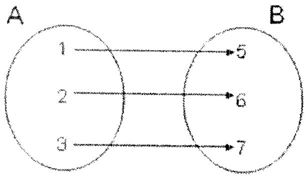


TYPES OF RELATIONS ON A MAPPING DIAGRAM

1. One-One Relation:

A relation A→B is said to be One-One relation if no two elements of A have the same image in B.

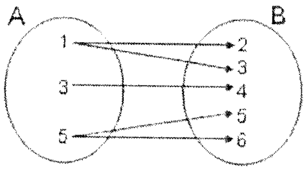
Example:



2. One to Many Relation:

A relation A→B is said to be One to Many relation if an element of A is related to two or more Elements of B.

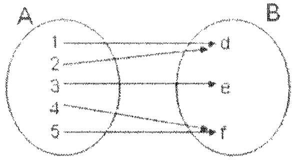
Example:



3. Many to One Relation:

A relation A→B is said to be Many to One relation if two or more elements of A are related to an element of B.

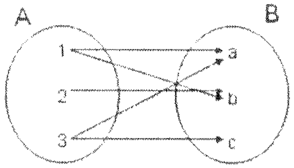
Example:



4. Many to Many Relation:

A relation A→B is said to be Many to Many relation if two or more elements of A are related to two or more element of B

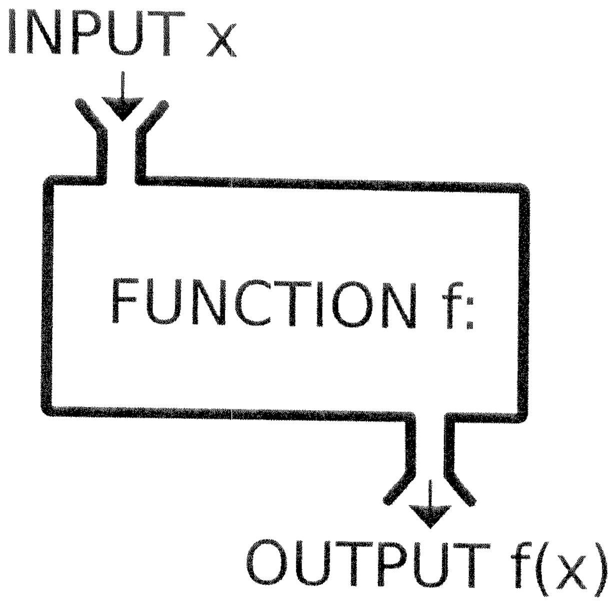
Example:

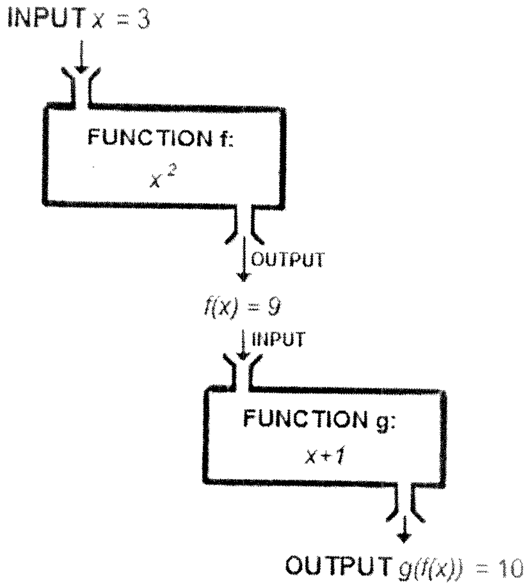


DEFINITION OF FUNCTION

A **function** is a relation for which each x value (source) is associated with <u>exactly one</u> y value (target). For a relation to be a function, there must be *only and exactly* one *y* that corresponds to a given *x*. If any input of a relation has more than one output, the relation is NOT a function.

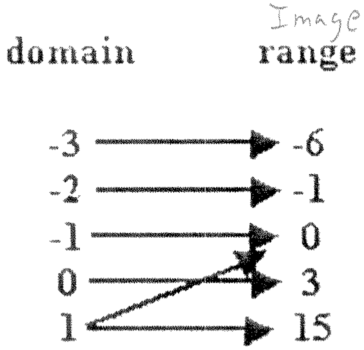
- An x value is written twice then it's not a function.


INPUT x
FUNCTION f:
OUTPUT f(x)


INPUT $x = 3$
FUNCTION f:
x^2
OUTPUT
$f(x) = 9$
INPUT
FUNCTION g:
$x+1$
OUTPUT $g(f(x)) = 10$

IN A MAPPING DIAGRAM

Is the following a relation only or a function as well?



(1, 0)

(1, 15)

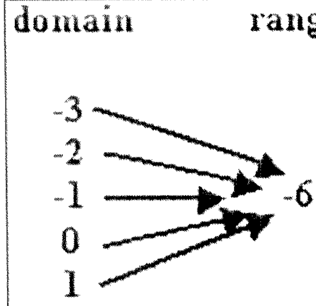
x's repeat

The image of 1 is 0.

The antecedent of -1 is -2.

What is x value when y is -1

This is a Relation



(-3, -6)

(-2, -6)

(-1, -6)

This is a Function

x's do not repeat

IN A SET

Is the following a relation only or a function as well?

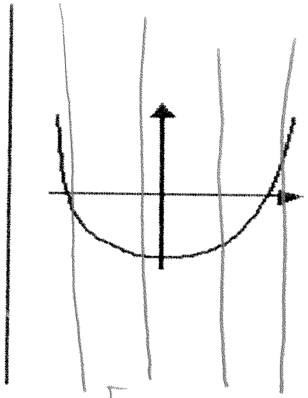
$R = \{(2,0), (3,8), (4,0), (5,4)\}$ ← Function

$R = \{(0,3), (0,4), (1,5)\}$ ← Relation

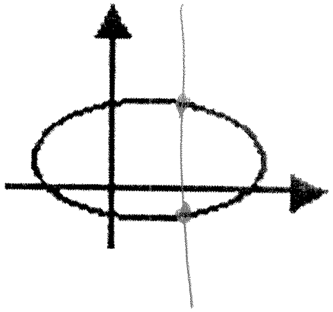
THE VERTICAL LINE TEST

Imagine a vertical line passing from left to right of the graph. If it touches 2 points simultaneously then it fails and the relation is not a function.

Is each a function?



Function



Not a Function (Relation)

Chap[illegible] 2: Functions

2.1 A function vs a relation pg. 47-48

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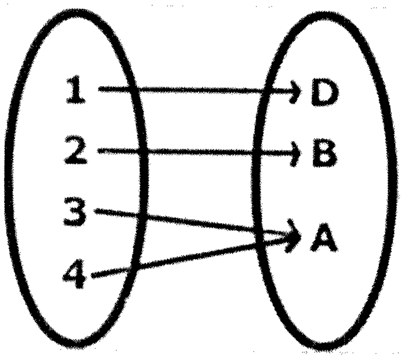

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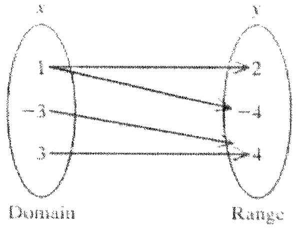

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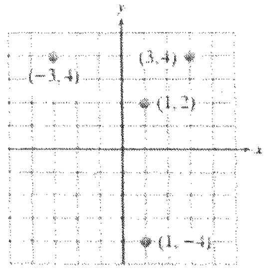

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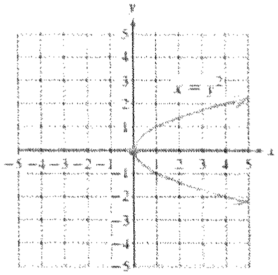

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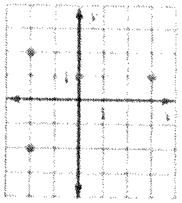

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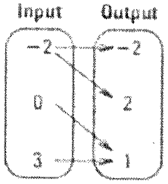

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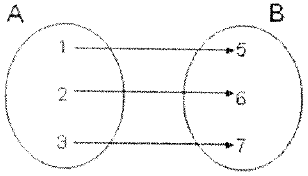

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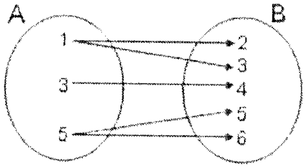

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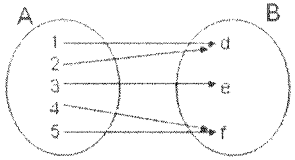

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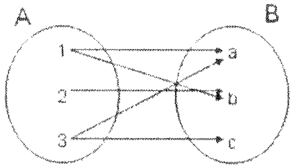

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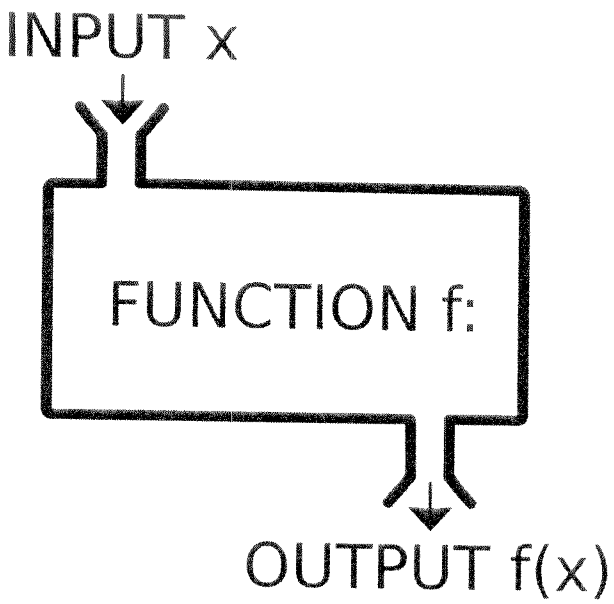

INPUT x

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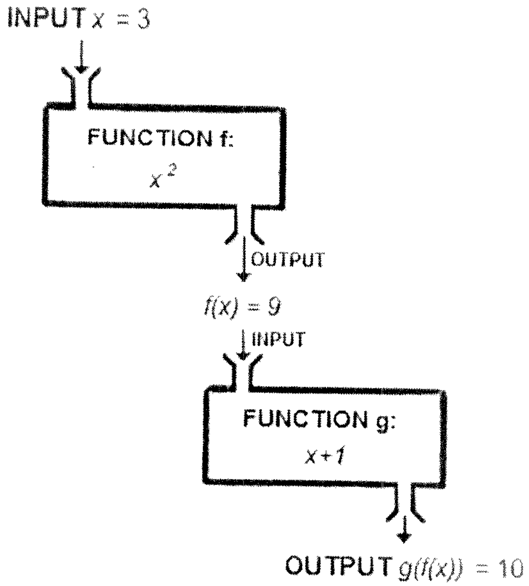
OUTPUT f(x)



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OUTPUT

$f(x) = 9$

INPUT

FUNCTION g:
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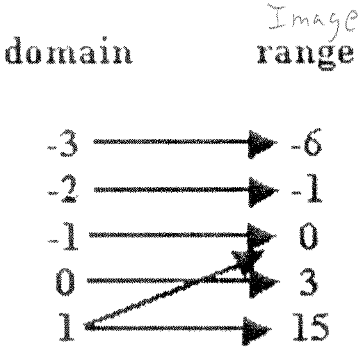

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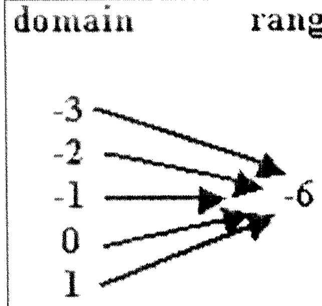

(-3, -6)
(-2, -6)
(-1, -6)

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IN A SET

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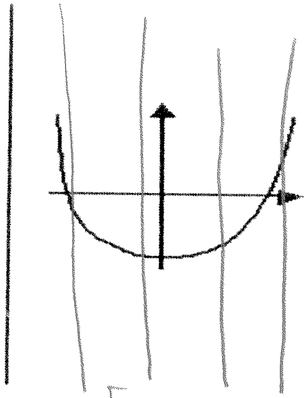
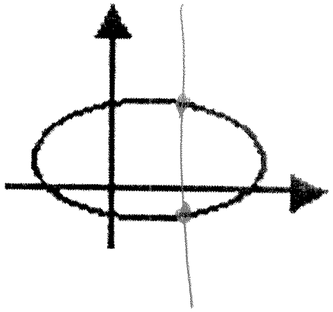
Is each a function?



Function



Not a Function (Relation)

Chapter 5: Analytic Geometry

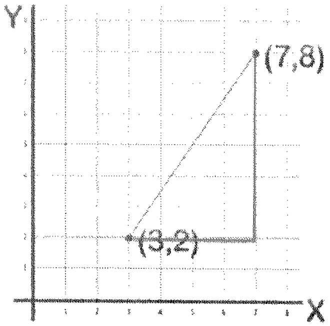
5.1 pg 133

How would you find the distance of the segment drawn? (Pyth.)

