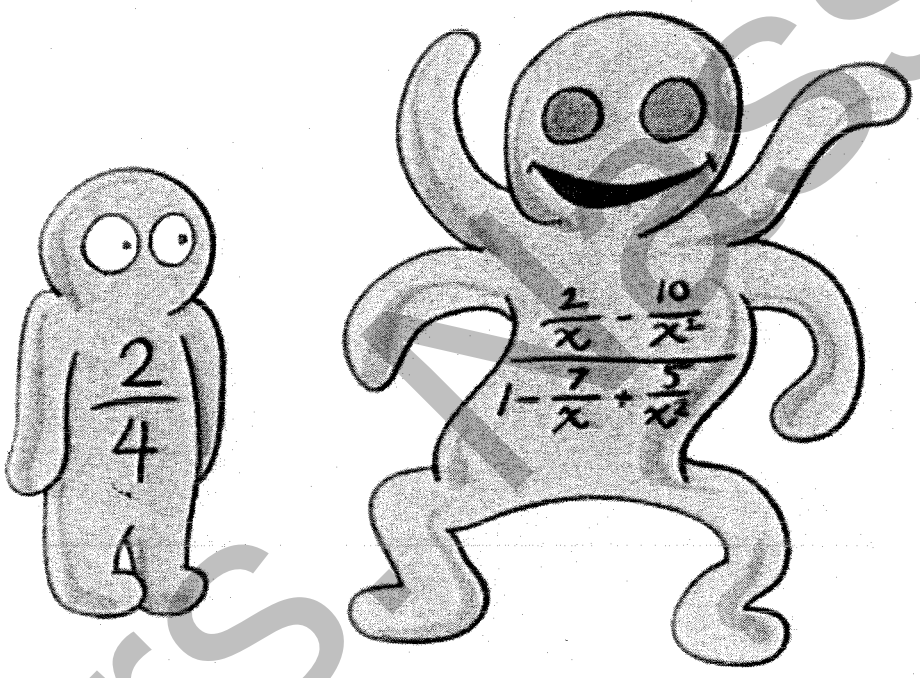
Chapter 1

Algebraic Expressions



Mrs. Nassif

Math 466

Summary Checklist

- ✓ **Factoring**
  - GCF
  - Sum/Product
  - Grouping
  - Perfect Square
  - Difference of Squares
- ✓ **Rational Expressions**
  - $+, -, \times, \div$
- ✓ **Solving Equations**
  - By Factoring & Zero Product Property
  - By the Quadratic Formula
- ✓ **Completing the Square**

Factoring Summary

Before factoring any polynomial, write the polynomial in **descending order** of one of the variables.

1. Factor out the Greatest Common Factor (GCF). Look for this in **every** problem. This includes factoring out a -1 if it precedes the leading term.

   Example: $-3x^2+12x-18=-3(x^2-4x+6)$

2. If there are **FOUR TERMS**, try to factor by grouping (GR).

   Example: $x^3+6x^2-2x-12$

   $x^3+6x^2 \quad -2x-12=$ *group the first two terms, last two terms*

   $x^2(x+6)-2(x+6)=$ *factor out GCF from each grouping*

   $(x+6)(x^2-2)$ *factor out the common grouping*

3. If there are **TWO TERMS**, look for these patterns:

x	x^2	x^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	
10	100	
11	121	
12	144	
13	169	
14	196	
15	225	

   a. The difference of squares (DOS) factors into conjugate binomials:

   $a^2-b^2=(a-b)(a+b)$

   Example: $9x^4-64y^2=(3x^2-8y)(3x^2+8y)$

   Note: a variable is a perfect square if the exponent is even

   b. The sum of squares does not factor:

   a^2+b^2 is prime

   Example: $9x^4+64y^2$ is PRIME

   c. The sum of cubes (SOC) or difference of cubes (DOC) factors by these patterns: each type contains a binomial (small bubble) times a trinomial (large bubble). Only the sign patterns differ between sum of cubes and difference of cubes.

   $a^3+b^3=(a+b)(a^2-ab+b^2)$

   Example : $(8x^3+27)=(2x+3)(4x^2-6x+9)$

   $a^3-b^3=(a-b)(a^2+ab+b^2)$

   Example : $(64x^6-125y^3)=(4x^2-5y)(16x^4+20x^2y+25y^2)$

   Note: a variable is a perfect cube if the exponent is a multiple of three

4. If there are **THREE TERMS**, look for these patterns:

   a. Quadratic trinomials of the form ax^2+bx+c *where* $a=1$ (QT $a=1$))factor into the product of two binomials (double bubble) where the factors of c must add to b.