$a^2 + b^2 = c^2$

$\sqrt{4^2 + 6^2} = \sqrt{c^2}$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = c$



$y_2 - y_1$

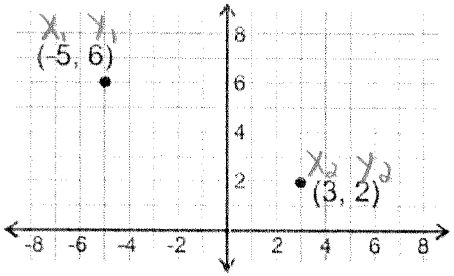
⑥

$x_2 - x_1$

④

The distance between points A and B:

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



✱ Any order

$\sqrt{[3-(-5)]^2 + (2-6)^2}$

$\sqrt{8^2 + (-4)^2}$

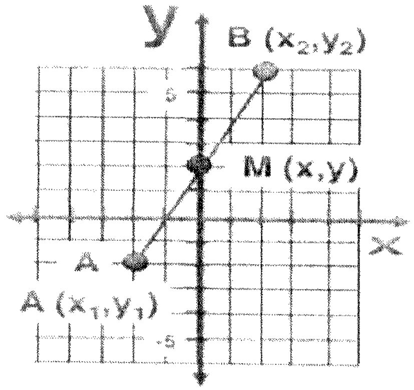
$\sqrt{64 + 16}$

$\sqrt{80}$

8.9 units

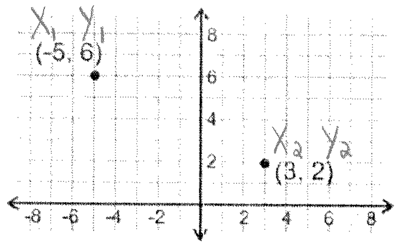
5.2 pg 134

The coordinates of the midpoint of segment AB if $A(x_1, y_1)$ and $B(x_2, y_2)$ are



$M(x, y) =$
$(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

Find the midpoint $M(x, y) = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$



$$\left(\frac{(-5)+3}{2}, \frac{6+2}{2}\right)$$

−1, 4

Midpoint Backwards (Midpoint Given)

Given the end point of A(-2, 5) and midpoint of (4, 4), what is the other endpoint, B.

$$\overset{M}{(4, 4)} = \left(\overset{\left(\frac{x_1+x_2}{2}\right)}{\frac{-2 + x_B}{2}}, \overset{\left(\frac{y_1+y_2}{2}\right)}{\frac{5 + y_B}{2}} \right)$$

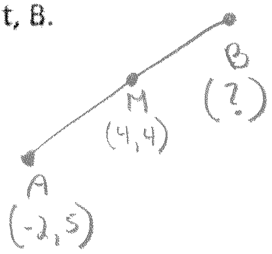
$$\frac{-2 + x_B}{2} = 4 \qquad \frac{5 + y_B}{2} = 4$$

$$-2 + x_B = 4(2) \qquad 5 + y_B = 4(2)$$

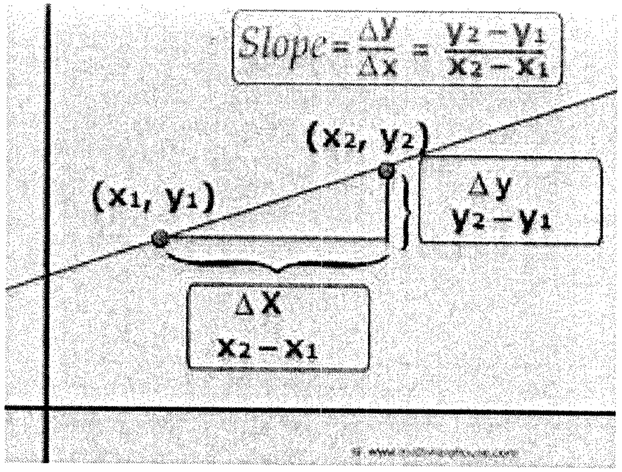
$$x_B = 8 + 2 \qquad y_B = 8 - 5$$

$$x_B = 10 \qquad y_B = 3$$

∴The other end point is B (10, 3)

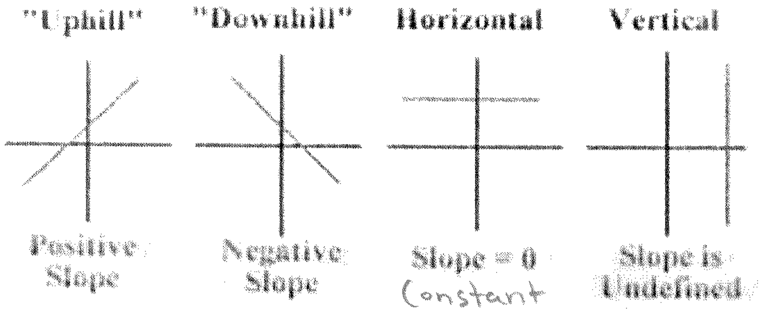


5.3 Slope of a line pg136-138

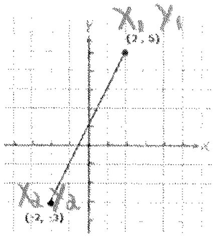


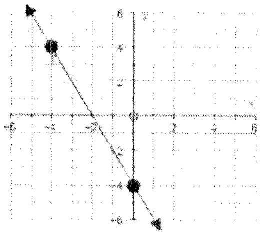
$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{rise}{run}$$



Find the slope





$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{(-3) - 5}{(-2) - 2}$$

$$\frac{-8}{-4}$$

$a = 2$

$$a = \frac{rise}{run}$$

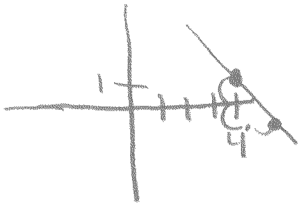
$$\frac{-8}{4}$$

$a = -2$

line goes down
that's wh
it's
negative

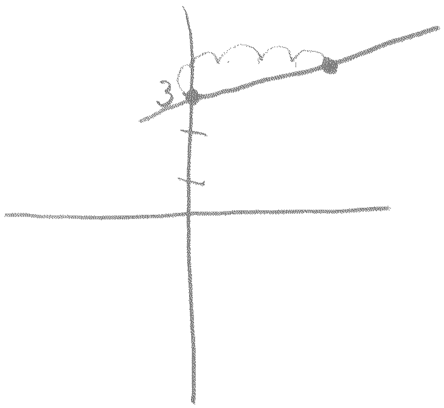
Graph the line passing at (4, 1) and has a slope of −2

$\frac{-2}{1} = \frac{\text{RISE (up, down)}}{\text{RUN (left, right)}}$



Graph the equation $y = \frac{1}{4}x \boxed{+ 3}$

Y-intercept, initial value



$\frac{1}{4} = \frac{\text{Rise}}{\text{Run}}$

5.4 Intercepts of a line pg140-141

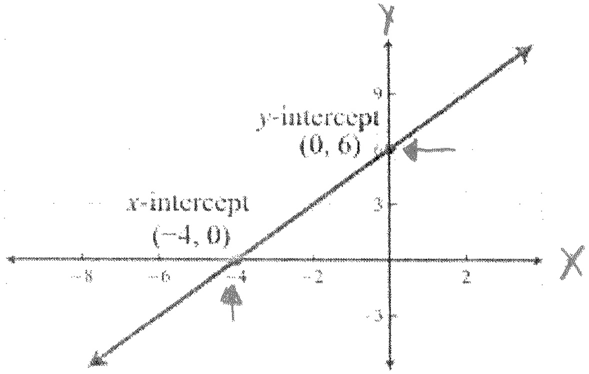
x-intercept: the point where the line crosses the x axis, if exists.

Coordinates of x int (#,0)

y-intercept: the point where the line crosses the y-axis, if exists.

Coordinates of y int (0, #)

Graphically



The intercepts in table of values

X	0	1	2	3
y	4	2.5	1	0

Y-intercept

(0,4)

X-intercept

(3,0)

From an equation

x-intercept: set y as 0 and solve for x	y-intercept: set x as 0 and solve for y
-2x+3y =12 $-2x+3(0)=12$ $\frac{-2x}{-2}=\frac{12}{-2}$ $x=-6$	-2x+3y =12 $-2(0)+3y=12$ $\frac{3y}{3}=\frac{12}{3}$ $y=4$

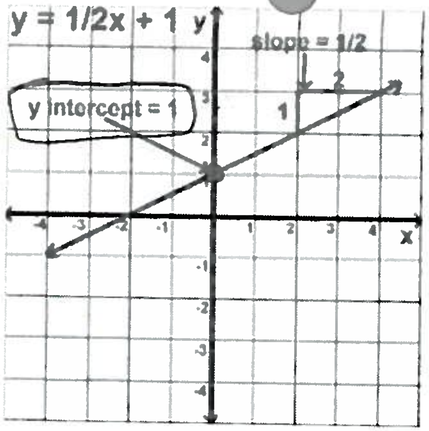
5.5 Functional form of the equation of a line (slope-intercept form) pg. 143

Functional form is

$y = ax + b$

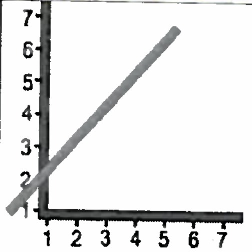
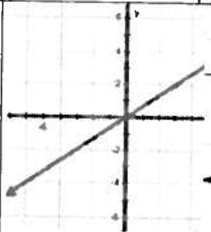
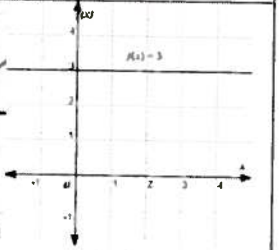
Note:

- $y = 1y$



Where a is slope or rate of change

Where b is y-intercept or initial value

Partial $f(x) = ax + b$	Direct $f(x) = ax$	Zero/Constant $f(x) = b$
 *does not pass through (0,0)	 *directly through (0,0)	 *horizontal

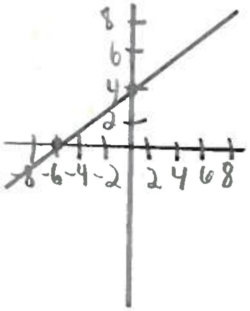
GRAPH AN EQUATION:

Two methods

Method 1: By finding the x and y intercepts and graphing.

*Suggested if not in functional form

$$-2x + 3y = 12$$



x-intercept	y-intercept
$-2x + 3(0) = 12$	$-2(0) + 3y = 12$
$\frac{-2x}{-2} = \frac{12}{-2}$	$\frac{3y}{3} = \frac{12}{3}$
$x = -6$	$y = 4$

Method 2: By finding 2 or more points and graphing.

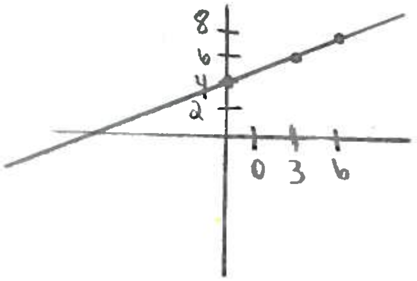
$-2x + 3y = 12$

$\frac{3y}{3} = \frac{12}{3} + \frac{2x}{3}$

$y = \frac{2}{3}x + 4$

go by the denominator

x	y
0	4
3	6
6	8

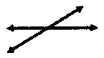
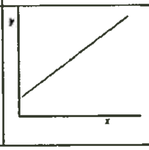
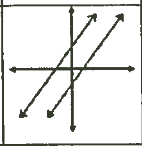
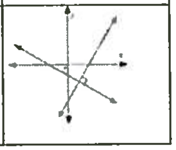


*Suggested if it is in functional form

RELATIVE POSITION OF TWO LINES PG. 144

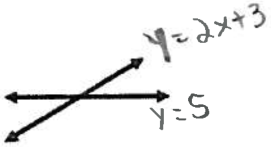
Given 2 lines with equations:

$y_1 = a_1x + b$ and $y_2 = a_2x + c$

Intersecting/ Secant Lines	Coincident Lines	Parallel Lines	Perpendicular Lines
			

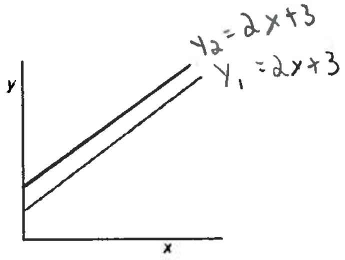
Case 1: Intersecting or Secant Lines

Slopes a_1 and a_2 are *different*



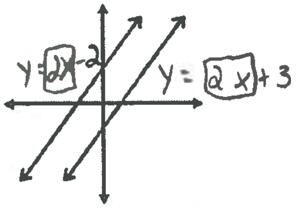
Case 2: Coincident Lines

Slopes a_1 and a_2 are equal & y-intercepts b and c are equal



Case 3: Parallel Lines

Slopes *a_1 and a_2 are equal* and y-intercepts are different



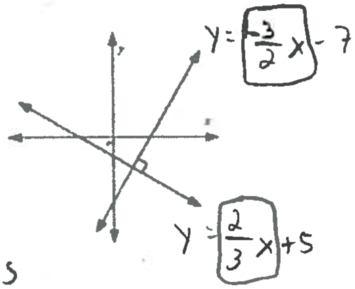
* Case 4: Perpendicular Lines

Slope a_1 is negative reciprocal to a_2.

$$a_1 \times a_2 = -1$$

The product of the 2 slopes equals -1

$-\frac{3}{2} \cdot \frac{2}{3} = -1$



Negative Reciprocal:

1. Flip fraction $\frac{a}{b} = \frac{b}{a}$
2. Switch sign $- \rightarrow +$ $+ \rightarrow -$

Ex:

$-\frac{2}{1} \rightarrow \frac{1}{2}$ $-100 \rightarrow \frac{1}{100}$

$\frac{1}{3} \rightarrow -\frac{3}{1} = -3$

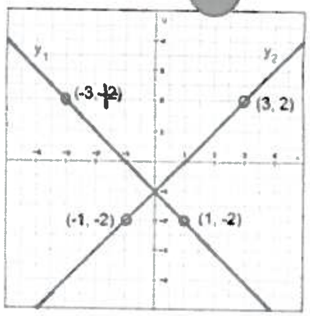
$-\frac{2}{3} \rightarrow \frac{3}{2}$

$4 \rightarrow -\frac{1}{4}$

$\frac{1}{5} = -5$

$6 = -1/6$

Show that l_1 and l_2 are perpendicular lines.



Step 1: slope l_1

$\frac{2-(-2)}{-3-1}$

$\frac{4}{-4}$

-1

Step 2: slope l_2

$\frac{-2-2}{-1-3}$

$\frac{-4}{-4}$

1

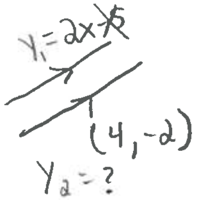
Step 3

$-1 \cdot 1 = -1$

$\therefore l_1 \perp l_2$

therefore

Find the equation of a line parallel to $f(x) = 2x - 5$ and passing thru (4, -2).



Step 1 $l_1 \parallel l_2 \therefore$ have the same slope ($a_1 = a_2 = 2$)

Step 2 Find equation l_2

$y_2 = 2x_2 + b$

Plug in $p(\underset{x}{4}, \underset{y}{-2})$

$-2 = 2(4) + b$

$-2 = 8 + b$

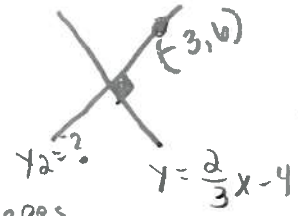
$\boxed{-10 = b}$

Find the equation of a line perpendicular to $2x - 3y - 12 = 0$and passing thru (-3, 6)

Step 1: Put equation in functional form

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{12}{-3}$

$y = \frac{2}{3}x - 4$



Step 2: $l_1 \perp l_2 \therefore$ they have negative reciprocal slopes

$\frac{2}{3} = -\frac{3}{2}$

Step 3: Find equation of line 2

$y = -\frac{3}{2}x + b$

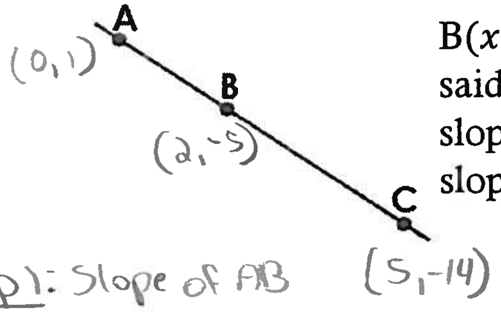
$6 = -\frac{3}{2}(-3) + b$

$6 = \frac{9}{2} + b$

1 5 = b

$\boxed{y = -\frac{3}{2}x + \frac{3}{2}}$

Note:



on the same line

Three points A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) are said to be collinear if slope(A,B) = slope(B,C).

Step 1: Slope of AB

$\frac{-5-1}{2-0}$

$\frac{-6}{2}$

-3

Step 2: Slope of B,C

$\frac{-14-(-5)}{5-2}$

$\frac{-9}{3}$

-3

Step 3

Slope AB = -3 and Slope BC = -3 ∴ it is collinear

5.6 GENERAL FORM OF THE EQUATION OF A LINE: $ax + by + c = 0$ PG. 146-148

a is not slope

Note:

- a is always positive
- Order is x, y, #
- No decimals or fractions

General→ Functional	Functional→General
$12x - 30y - 120 = 0$ $\frac{-30y}{-30} = \frac{-12x}{-30} + \frac{120}{-30}$ $y = \frac{2}{5}x - 4$	$^{3}\left(y = \frac{2}{3}x - 4\right)^{3}$ $3y = 2x - 12$ $0 = 2x - 12 - 3y$ $0 = 2x - 3y - 12$

Determine if point (4, 0.4) is on the line $3x - 5y - 10 = 0$.

$3(4) - 5(0.4) - 10 = 0$

$12 - 2 - 10 = 0$

$0 = 0$

Yes, the point is on the line

→ plug in the point given and check if the equation is true

Note: (pg.149) → Quick way of determining the relative position when equations are in general form:

$$l_1: a_1x + b_1y + c_1 = 0$$
$$l_2: a_2x + b_2y + c_2 = 0$$

Intersecting	Parallel	Coincident
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

5.7 SYMMETRIC FORM OF THE EQUATION OF A LINE PG 150-152

Symmetric form: $\frac{x}{a} + \frac{y}{b} = 1$ a is the x-intercept, b is the y-intercept and $\underbrace{a, b \neq 0}_{\text{restrictions}}$

$\frac{x}{a} + \frac{y}{b} = 1$ $y = ax + b$ $ax + by + c = 0$

Symmetric	Functional	General
$-6\left(\frac{x}{2} + \frac{y}{-6} = 1\right)-6$ $-3x + y = -6$	$-3x + y = -6$ $y = 3x - 6$	$-3x + 1y = -6$ $\boxed{y} = 3x - 6$ $3x - y - 6 = 0$
$y = -2x + 9$ isolate the constant and divide by the constant $\frac{y}{9} + \frac{2x}{9} = \frac{9}{9}$	$y = -2x + 9$	$y = \boxed{-2x + 9}$ $2x - 9 + y = 0$

$\frac{y}{9} + \frac{x}{9/2} = 1$

$y = \frac{3}{5}x + \frac{6}{5}$ *Find x and y-intercepts x-intercept $0 = \frac{3}{5}x + \frac{6}{5}$	$\frac{-5y}{-5} = \frac{-3x}{-5} \frac{-6}{-5}$ $y = \frac{3}{5}x + \frac{6}{5}$	$3x - 5y + 6 = 0$

$\frac{-\frac{6}{5}}{\frac{3}{5}} = \frac{3}{5}x$

$-2 = x$

Yintercept

$y = \frac{3}{5}(0) + \frac{6}{5}$

$y = \frac{6}{5}$

$\frac{x}{-2} + \frac{y}{\frac{6}{5}} = 1$

Ex : Find the equation of a line in symmetric form if points A(0,4) and B(5,10) are given.

x y

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} + \frac{y}{4} = 1$$

$$\frac{5}{a} + \frac{10}{4} = 1$$

$$\frac{5}{a} = 1 - \frac{10}{4}$$

$$\frac{5}{a} = -\frac{3}{2} \quad \frac{\;}{5} = \frac{1}{\frac{3}{5}}$$

$$\frac{-3a}{-3} = \frac{10}{-3}$$

$$a = \frac{10}{-3}$$

$$\frac{x}{-\frac{10}{3}} + \frac{y}{4} = 1$$

Ex: Find the intercepts for $\frac{3x}{5} + \frac{5y}{4} = 1$

$$x = \frac{5}{3} \qquad y = \frac{4}{5}$$

5.8 FINDING THE EQUATION OF A LINE

PG. 154-155

Ex 1: Find the equation of a line which passing thru A(2, 5)and B(−3, 15).

$x_1\ y_1 \qquad x_2\ y_2$

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15-5}{-3-2} = \frac{10}{-5} = -2$$

$y = -2x + b$

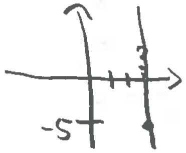
$5 = -2(2) + b$

$5 = -4 + b$

$9 = b$

$$\boxed{y = -2x + 9}$$

Ex 2: Find the general form of the equation of a line which passes vertically through (3, −5).



$x = 3$

$$\boxed{x - 3 = 0}$$

Ex 3: Find the functional form of an equation of a line with x-intercept 4 and y-intercept -2.

$$-4\left(\frac{x}{4} + \frac{y}{-2} = 1\right)$$

$$-1x + 2y = -4$$

$$x - 2y - 4 = 0$$

$$\frac{-2y}{-2} = \frac{-x}{-2} + \frac{4}{-2}$$

$$\boxed{y = \frac{1}{2}x - 2}$$

5.9 Distance from point to a line pg.156-157

Find the distance from the point to the line without using the formula.

Step 1: Find equation l_1

$\boxed{y_1 = -\frac{2}{3}x + 2}$

Step 2: Find equation l_2

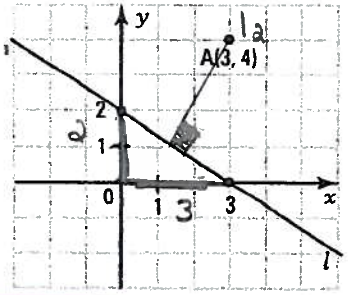
$l_1 \perp l_2$ ∴ slopes are negative reciprocal

$-\frac{2}{3} \rightarrow \frac{3}{2}$

$y = \frac{3}{2}x + b$

$4 = \frac{3}{2}(3) + b$

$-\frac{1}{2} = b \rightarrow \boxed{y = \frac{3}{2}x - \frac{1}{2}}$



Step 3: Find coordinate B by comparison

$^{6}\left(-\frac{2}{3}x + 2 = \frac{3}{2}x - \frac{1}{2}\right)^{6}$

$\boxed{-4x} + 12 = 9x \boxed{-3}$

$\frac{15}{13} = \frac{13x}{13}$ $x = 1.15$

$y = \frac{3}{2}x - \frac{1}{2}$

$y = \frac{3}{2}(1.15) - \frac{1}{2}$

$y = 1.23$

Note: The line must be in general form.

Note: This formula gives you the shortest distance from the point to the line. It is the length of the line segment which joins the point to the line and is **perpendicular** to the line.

The distance between a point P and a line l is the length of the perpendicular segment joining point P to line l.

Step 4: Distance (A,B)

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(3 - 1.15)^2 + (4 - 1.23)^2}$

$\sqrt{(1.85)^2 + (2.77)^2}$

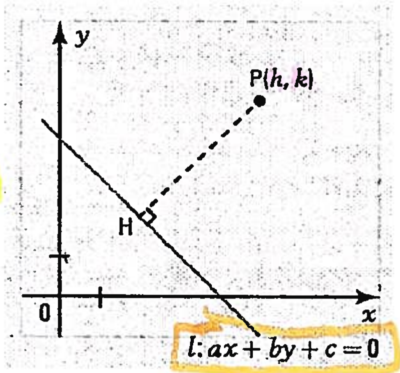
$\sqrt{3.42 + 7.67}$

$\sqrt{11.09}$ D = 3.33

(1.15, 1.23)

$$d(P, l) = \frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$$

absolute value turns all values positive in final result



General

Ex: Find the distance from point (3, -4) and line $y = 5x - 6$

h k

$0 = 5x - y - 6$

$\frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

h = 3
k = -4
a = 5
b = -1
c = -6

$\frac{|(5)(3) + (-1)(-4) + (-6)|}{\sqrt{(5)^2 + (-1)^2}}$

$\frac{|15 + 4 - 6|}{\sqrt{25 + 1}}$

$\frac{|13|}{\sqrt{26}} = \frac{|13|}{5.09} = 2.55$

$\boxed{D = 2.55}$

Ex: Find the distance between two lines

$$l_1 : y = 3x - 5$$

$$l_2 : y = 3x + 4$$

Hint: Find a point on one of the lines.

Step 1: Find a random point on l_1

$(1, -2)$ $\quad$ $y = 3(1) - 5$

$y = -2$

Step 2: put l_2 in general form

$0 = 3x - y + 4$

270

Step 3: Find distance

$h = 1$

$k = -2$

$a = 3$

$b = -1$

$c = 4$

$$\frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$$

$$\frac{|(3)(1) + (-1)(-2) + (4)|}{\sqrt{(3)^2 + (-1)^2}}$$

$$\frac{|9|}{\sqrt{10}}$$

$$\boxed{D = 2.85}$$

Find the x and y intercepts

1. **Linear:** $f(x) = \frac{2}{5}x - 6$

x-intercept

let f(x)=0 and solve for x.

$0 = \frac{2}{5}x - 6$

$\frac{6}{1} = \frac{2}{5}x$

imaginary

$30 = 2x$

$15 = x$

(15,0)

y-intercept

let f(0)

$f(0) = \frac{2}{5}(0) - 6$

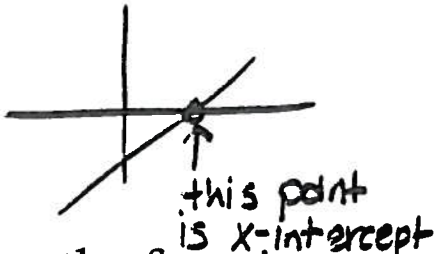
$f(0) = -6$

(0,-6)

INTERCEPTS PG. 52

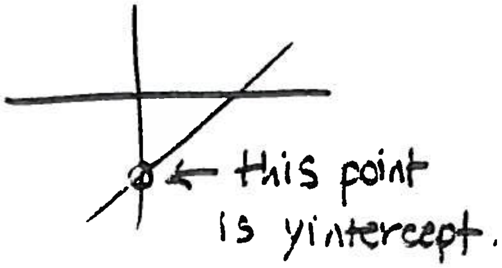
Zeros (x-intercepts): where the function crosses the x axis (let f(x)=0)

means let y=0 and solve for x.


this point is x-intercept

Y-intercept (initial value): where the function crosses the y-axis (find f(0))

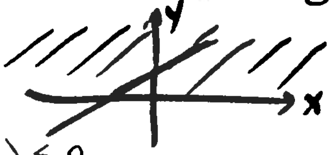
this notation means plug x as 0 and solve for y.


this point is yintercept.

Sign pg. 53 #13-15

$f(x) \geq 0$

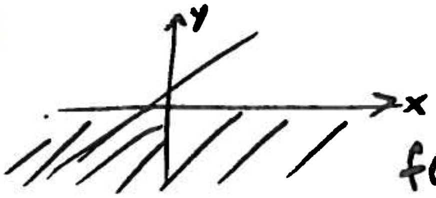
A function is POSITIVE for all values of x from left to right for which the graph is ABOVE the x-axis.



$f(x) \geq 0$

$f(x) \leq 0$

A function is NEGATIVE for all values of x from left to right for which the graph is BELOW the x-axis



$f(x) \leq 0$

Find the x and y intercepts

2. 2nd degree: $f(x) = x^2 - 1x - 30$

x-intercept

let $f(x)=0$

$0 = x^2 - x - 30$

$0 = (x+5)(x-6)$

$\swarrow$ $\searrow$

$x=-5$ $x=6$

$(-5,0)$ and $(6,0)$

y-intercept

find $f(0)$

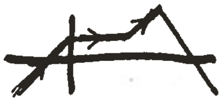
$f(0) = (0)^2 - (0) - 30$

$f(0) = -30$

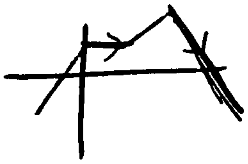
$(0,-30)$

Variation of a function pg 55 #16-19

A function is INCREASING for all values of x from left to right for which the line goes up (the y values are increasing) or remains constant.



A function is DECREASING for all values of x from left to right for which the line goes down (the y values are decreasing) or remains constant.