      Example: $x^2-4x-12=(x-6)(x+2)$

   b. Quadratic trinomials of the form ax^2+bx+c *where* $a\neq 1$ (QT $a\neq 1$) eventually factor into the product of two binomials (double bubble), but you must first find the factors of *ac* that add to *b*, rewrite the original replacing *b* with these factors of *ac*, then factor by grouping to finally get to the double bubble.

      Example:

      $9x^2+15x+4 \qquad ac=(9)(4)=36$

      factors of 36 *that add to* 15: 12 *and* 3

      $9x^2+12x+3x+4=$

      $3x(3x+4)+1(3x+4)=$

      $(3x+4)(3x+1)$

   c. Quadratic square trinomials (QST) of the form ax^2+bx+c may factor into the square of a binomial. Look for the pattern where two of the terms are perfect squares, and the remaining term is twice the product of the square root of the squares:

      $a^2\pm 2ab+b^2=(a\pm b)^2$

      Example: $16x^2-40x+25=(4x-5)^2$

5. Factor all expressions completely. Sometimes, you will need to use two or three types of factoring in a single problem.

   Example:

   $-2x^4+32=$ *factor out the GCF of* -2

   $-2(x^4-16)=$ *factor the difference of squares*

   $-2(x^2-4)(x^2+4)=$ *factor the remaining difference of squares*

   $-2(x-2)(x+2)(x^2+4)$ (*remember that the sum of squares is prime*)

Skill Builder

8 Factoring Review Level 1

EVALUATED

Completely factor the following polynomials. SOLUTIONS

1. $x^3+3x^2+2x=?$ $x(x+2)(x+1)$
2. $6x^2y-6xy-12y=?$ $6y(x-4)(x+3)$
3. $2x^2-72=?$ $2(x-6)(x+6)$
4. $36x^2-4x^4y^2=?$ $4x^2(3-xy)(3+xy)$
5. $50x^3-40x^2+8x=?$ $2x(5x-2)(5x-2)$
6. $3x^3y+6x^2y^2+3xy^3=?$ $3xy(x+y)^2$
7. $32a+8ab-12b-48=?$ $4(4+b)(2a-3)$
8. $x^7-x^6+x^3-x^2=?$ $x^2(x^4+1)(x-1)$
9. $x^8-1=?$ $(x-1)(x+1)(x^2+1)(x^4+1)$
10. $4-y^2+x^2y^2-4x^2=?$ $(2-y+2x)(2+y-2x)$

9 Factoring Review Level 2

EVALUATED

Completely factor the following polynomials.

1. $(x-3)^2-16=?$ $(x+1)(x-7)$
2. $25-(x-2)^2=?$ $(7-x)(3+x)$
3. $(2x+1)^2-(3x-y)^2=?$ $(-x+1+y)(5x+1-y)$
4. $(x+3)^2-(x-y)^2=?$ $(3+y)(2x+3-y)$
5. $x^2+6xy+9y^2-25=?$ $(x+3y-5)(x+3y+5)$
6. $x^2-2xy+y^2-t^2=?$ $(x-y-t)(x-y+t)$
7. $4x^2-9y^2+12y-4=?$ $(2x-3y+2)(2x+3y$
8. $9w^2-x^2-4xy-4y^2=?$ $(3w-x-2y)(3w+x+2y$
9. $(x+3)^2-5x-15=?$ $(x+3)(x-2)$
10. $1-(x^2+y^2)-2xy=?$ $(1-x-y)(1+x+y)$

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Skill Builder

13 Factoring Review Level 3 ENRICHMENT

Factor these polynomials completely.

1. $x^8 - y^8 = ?$ $(x-y)(x+y)(x^2+y^2)(x^4+y^4)$
2. $x^2 - \frac{1}{4} = ?$ $(x-1/2)(x+1/2)$
3. $x^3 - \frac{8}{27} = ?$ $(x-2/3)(x^2+2/3x+4/9)$
4. $x^2 - 2xy + y^2 - t^2 = ?$ $(x-y-t)(x-y+t)$
5. $x^2 + 6xy + 9y^2 - 25 = ?$ $(x+3y-5)(x+3y+5)$
6. $9w^2 - x^2 - 4xy - 4y^2 = ?$ $(3w-x-2y)(3w+x+2y)$
7. $4x^2 - 9y^2 + 12y - 4 = ?$ $(2x-3y+2)(2x+3y-2)$
8. $(x+y)^2 - x - y = ?$ $(x+y)(x+y-1)$
9. $(x+3)^2 - 5x - 15 = ?$ $(x+3)(x-2)$
10. $1 - (x^2 + y^2) - 2xy = ?$ $(1-x-y)(1+x+y)$

14 Stating Restrictions EVALUATED

State the Restrictions on x.