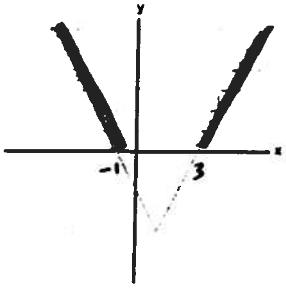


$f(x) > 0$ $f(x) < 0$

STRICTLY POSITIVE OR NEGATIVE

means that we do NOT include the zeros.

do not include values on x-axis



$f(x) \geq 0$ for $x \in]-\infty,-1] \cup [3,\infty[$

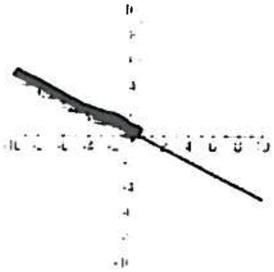
$f(x) > 0$ for $x \in]-\infty,-1[\cup]3,\infty[$

$f(x) \leq 0$ for $x \in [-1,3]$

$f(x) < 0$ for $x \in]-1,3[$

element (part of)

The function is st. $\ominus$ for the values of x which are $]-1,3[$

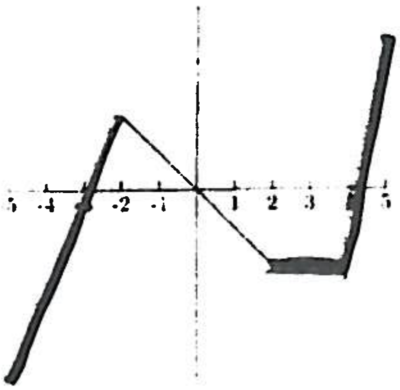


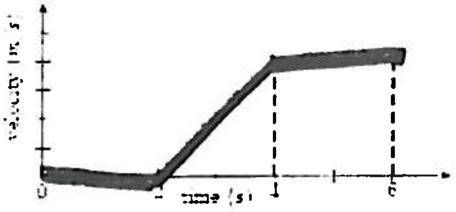
$f(x) \geq 0$ for $x \in]-\infty,0]$

$f(x) > 0$ for $x \in]-\infty,0[$

$f(x) \leq 0$ for $x \in [0,\infty[$

$f(x) < 0$ for $x \in]0,\infty[$





f increasing over $[-5,-2] \cup [2,5]$

f decreasing over $[-2,4]$

$f\uparrow$ over $[-5,-2] \cup [4,5]$

$f\searrow$ over $[-2,2]$

f increasing over $[0,6]$

f decreasing over $[0,2] \cup [4,6]$

$f\nearrow$ over $[2,4]$

$f\searrow$ over $\{\emptyset\}$ NONE

Note: a constant function is considered both increasing and decreasing.

A constant function is a horizontal line.

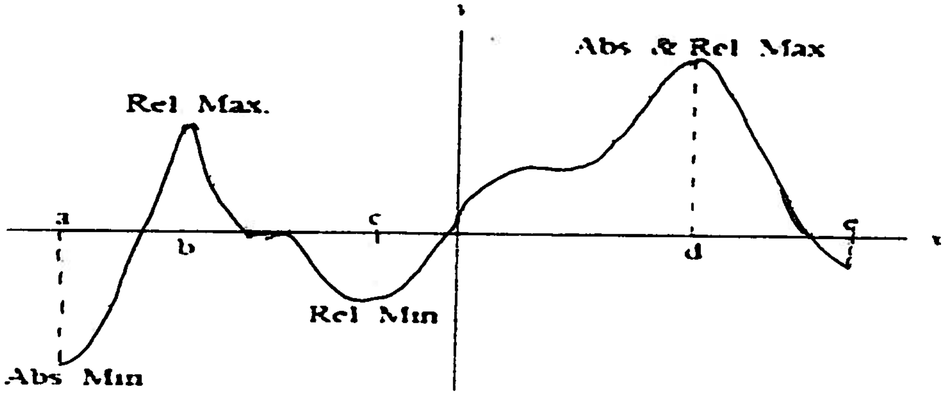
Constant functions are considered both increasing and decreasing.

When the term]strictly[is used, do not include the values where the function is constant.

Notation
Strictly increasing $f\nearrow$
Strictly decreasing $f\searrow$

Absolute Minimum: is the lowest y value, if it exists

Relative Minimum is the second lowest y value, if it exists



Extema of a function pg 58 #20-22, pg 70#1-4 , pg 45 #1-2

Absolute Maximum: is the highest y value, if it exists

Relative Maximum: is the second highest y value, if it exists

[ ]

[ ]

x #

t : y #

Max : y #

Min : y #

↑ : x [ ]

↓ : x [ ]

} strictly means exclude constant

⊕ : x [ ]

⊖ : X [ ]

} strictly means exclude values on x-axis

X : left to right

Y : bottom to top

---

f(x) ≥ 0 = positive

f(x) ≤ 0 = negative

f(x) > 0 = strictly positive

f(x) < 0 = strictly negative

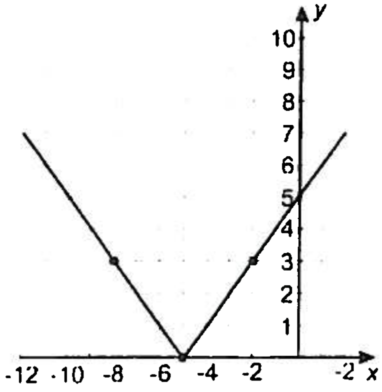
f inc = increasing

f ↗ = strictly increasing

- f dec = decreasing
- f ↓ = strictly decreasing

✱ strictly increasing ⊕ decreasing = [ ]

✱ strictly positive ⊕ negative =] [

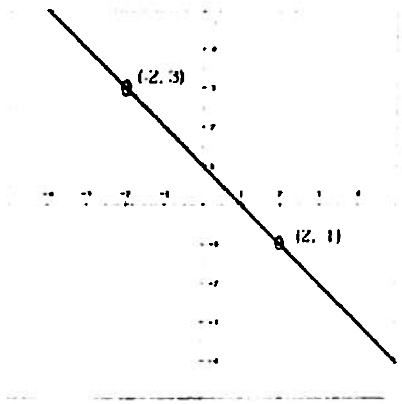


Abs Max NONE

Abs Min 0

Rel Max NONE

Rel Min NONE



Abs Max NONE

Abs Min NONE

Rel Max NONE

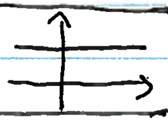
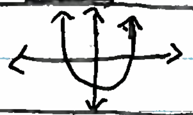
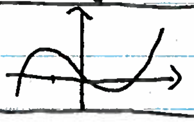
Rel Min NONE

NOTE! Never include infinite

Chapter 3: Polynomial Functions

3.1 Polynomial Functions – general idea

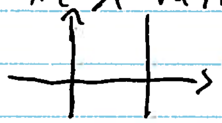
Degree of a function: is the value of the highest exponent

degree	example	Name	Sketch
0	$f(x) = 40$	Constant	
1	$f(x)=2x-3$	Linear	
2	$f(x)=8x^2-2x$	Quadratic	
3	$f(x)=3x^3+2x+5$	Cubic	

- The following is a relation, not a function

$x = k, k \in \mathbb{R}$

- All the X values are 4, the y values are different



3.2 Constant Function pg. 79 #1-5 omit 4

A constant function is a zero degree polynomial function. Rule is $f(x) = b,\ b \in \mathbb{R}$

Ex: The cost of a metro ticket is $3 $\overset{\text{constant}}{\boxed{\text{no matter}}}$ the distance you travel.

$$f(x) = 3$$

3.3 Linear Function pg. 83-87 omit #4

The equation of a line is $f(x) = ax+b$

a is called slope and rate of change

If $a > 0$, then line is increasing ↗

If $a < 0$, then line is decreasing ↘

Nilroy

Use two coordinates, labelled as (X_1, Y_1) and (X_2, Y_2), plug in formula:

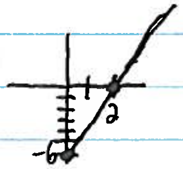
$$\boxed{a = \frac{Y_2 - Y_1}{X_2 - X_1}}$$

How to graph

Find the X and Y intercepts and plot them

Ex: $f(x) = 3x - 6$

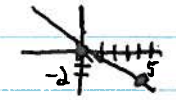
x-int	y-int
$0 = 3x - 6$	$y = -6$
$x = 2$	



Ex: $f(x) = -\frac{2}{5}x$

$X = 0$

$Y = 0$



x	y
0	0
5	-2

Word Problems

A pool containing 2000L is being emptied. After 10 min, it contains 1500L

a) Name the variables and find the rule

X: time Y: amount of water

b) Find the amount of water after 15 min

1250 L

c) Find the time it takes to empty the pool

40 minutes

Rule

10, 1500

0, 2000

$$\frac{2000 - 1500}{0 - 10}$$

$$\frac{500}{-10}$$

$$-50$$

$f(x) = -50x + 2000$

b) $f(15) = -50(15) + 2000$

$f(15) = 1250$ L

c) $0 = -50x + 2000$

$\frac{-2000 = -50x}{-50}$

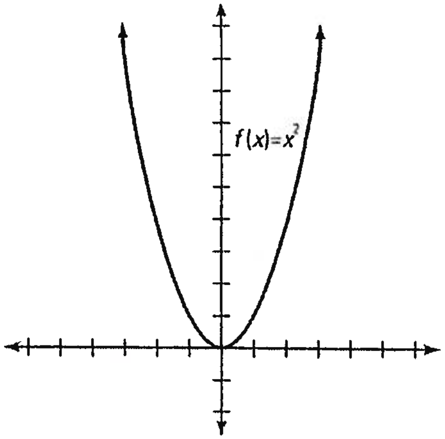
$x = 40$

Quadratic Function

Youtube:

Quadratic Functions and Parabolas in the Real World
https://youtu.be/He42k1xRpbQ

3.4 Quadratic function pg 89#1-20 omit 4, 11

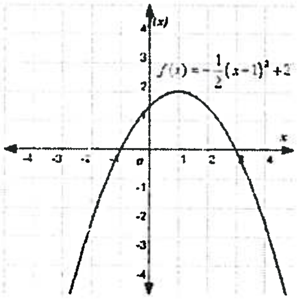


Basic quadratic function is $f(x) = 1x^2$

Transformed quadratic function is when the function is not going through the origin and has shifted left/right and up/down.

The standard form of a quadratic equation is

$f(x) = a(x - h)^2 + k$



The U-curve is called Parabola

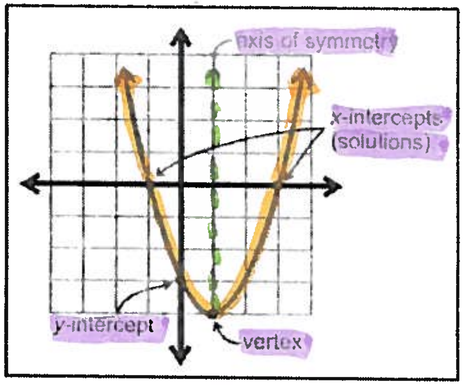
Role of Parameters in Standard Form

$f(x) = a(x - \underline{h})^2 + \underline{k}$

Vertex V(h, k)

Axis of symmetry

$x = h$

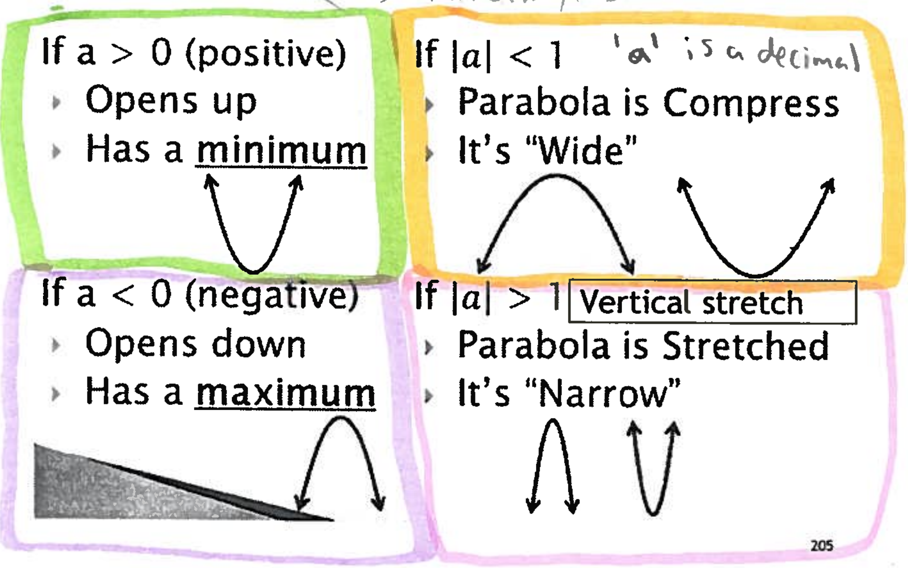


slope

x Y

Parameter *a* $f(x) = a(x - h)^2 + k$

random part



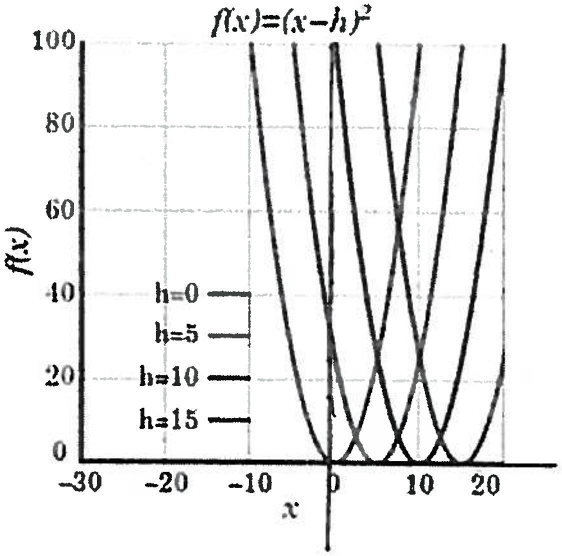
If a > 0 (positive)
- Opens up
- Has a minimum

If |*a*| < 1 'a' is a decimal
- Parabola is Compress
- It's "Wide"

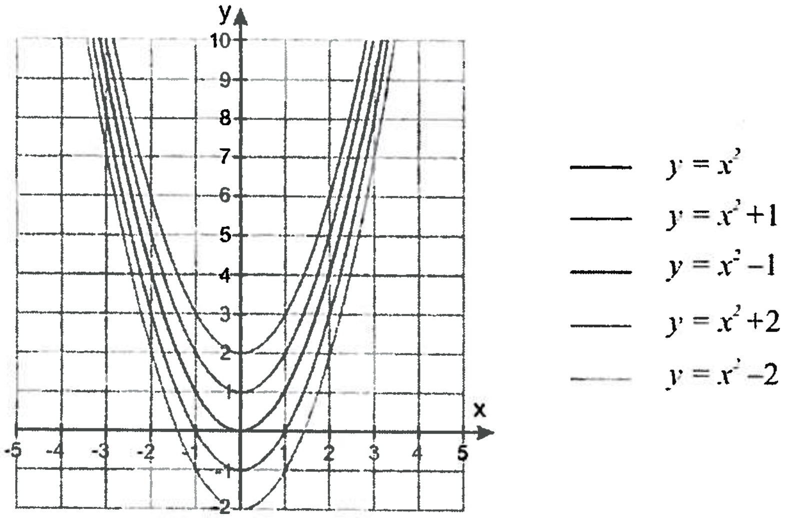
If a < 0 (negative)
- Opens down
- Has a maximum

If |*a*| > 1 Vertical stretch
- Parabola is Stretched
- It's "Narrow"

Parameter *h*: shifts the parabola left and right on the x axis (horizontal translation)



Parameter k: shifts the parabola up and down on the y-axis (vertical translation)



$f(x) = a(x-h)^2 + k$

$h \pm \sqrt{\frac{-k}{a}}$

$f(x) = 1(x-2)^2 + 9$

$a = 1$
$h = 2$
$k = 9$

$2 \pm \sqrt{\underline{-9}}$

Graphing the Quadratic function

Steps: Find the vertex, the zeros and initial value and plot on graph.

$y = a(x-h)^2 + k$

Ex: $f(x) = 1(x-2)^2 + 9$

Vertex V: $(h,k) = (2,9)$

Zeros: let $f(x) = 0$

$0 = 1(x-2)^2 + 9$

$\sqrt{-9} = \sqrt{(x-2)^2}$

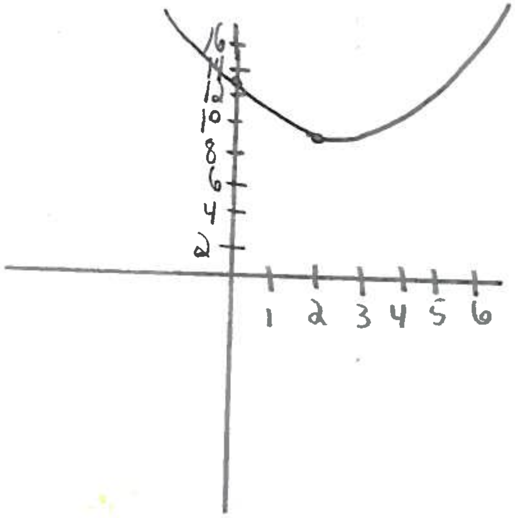
Does not exist! So no zeros

Initial Value: let $x = 0$

$f(0) = 1(0-2)^2 + 9$

$f(0) = 1(4) + 9$

$f(0) = 15$



Ex: $f(x) = -4(x+3)^2 + 9$

Vertex: $(h,k) = (-3, 9)$

Zeros: let $f(x) = 0$

$0 = -4(x+3)^2 \boxed{+9}$

$-9 = -4(x+3)^2$

$\frac{-9}{-4} = \frac{-4(x+3)^2}{-4}$

$\sqrt{\frac{9}{4}} = \sqrt{(x+3)^2}$

$\frac{3}{2} = x \boxed{+3}$ | $-\frac{3}{2} = x \boxed{+3}$

$-1.5 = x$ | $-4.5 = x$

$a = -4$

$h = -3$

$k = 9$

$h \pm \sqrt{\frac{-k}{a}}$

$-3 \pm \sqrt{\frac{-9}{-4}}$

$-3 \pm \sqrt{2.25}$

-3 ± -1.5

$-3 + -1.5$

$-3 - 1.5$

$\boxed{-4.5}$

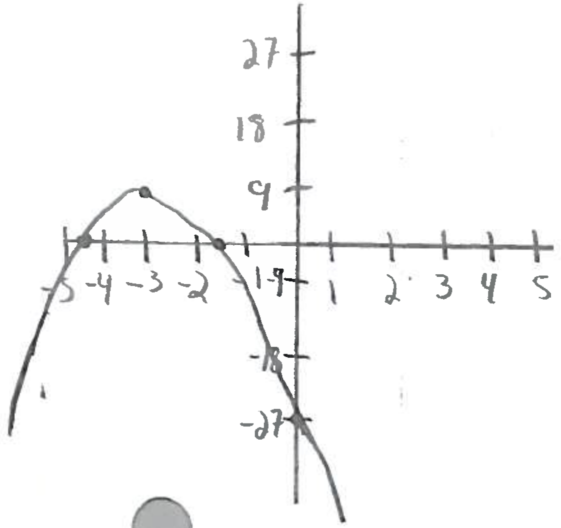
or $-3 - -1.5$

$-3 + 1.5$

$\boxed{-1.5}$

Initial value: $y = -4(0+3)^2 + 9$

$y = -27$



Ex: $f(x) = 3(x-1)^2$

Vertex: $(1, 0)$

Zeros: $\frac{0}{3} = \frac{3(x-1)^2}{3}$

$0 = (x-1)^2$

$0 = x - 1$

$x = 1$

$a = 3$

$h = 1$

$k = 0$

$1 \pm \sqrt{\frac{-0}{3}}$

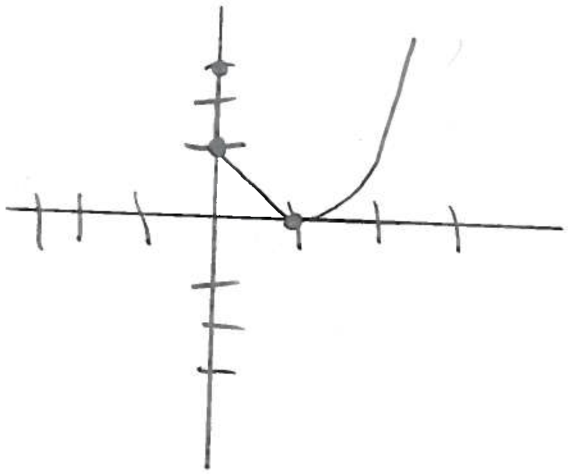
$1 \pm \sqrt{0}$

1 ± 0

$\boxed{1}$

Initial value: $f(0) = 3(0-1)^2$

$f(0) = 3$



Determine 10 properties

Domain ℝ

Range $]-\infty, 2]$

Zeros $\{0, 6\}$

Y intercept $\{0\}$

Max 2

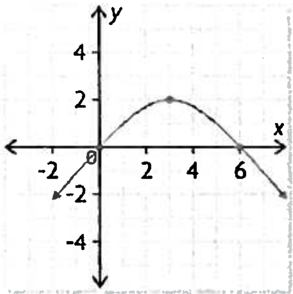
Min NoNe

$f(x) \geq 0$ $[0, 6]$

$f(x) \leq 0$ $]-\infty, 0] \cup [6, \infty[$

↑ $]-\infty, 3]$

↓ $[3, \infty[$



Find the rule – given vertex and a point

pg 98#21-28

Find the rule in standard form $f(x) = a(x-h)^2 + k$

$v: (h, k)$

$(3, 2)$

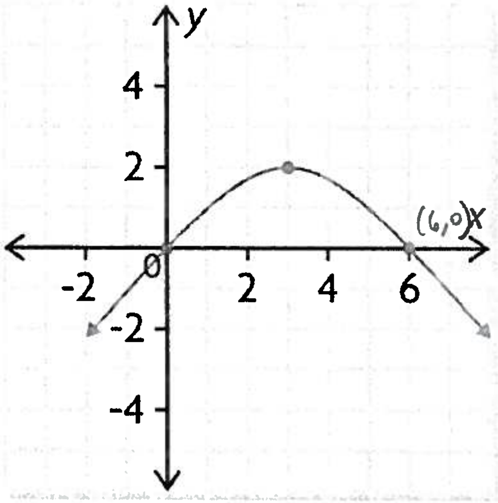
$y = a(x-h)^2 + k$

$0 = a(6-3)^2 + 2$

$0 = 9a \boxed{+2}$

$\frac{-2}{9} = \frac{9a}{9}$

$a = -\frac{2}{9}$



$$\boxed{y = -\frac{2}{9}(x-3)^2 + 2}$$

Ex: A parabola has a vertex (6, 10) and passes thru P(16,4)

h k

x y

a) Find the equation

$y = a(x-h)^2 + k$

$4 = a(16-6)^2 + 10$

$4 = 100a + 10$

$\frac{-6}{100} = 100a$

$a = -0.06$

$$\boxed{y = -0.06(x-6)^2 + 10}$$

lug x as 0

b) Find the y intercept

$y = -0.06(0-6)^2 + 10$

$y = -0.06(36) + 10$

$$\boxed{y = 7.84 \text{ or } \frac{196}{25}}$$

x

c) Find f(12)

$f(12) = -0.06(12-6)^2 + 10$

$f(12) = -0.06(36) + 10$

$$\boxed{f(12) = 7.84}$$

Ex: A parabola has a vertex $(\overset{h}{4},\overset{k}{18})$ and a y-int of 6.

a) Find the equation

$$y = a(x-h)^2+k$$

$$6 = a(0-4)^2+18$$

$$6 = 16a\boxed{+18}$$

$$\frac{-12}{16} = \frac{16a}{16}$$

$$a = -\frac{3}{4}$$

$$y = -\frac{3}{4}(x-4)^2+18$$

b) Find the zeros

$$0 = a(x-h)^2+k$$

$$0 = -\frac{3}{4}(x-4)^2+18$$

$$h \pm \sqrt{-\frac{k}{a}}$$

$$-4 \pm \sqrt{-\frac{18}{-\frac{3}{4}}}$$

$$-4 \pm \sqrt{24}$$

$$-4+\sqrt{24} \qquad -4-\sqrt{24}$$

$x = -0.9$ $\qquad$ $x = 8.9$

$$S = \{-0.9, 8.9\}$$

Ex: A parabola has V(2,6) and point (3,5). Find the difference in x values when $f(x) = -43$.

① $5 = a(3-2)^2 + 6$

$5 = a(1) \textcircled{+6}$

$\frac{-1}{1} = 1a$

$a = -1$

$y = -1(x-2)^2 + 6$

② $\textcircled{-43} = -1(x-2)^2 + 6$

$F(x) = -1(x-2)^2 + 49$

$h \pm \sqrt{-\frac{k}{a}}$

$-2 \pm \sqrt{-\frac{49}{-1}}$

-2 ± 7

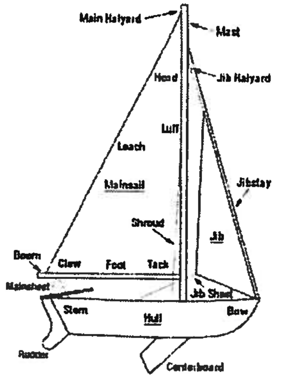
$-2+7 \quad -2-7$

$x = 5 \quad x = -9$

Difference $= |-9-5|$

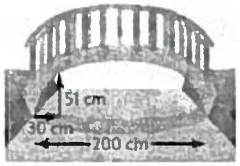
↳ $=14$

~~Quadratic Word Problems~~



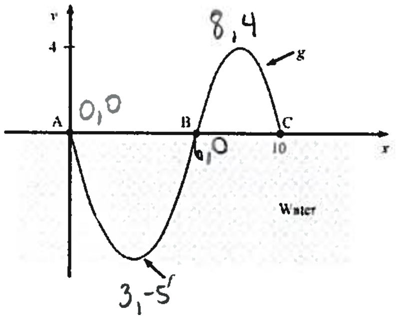
~~The underside of a bridge forms a parabolic arch. The arch has a maximum height of 30 m and a width of 50 m. Can a sailboat pass under the bridge, 8 m from the axis of symmetry, if the top of its mast is 27 m above the water? Justify your solution.~~

~~Kayli wants to build a parabolic bridge over a stream in her backyard as shown at the left. The bridge must span a width of 200 cm. It must be at least 51 cm high where it is 30 cm from the bank on each side. How high will her bridge be?~~



QUADRATIC WORD PROBLEMS MIXED

The following graph represents the side view of the path of a dolphin as it performs a trick during a show at an aquarium. This path is composed of portions of two parabolas associated with function f and g respectively. The scale of the graph is in metres. The rule $f(x)=\frac{5}{9}(x-3)^2-5$ represents the dolphin's path when it is in the water. When it is out of the water, the dolphin reaches a maximum height of 4 metres. The distance between points A and C is 10 metres. What is the rule of the function g?



Point B

$0 = \frac{5}{9}(x-3)^2 - 5$

$\frac{5}{5/9} = \frac{\frac{5}{9}(x-3)^2}{5/9}$

$\sqrt{9} = \sqrt{(x-3)^2}$

$3 = x-3$

$-3 = x-3$ $\quad$ $3 = x-3$

$x = 0$ $\quad$ $x = 6$

Vertex f(g)

$\frac{6+10}{2} = 8$

$(8,4)$

Rule f(g)

$0 = a(6-8)^2 + 4$

$\frac{-4}{4} = 4a$

$a = -1$

$\boxed{y = -1(x-8)^2 + 4}$

Determine which form of the quadratic would be best to use in each case.

Determine the equation of the second-degree function associated with the description provided.

a) The vertex is located at $V(3, 2)$ and the graph passes through the point $P(4, 3)$.

b) The two zeros are -3 and 1 and $f(-1)=2$.

c) The equation of the axis of symmetry is $x=-1$. The maximum is 2 and the graph passes through the point $P(4, -123)$.

d) The only zero of the function is -2 and $f(-1)=-1$.

e) Points $P(-1, 7)$, $Q(-9, 7)$ and $R(-3, 1)$ are on the parabola representing the function.

f) The y-intercept is greater than or equal to the zeros, which are -1 and 5.

CONVERTING FORMS

Use these methods only when asked to convert.

Standard → General

MULTIPLY

$$f(x) = a(x-h)^2 + k \rightarrow f(x) = ax^2 + bx + c$$

$$f(x) = 2(x-3)^2 - 12$$

$$f(x) = 2(x^2 - 6x + 9) - 12$$

$$\boxed{f(x) = 2x^2 - 6x + 6}$$

Standard → Factored

Find zeros

$$f(x) = a(x-h)^2 + k \rightarrow f(x) = a(x-x_1)(x-x_2)$$

$$f(x) = 2(x-3)^2 - 12$$

$$\boxed{f(x) = 2(x-5.4)(x-0.6)}$$

$$3 \pm \sqrt{\frac{- -12}{2}}$$

$$3 \pm \sqrt{6}$$

$3+\sqrt{6}$ $3-\sqrt{6}$

$x = 5.4$ $x = 0.6$

General→Standard

$$f(x) = ax^2 + bx + c \rightarrow f(x) = a(x - h)^2 + k$$

$$f(x) = 2x^2 - 12x + 6$$

$2(x^2 - 6x + 3)$

$2\ (x^2 - 6x + 9) + 3 - 9]$

$2[(x-3)^2 - 6]$

$2(x-3)^2 - 12$

$a(x-h)^2 + k$

$\boxed{f(x) = 2(x-3)^2 - 12}$

Step 1: Make the coefficient of x^2=1

Step 2: Find $\left(\frac{b}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = 9$

Step 3: Add (and subtract) this constant

Step 4: Factor the perfect square

Step 5: Simplify

General→Factored

Factor to find zeros

$$f(x) = ax^2 + bx + c \rightarrow f(x) = a(x - x_1)(x - x_2)$$

$f(x) =$ ~~$2x^2 - 12x + 6$~~ $3x^2 + 3x - 6$

$f(x) = 3x^2 + 3x - 6$

$f(x) = 3(x^2 + x - 2)$

$\boxed{f(x) = 3(x+2)(x-1)}$

Factored → Standard

Find vertex

$f(x) = a(x - x_1)(x - x_2) \to f(x) = a(x - h)^2 + k$

$f(x) = 3(x - 1)(x + 2)$

$V = \left(\frac{x_1 + x_2}{2}, \text{plug in}\right)$

$= \frac{1 + (-2)}{2}, y$

$= -0.5, y$

$f(x) = 3(-0.5 - 1)(-0.5 + 2)$

$f(x) = -6.75$

$V = (-0.5, -6.75)$

$\boxed{f(x) = 3(x + 0.5)^2 - 6.75}$

Factored → General

JUST DO IT

$f(x) = a(x - x_1)(x - x_2) \to f(x) = ax^2 + bx + c$

$f(x) = 3(x - 1)(x + 2)$

$3(x^2 + x - 2)$

$\boxed{f(x) = 3x^2 + 3x - 6}$

LINEAR AND QUADRATIC INEQUALITIES

REGIONS OF THE CARTESIAN PLANE

(SECTION 5.10 IN WORKBOOK) PG.159

$<$ Less than Shade below/left Dashed line	$>$ exceeds Greater than Shade above/right Dashed line
$\leq$ at most, maximum, does not exceed Less than or equal Shade below/left Solid line	$\geq$ at least, minimum Greater than or equal Shade above/right Solid line

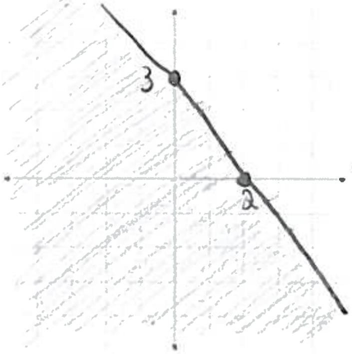
Note: To use the table to shade, y must be positive on the left side of the equation.

Another method is to plug in a point, usually (0, 0). If the point verifies the inequality, shade towards the origin (or the point you plugged in).

solid, shade below

Ex: Graph $3x + 2y - 6 \leq 0$

Linear



To graph:

Find x-int : $y = 0$

$3x + 2(0) - 6 \leq 0$

$\frac{3x}{3} \leq \frac{6}{3}$

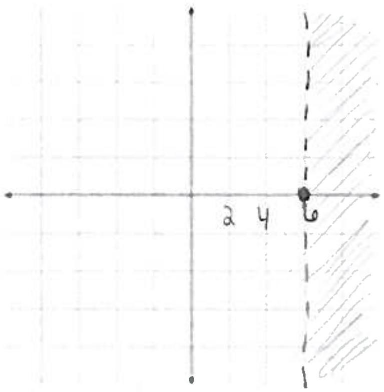
$x \leq 2$

Find y-int: $3(0) + 2y - 6 \leq 0$

$\frac{2y}{2} \leq \frac{6}{2}$

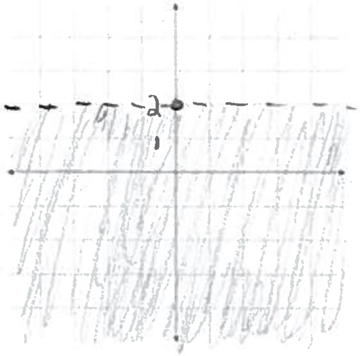
$y \leq 3$

Ex : Graph $x > 6$



Points on the line don't satisfy the inequality

Ex : Graph $y < 2$



Method 2

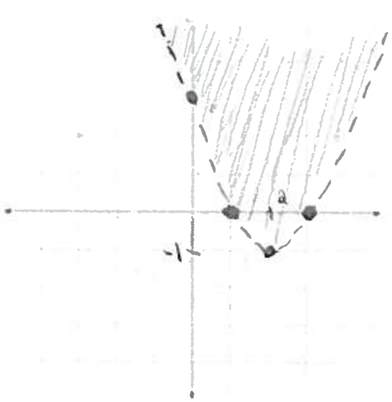
(0,0)

$0 > 0+0+3$

$0>3$

False (0,0) is not a solution to the inequality → shade away from (0,0)

Graph $y \overset{\text{dashed}}{>} x^2 - 4x + 3$



$V = (-\frac{b}{2a}, y)$

$V: \frac{-(-4)}{2(1)} = 2$

$y > (2)^2 - 4(2) + 3$

$y > -1$

$V = (2,-1)$

Zeros = $0 > x^2 - 4x + 3$

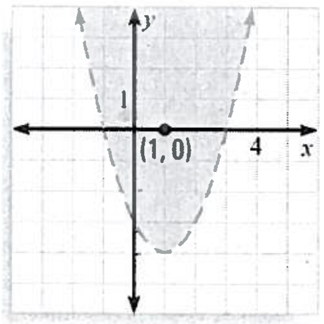
$0 > (x-3)(x-1)$

$x = 3 \quad x = 1$

Y-int = $y > (0)^2 - 4(0) + 3$

$y = 3$

Ex: Write the equation associated with this graph.



$V = (1,-4)$

Point = $(-1,0)$

$0 = a(-1-1)^2 \boxed{-4}$

$\frac{4}{4} = \frac{4a}{4}$

$a = 1$

$y = 1(x-1)^2 - 4$

- Dashed
- Shaded above

$y > 1(x-1)^2 - 4$

Ex : Name the variables and write the equation for each.

1. A parking lot has a surface area of 500m^2. Each truck occupies an area of 5m^2 and each car occupies 3m^2.

X: truck's amount

Y: car's amount

$5x + 3y \leq 500$

2. In a classroom, there are at least three times as many boys as girls.

B: # of boys

G: # of girls

$B \geq 3G$

Example to watch out

- $\frac{-y}{-1} > \frac{x^2}{-1}$

  $y < -x^2$

*Dividing or multiplying by negative one to an inequality, the symbol changes.

- $x^2 > y$

  $\frac{-y}{-1} > \frac{-x^2}{-1}$

  $y < x^2$

OR

$x^2 > y$

$y < x^2$

Quadratic Word Problems

1) Melanie was playing with a remote-controlled toy airplane. The plane took off from a balcony and landed on the ground 8 minutes later. Three minutes after taking off, the plane reaching a maximum altitude of 10 meters. In the diagram, the plane's altitude as a function of time is represented by a portion of a parabola. How high off the ground is the balcony located?

① Rule

$0 = a(8-3)^2 \boxed{+10}$ $\quad y = -\frac{2}{5}(x-3)^2+10$

$\frac{-10}{25} = \frac{a(25)}{25}$

$a = -\frac{2}{5}$

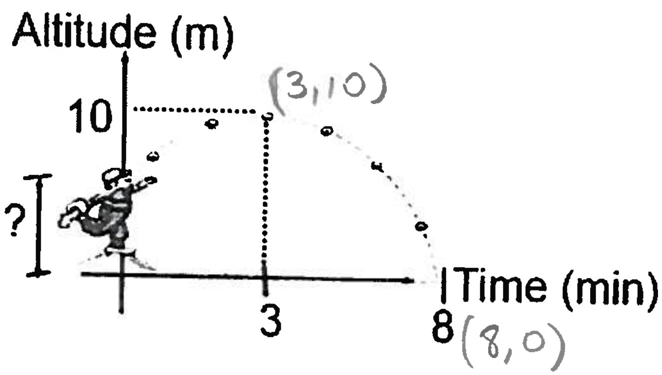
② Y-intercept

$y = -\frac{2}{5}(0-3)^2+10$

$y = -\frac{2}{5}(9)+10$

$y = 6.4$

It is located 6.4 m off the ground.

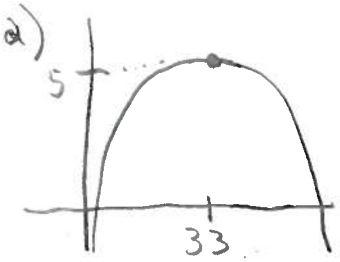


2) **Flying fish use their pectoral fins like airplane wings to glide through the air. The path of the fish can be modeled by the quadratic function**

$$f(x) = -\frac{5}{1089}(x-33)^2+5$$

When does the fish reach its maximum height, what is the fish's maximum height, and how far can it fly before it reenters the water? zeros

2)



max = 5

① Find zeros

$0 = \frac{-5}{1089}(x-33)^2+5$

$33 \pm \sqrt{\frac{-5}{-5/1089}}$

33 ± 33

$33+33$ $33-33$

$X=66$ $X=0$

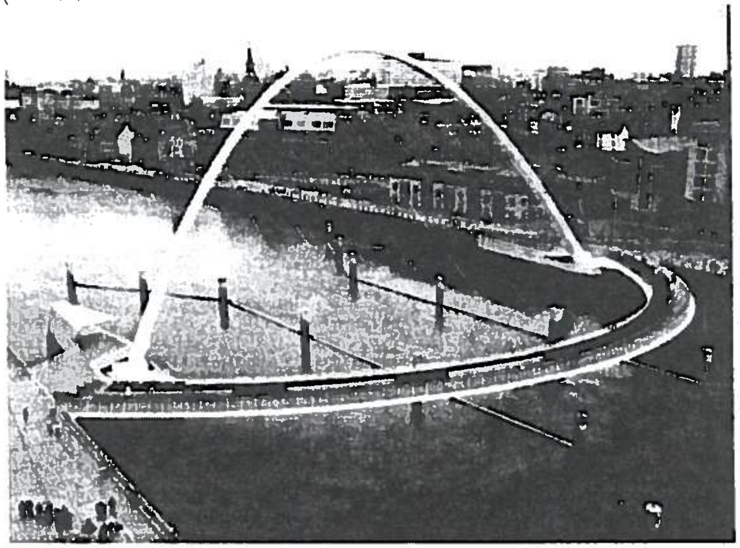
② Seconds

$66-0$

66 seconds

It can fly 66 seconds before it falls.

3) The arch of the Gateshead Millennium Bridge forms a parabola with the equation $f(x) = -0.016(x-52.5)^2 + 45$ where x is the horizontal distance in meters from the arch's left end and y is the distance in meters from the base of the arch. What is the width of the arch?

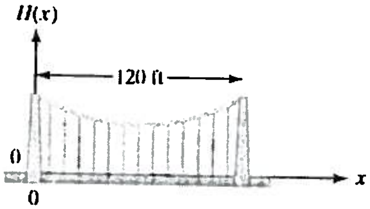


4) A suspension bridge is 120 ft long. Its supporting cable hands in a shape that resembles a parabola. The function defined by $H(x) = \frac{1}{90}(x-50)^2 + 30$ approximates the height of the supporting cable a distance of x ft from the end of the bridge.

a. What is the location of the vertex of the parabolic cable?

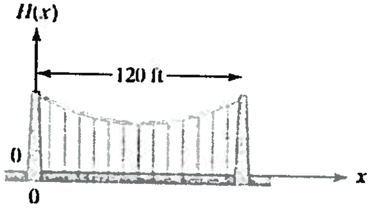
b. What is the minimum height of the cable?

c. How high are the towers at either end of the supporting cable?



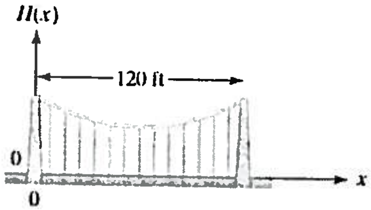
a. What is the location of the vertex of the parabolic cable?

$$H(x) = \frac{1}{90}(x-50)^2 + 30$$



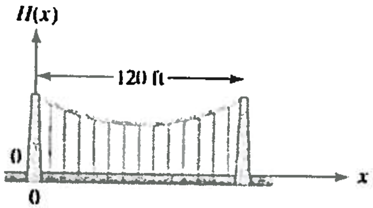
b. What is the minimum height of the cable?

$$H(x) = \frac{1}{90}(x-50)^2 + 30$$

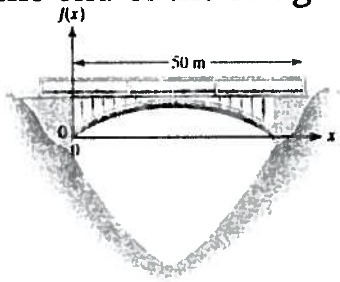


c. How high are the towers at either end of the supporting cable?

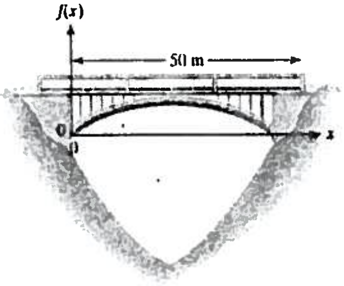
$$H(x) = \frac{1}{90}(x-50)^2 + 30$$



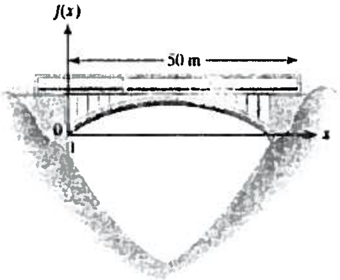
5) A 50-m bridge over a crevasse is supported by a parabolic arch. The function defined by $f(x) = -0.61(x - 25)^2 + 100$ approximates the height of the supporting arch x meters from the end of the bridge.



a. What is the location of the vertex of the arch? $f(x) = -0.61(x - 25)^2 + 100$



b. What is the maximum height of the arch (relative to the origin)?
$f(x) = -0.61(x - 25)^2 + 100$



3.5 QUADRATIC FUNCTIONS - (GENERAL FORM) PG101#1-13

The general form of an equation is

$$f(x) = ax^2 + bx + c$$

Vertex = $(-\frac{b}{2a}, y)$

Note: to find the y value, plug in your x value.

Note: We never have to find this form. It's given in a problem.

$$K = \frac{4ac - b^2}{4a}$$

Ex: For $f(x) = 2x^2 - 7x + 3$

a) Find the vertex
b) Find the zeros
c) Find the y-intercept
d) sketch
e) find the sign

a) $V: \left(-\frac{b}{2a}, y\right)$

$V: -\frac{(-7)}{2(2)} = 7,4$

$V = \left(\frac{7}{4}, y\right)$

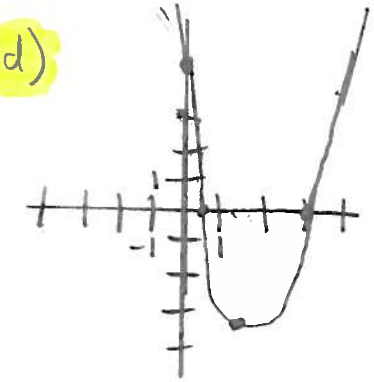
c) $F(0) = 2(0)^2 - 7(0) + 3$

$f(0) = 3$

Ex: For $f(x) = 2x^2 - 7x + 3$

a) Find the vertex
b) Find the zeros
c) Find the y-intercept
d) Sketch
e) Find the sign.

d)



e) ⊕ : $]-\infty, 0,5] \cup [3, \infty[$

⊖ : $[0,5, 3]$

$\left(\frac{7}{4}\right) = 2\left(\frac{7}{4}\right)^2 - 7\left(\frac{7}{4}\right) + 3$

$f\left(\frac{7}{4}\right) = -\frac{25}{8}$

For $f(x) = 2x^2 - 7x + 3$

a) Find the vertex
b) Find the zeros
c) Find the y-intercept
d) sketch
e) find the sign

$V = \left(\frac{7}{4}, -\frac{25}{8}\right)$

b) $0 = 2x^2 - 7x + 3$

$2x^2 - 6x - x + 3$

$2x(x-3) - 1(x-3)$

$(2x-1)(x-3)$

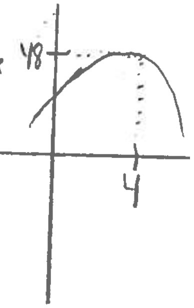
$x = \frac{1}{2}$ $\quad$ $x = 3$

$S = \left\{\frac{1}{2}, 3\right\}$

WORD PROBLEM

1) The polynomial function $h(t) = 24t - 3t^2$ describes the height $h(t)$ of a ball, expressed in meters, in relation to time t, expressed in seconds. [What is the greatest height reached by the ball?]

Find max



$h(t) = \underbrace{-3t^2}_{a} + \underbrace{24t}_{b} \underbrace{\quad}_{c=0}$

$V = \left(-\frac{b}{2a}, y\right)$

$V = -\frac{24}{2(-3)} = 4$

$V = (36, 48)$

$h(t) = -3(4)^2 + 24(4)$

$h(t) = 48$

Max = 48

2) On a forward somersault dive, Mike's height h meters above the water is given by $h = -5t^2 + 6t + 3$, where t is the time in seconds after he leaves the board.

a. Find mike's maximum height above the water. 4.8

b. How long does it take him to reach the maximum height? 0.6

c. How long is it before he enters the water? 1.58 seconds

d. How high is the board above the water? 3

$V = \left(\frac{-b}{2a}, y\right)$

$V: \frac{-6}{2(-5)} = 0.6$

$y = -5(0.6)^2 + 6(0.6) + 3$

$Y = 4.8$

$V: (0.6, 4.8)$

c) Find zeros

$0 = -5t^2 + 6t + 3$

$x = \frac{-6 \pm \sqrt{6^2 - 4(-5)(3)}}{2(-5)}$

$x = \frac{-6 \pm \sqrt{96}}{-10}$

$\frac{-6 + 9.8}{-10}$ $\frac{-6 - 9.8}{-10}$

$x = 0.38$ $\boxed{x = 1.58}$

d) Find Y

$h = 5(0)^2 + 6(0) + 3$

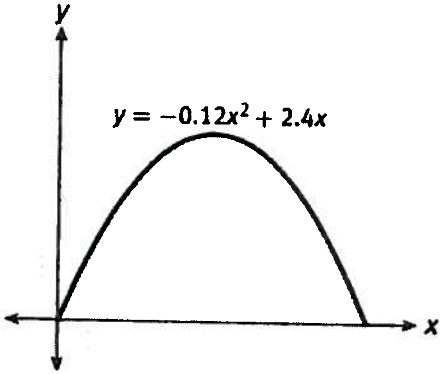
$Y = 3$

$h = -5t^2 + 6t + 3$

a. Find mike's maximum height above the water.
b. How long does it take him to reach the maximum height?
c. How long is it before he enters the water?
d. How high is the board above the water?

3) An architect is designing a tunnel and is considering using the function $f(x) = -0.12x^2 + 2.4x$ to determine the shape of the tunnel's entrance, as shown in the figure. In this model, $f(x)$ is the height of the entrance in feet and x is the distance in feet from one end of the entrance.

a) How wide is the tunnel's entrance at its base?



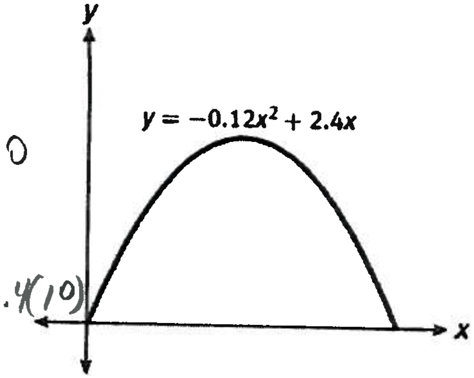
$y = -0.12x^2 + 2.4x$

$0 = -0.12x(x - 20)$

$x = 0 \quad x = 20$

✓ Width = 20

a) What is the vertex? What does it represent?



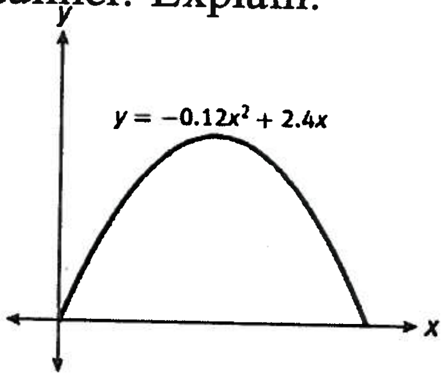
$V: \frac{-2.4}{2(-0.12)} = 10$

$y = -0.12(10)^2 + 2.4(10)$

$y = 12$

✓ $V = (10,12)$

b) Could a truck that is 14 feet tall pass through the tunnel? Explain.



✓ No, because the highest value is 12.

3.6 QUADRATIC FUNCTIONS - (FACTORED FORM)

PG105#1-16, PG109-1-18, PG.75

$\boxed{f(x) = a(x - x_1)(x - x_2)}$

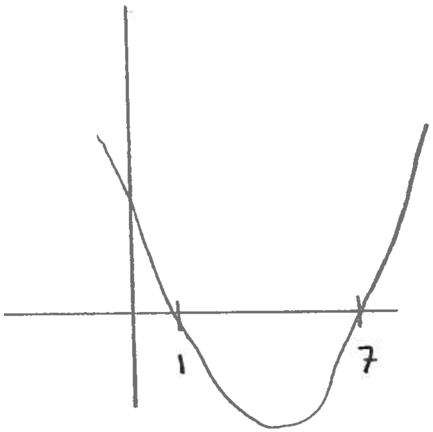
where x_1 and x_2 are zeros.

Note: To use this form, you need to have the 2 zeros and another random point.

Ex: Find the vertex for $f(x) = 3(x-1)(x-7)$

Zeros: 1, 7

Vertex $= \frac{7+1}{2} = \frac{8}{2} = 4$



To find k: plug in $x=4$

$f(4) = 3(4-1)(4-7)$

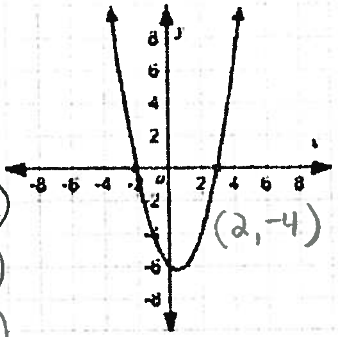
$f(4) = -27$

V: $(4, -27)$

Finding the rule – given the zeros and a point

Zeros: $-2, 3$

Point: $(2, -4)$



$f(x) = a(x-x_1)(x-x_2)$

$f(x) = a(x+2)(x-3)$

$-4 = a(2+2)(2-3)$

$-4 = a(4)(-1)$

$-4 = -4a$

$\frac{-4}{-4}$

$a = 1$

$f(x) = 1(x+2)(x-3)$

Find the rule:

x	y
-2	4
-1	0
2	0
3	4

Zeros: $-1, 2$

Point: $(-2, 4)$

$f(x) = a(x-x_1)(x-x_2)$

$f(x) = a(x+1)(x-2)$

$4 = a(-2+1)(-2-2)$

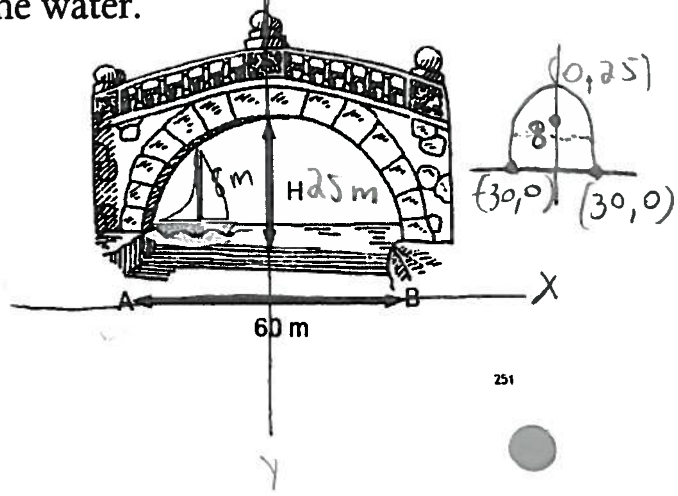
$\frac{4}{4} = \frac{4a}{4}$

$a = 1$

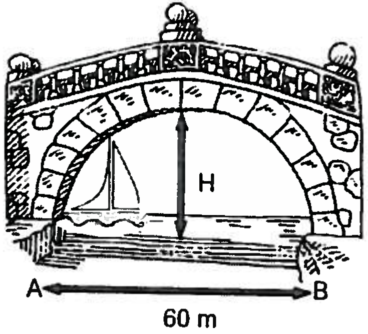
$f(x) = 1(x+1)(x-2)$

Word Problem

1) A bridge is supported by an arch in the shape of a parabola. The distance between points A and B is 60 m and the height H of the arch is 25 m. The top of a sailboat mast is 8m above the surface of the water.



a. What is the minimum distance (horizontal) from point A at which the sailboat can pass?



Zeros: -30, 30

Point: (0, 25)

$f(x) = a(x - x_1)(x - x_2)$

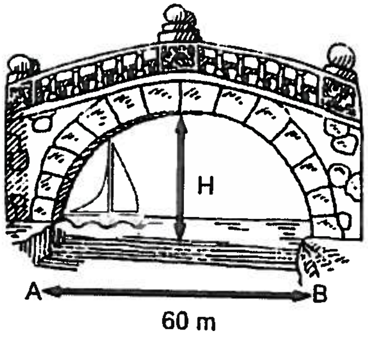
$25 = a(0+30)(0-30)$

$\frac{25}{-900} = -900a$

$a = -\frac{1}{36}$

$f(x) = -\frac{1}{36}(x+30)(x-30)$

b. What is the width of the arch at 8 m above the surface of the water?



$8 = -\frac{1}{36}(x+30)(x-30)$

$\frac{8}{-\frac{1}{36}} = \frac{-\frac{1}{36}(x^2 - 900)}{-\frac{1}{36}}$

$-288 = x^2 \boxed{-900}$

$\sqrt{612} = \sqrt{x^2}$

x = 24.7 m ~ 25 m

30 - 25 = 5 m from point A

SUMMARY: THE 3 FORMS OF THE QUADRATIC EQUATION ARE:

Standard form

$$f(x) = a(x - h)^2 + k$$

Must be given:

General $f(x) = ax^2 + bx + c$

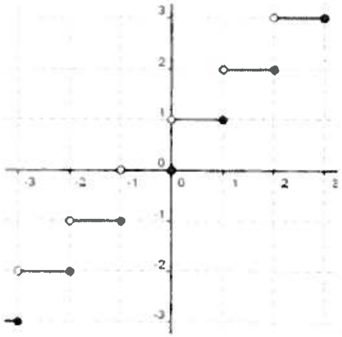
Factored

$$f(x) = a(x - x_1)(x - x_2)$$

Must be given:

CHAPTER 4:
GREATEST INTEGER FUNCTION

4.1 Step Function pg 116#1



f(2) = 2

f(-1)= -1

Find the interval of x for which f(x) = 2.

]3, -1]

- ● closes = included
- ○ opened = excluded

4.2 Greatest Integer of real number pg117#1-5

The greatest integer of a real number x noted $[x]$, returns the biggest integer less than or equal to x. It rounds down a real number to the nearest integer.

Round down to the next integer if it's a decimal

$[3] =$ 3

$[0.953] =$ 0

$[4.3] =$ 4

$[-4.3] =$ -5

$[-\frac{3}{4}] =$ -1

Calculate $f(-4.3)$, given

$$f(x) = -2[4(x+3)] - 5$$

$$f(-4.3) = -2[4(-4.3+3)] - 5$$

$$= -2[4(-1.3)] - 5$$

$$= -2[-5.2] - 5$$

Round down

$$= -2(-6) - 5$$

$$f(-4.3) = 7$$

What could the values of the question mark be?

$[?] = 4$

$[4, 5[$

$[?] = 10$

$[10, 11[$

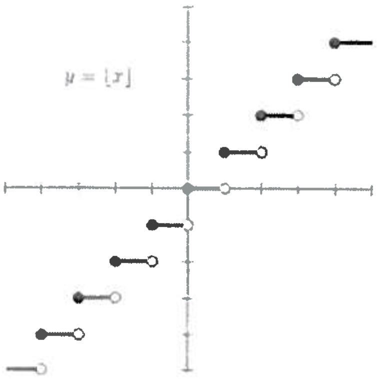
$[?] = 13$

$[13, 14[$

$[?] = a$

$[a, a+1[$

4.3 Basic Greatest Integer Function (GIF)



Find the properties

Dom f = $\mathbb{R}$

Ran f = $\mathbb{Z}$

Max, Min = None

y-int = 0

zeros = $[0,1[$

Inc= $\mathbb{R}$

Dec= None

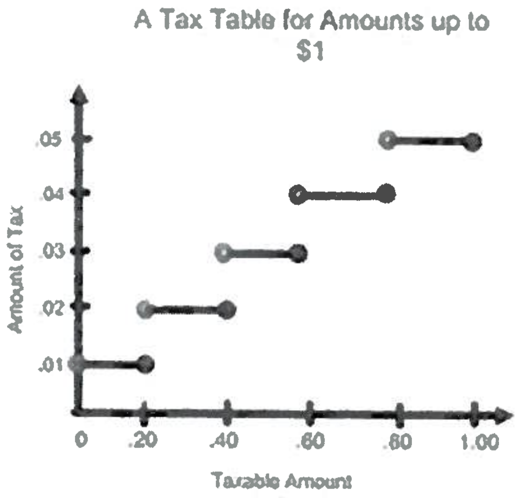
Pos= $[0,\infty[$

Neg= $]-\infty,0]$

4.4 Transformed greatest integer function

pg 123#1-12, pg127, pg 115

$$f(x) = a[b(x-h)] + k$$



To solve $[x] = a$,

we say x could be anything between a and the next integer $a + 1$, noted as

$$a \le x < a + 1,$$
$$a \in \mathbb{Z}$$

Therefore, to solve for x in $[x] = 2$,
The solution is

$$2 \le x < 3$$
OR
[2,3[

Solve for x

1) $\frac{-2[3x - 1]}{-2} = \frac{-10}{-2}$

$[3x-1] = 5$

$5 \le 3x-1 < 6$

* (3x-1) has to be between
5 and 6

$\overset{+1}{5} \le 3x-1 < \overset{+1}{6}$

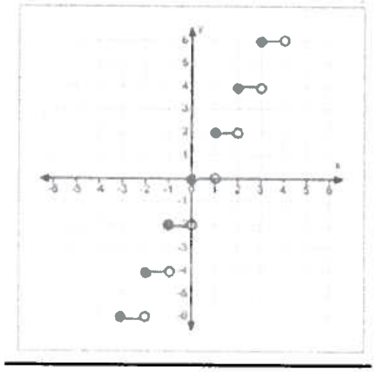
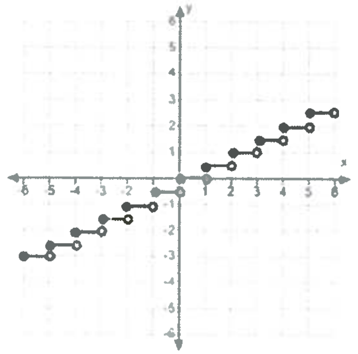
$\frac{6}{3} \le \frac{3x}{3} < \frac{7}{3}$

$2 \le x < 2.33$

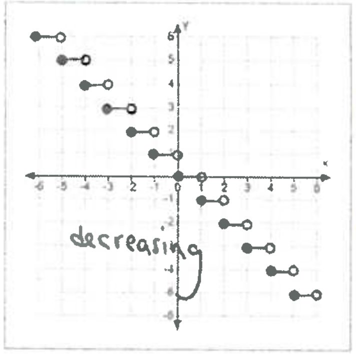
$[2, \frac{7}{3}[$

2) $[x - 2] = -1$

Parameter “a”:

$f(x) = 2[x]$	$f(x) = \frac{1}{2}[x]$
Vertical Stretch	Vertical Compression
	

$f(x) = -[x]$



Summary: Parameter a

height

determines the distance between each step (called the counterstep).

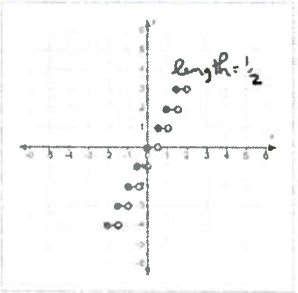
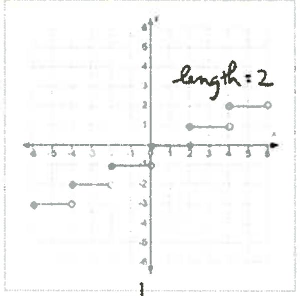
The counterstep = $|a|$ absolute value

If a , the function is increasing.

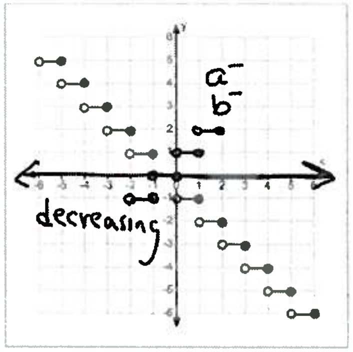
If a , the function is reflected about the x-axis and is decreasing.

Steps: ●—○

Parameter "b" : determines length of each step

$f(x) = [2x]$	$f(x) = [\frac{1}{2}x]$
Horizontal Compression	Horizontal Compression
 $b = 2$ length $= \frac{1}{2}$ RECIPROCAL	 $b = \frac{1}{2}$ length $= 2$

$f(x) = [-x]$



Summary: Parameter b determines the length of each step.

The length $= \frac{1}{|b|}$ i.e., the reciprocal of b

If b^+, the function is increasing.

Steps: ●—○

If b^-, the function is decreasing and is reflected about the y-axis.

Steps: ○—●

Notice that the origin, $(0,0)$, has been a solid (coloured) dot.

Parameter h translates the function h units left or right;
Parameter k translates the function k units up or down.

This means that (h,k) will always be a solid dot.

Graph

$$f(x) = 2\left[\frac{1}{4}(x-3)\right] + 1$$

Solution:

Establish a, b, h, k

$a = 2 \quad b = \frac{1}{4} \quad h = 3 \quad k = 1$

Starting dark dot: (h,k) (3,1) ●

Step height = $|a|$ $= |2| = 2$

Step length = $\frac{1}{|b|}$ = Reciprical of $\frac{1}{4} = 4$

If b<0 then ○——●

If b>0 then ●——○

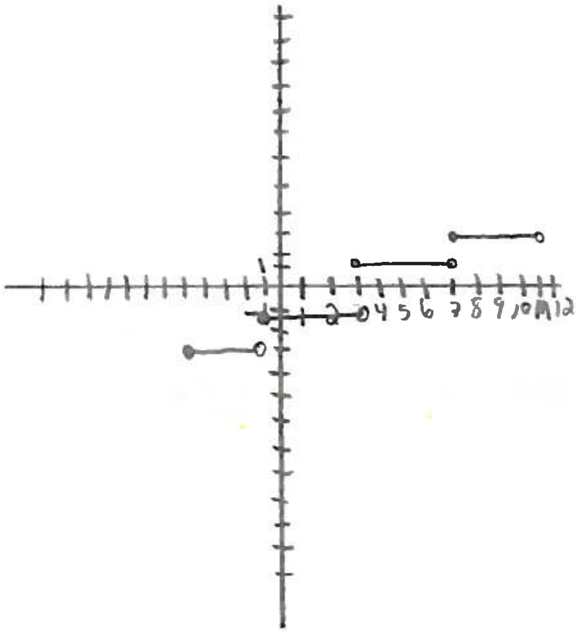
$b = \frac{1}{4}$

If ab>0 then ↗

If ab<0 then ↘

⊕ · ⊕

$2 \cdot \frac{1}{4} = ⊕$



Graph $f(x) = -1\left[\frac{1}{2}x + 1\right] - 1$

a = -1

$b = \frac{1}{2}$

h = -2

k = -1

$-1\left[\frac{1}{2}(x+2)\right]-1$

(h,k) = (-2,-1) ●

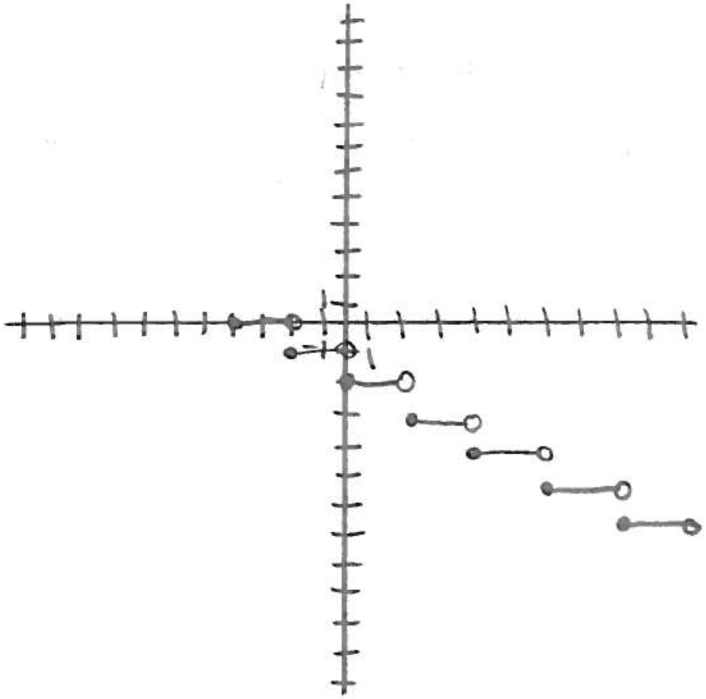
height = |a| = 1

length = $\frac{1}{|b|}$ = 2 ← reciprical of $\frac{1}{2}$

b > 0 = ●——○

ab < 0 = ↙

$(-1)\left(\frac{1}{2}\right) =$ ⊖



To find the equation, follow these steps:

1) Starting point. Choose any closed dot and call it (h, k)

2) Step height = $|a|$

3) Step length = $\frac{1}{|b|}$

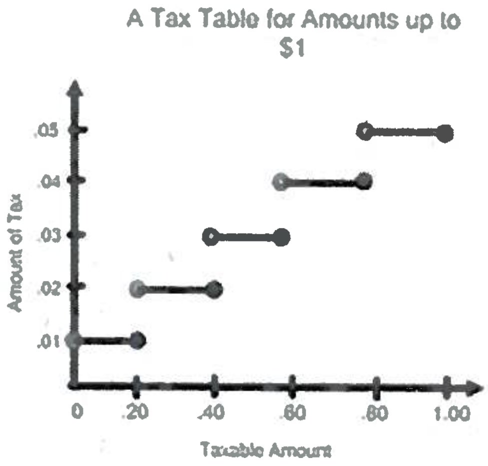
4) If $b < 0$ then ○—●

If b>0 then ●—○

5) If $ab > 0$ then ↗

If $ab < 0$ then ↘

Find the rule of this function:



① $(h,k) = (0.20, 0.01)$

② height $= |a| = 0.01$

③ length $= \frac{1}{|b|} = \frac{1}{0.20}$ ← isolate for $|b|$

length $= 5$

○—● $b < 0 = -5$

↗ ∴ $ab > 0$ → $a = -0.01$

⊖ ⊖ = ⊕ $b = -5$

FINAL ANSWER: $F(x) = -0.01[-5(x-0.2)] + 0.0$

Word Problem

A salesman is paid \$400 weekly plus a bonus of 100\$ for every 500\$ in sales.

k

a

b

$f(x) = 100\left[\frac{1x}{500}\right] + 400$

a) Write the equation.
b) Draw the graph.
c) If he sells 15 433\$ of merchandise in a week. What is his salary?
d) Is it possible for the salesman to receive a salary equal to \$672?

a) $f(x) = 100\left[\frac{1x}{500}\right] + 400$

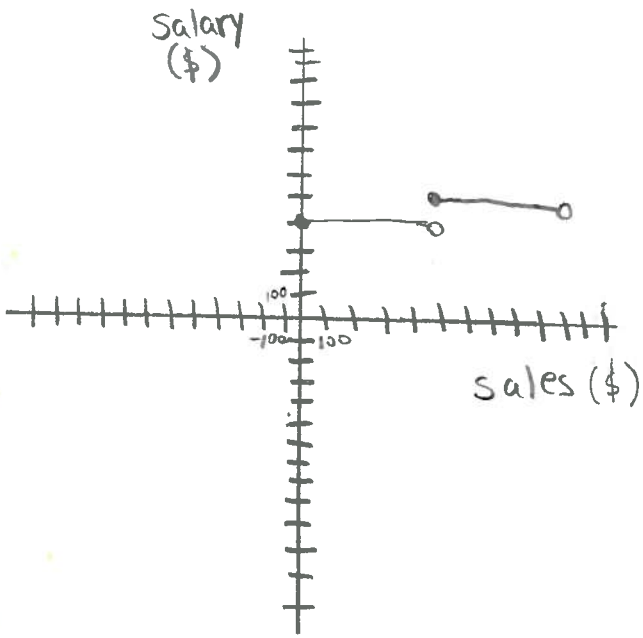
b) $(h,k) = (0, 400)$ •

$|a| = 100$

$|b| = 500$

$b > 0 =$ ●–○

$ab > 0 =$ ↗



A salesman is paid \$400 weekly plus a bonus of 100\$ for every 500\$ in sales.

a) Write the equation.
b) Draw the graph.
c) If he sells 15 433\$ of merchandise in a week. What is his salary?
d) Is it possible for the salesman to receive a salary equal to \$672?

c) $f(x) = 100\left[\frac{15\,433}{500}\right] + 400$

$f(x) = 100\left[30.86\right] + 400$

$f(x) = 100(30) + 400$

$f(x) = \$3400$

d) $672 = \overbrace{100\left[\frac{1x}{500}\right] \boxed{+400}}$

$\frac{272}{100} = \frac{100\left[\frac{x}{500}\right]}{100}$

$2.72 = \left[\frac{x}{500}\right]$

$\boxed{\text{Impossible}}$ the result of a greatest integer function cannot be a decimal (always integer)

A salesman is paid $400 weekly plus a bonus of 100$ for every 500$ in sales.

e) Write the equation.
f) Draw the graph.
g) If he sells 15 433$ of merchandise in a week. What is his salary?
h) Is it possible for the salesman to receive a salary equal to $672?