1. $\frac{x+3}{x-4}$ $x \neq 4$
2. $\frac{x-5}{x}$ $x \neq 0$
3. $\frac{x+1}{4x}$ $x \neq 0$
4. $\frac{x-3}{-2x^2}$ $x \neq 0$
5. $\frac{x+5}{x^2-1}$ $x \neq 1, -1$
6. $\frac{x^2+6x+9}{x^2-5x+6}$ $x \neq 3, 2$
7. $\frac{3x^2-x-4}{4x^3-9x}$ $x \neq 0, 3/2, -3/2$
8. $\frac{x^2-25}{2x^3-x^2-6x}$ $x \neq 0, -3/2, 2$
9. $\frac{4x^3}{x^4-16}$ $x \neq 2, -2$
10. $\frac{3x^2}{4x^4-5x^3-6x^2}$ $x \neq 0, -3/4, 2$

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Section 8

1) $x(x^2+3x+2)$

$x(x+2)(x+1)$

2) $6x^2y-6xy-12y$

$6y(x^2-x-12)$

$6y(x-4)(x+3)$

3) $2x^2-72$

$2(x^2-36)$

$2(x-6)(x+6)$

4) $4x^2(9-x^2y^2)$

$4x^2(3-xy)(3+xy)$

5) $50x^3-40x^2+8x$

$2x(\cancel{50x^2}-40x+8)$

$2x(25x^2-20x+4)$

$2x(\cancel{5}x-2)(\cancel{5}x-2)$

6) $3xy(x^2+2xy+y^2)$

$3xy\cdot(x+y)(x+y)$

7) $4(8a+2ab-3b-12)$

$4(2a(4+b)-3(b+4))$

$4((4+b)(2a-3))$

8) $x^2(x^5-x^4+x-1)$

$x^2(x^4(x-1)+(x-1))$

$x^2(x^4+1)(x-1)$

9) x^8-1

$(x^4-1)(x^4+1)$

$(x^2-1)(x^2+1)(x^4+1)$

$(x-1)(x+1)(x^2+1)(x^4+1)$

10) $4-y^2+x^2y^2-4x^2$

$\cancel{4}-(y-2x)(y+2x)$

$4-(y^2-x^2y^2+4x^2)$

$4-(y-2x)^2$

$(2+(y-2x))(2+y-2x)$

$(2-y+2x)(2+y-2x)$

Section 9

① $((x-3)+4)(x-3-4)$
$(x+1)(x-7)$

② $25-(x-2)^2$

$(5-(x-2))(5+(x-2))$
$(5-x+2)(5-2+x)$
$(7-x)(3+x)$

③ $(2x+1-(3x-y))(2x+1+3x-y)$
$(x+1+y)(5x+1-y)$

④ $(x+3-(x-y))(x+3+x-y)$
$(3+y)(2x+3-y)$

⑤ $(x+3y)^2-25$
$(x+3y-5)(x+3y+5)$

⑥ $\frac{x^2-2xy+y^2-t^2}{(x-y)^2-t^2}$
$(x-y-t)(x-y+t)$

⑦ $4x^2-9y^2+12y-4$
$4x^2-(9y^2-12y+4)$
$4x^2-(3y-2)^2$
$(2x-(3y-2))(2x+3y-2)$

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(8) $9w^2 - x^2 - 4xy - 4y^2$

$9w^2 - (x^2 + 4xy + 4y^2)$

$9w^2 - (x+2y)^2$

$a^2 - b^2 = (a-b)(a+b)$

$(3w - (x+2y))(3w + (x+2y))$

(9) $(x+3)^2 - 5(x+3)$

$(x+3)[x+3-5]$

$(x+3)(x-2)$

(10) $1 - (x+y)^2 - 2xy$

$1 - x^2 - y^2 - 2xy$

$1 - (x^2 + 2xy + y^2)$

$1 - (x+y)^2$

$(1 + (x+y))(1 - (x+y))$

$(1 + x + y)(1 - x - y)$

Section B

① $x^8 - y^8$

$(x^4 - y^4)(x^4 + y^4)$

$(x^2 - y^2)(x^2 + y^2)(x^4 + y^4)$

$(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$

② $(x - 1/2)(x + 1/2)$

③ $x^3 - \left(\frac{2}{3}\right)^3 = \left(x - \frac{2}{3}\right)\left(x^2 + \frac{2}{3}x + \frac{4}{9}\right)$

④ $\dfrac{x^2 - 2xy + y^2 - t^2}{(x - y)^2 - t^2}$

$(x - y - t)(x - y + t)$

⑤ $x^2 + 6xy + 9y^2 - 25$

$(x + 3y)^2 - 25$

$(x + 3y - 5)(x + 3y + 5)$

⑥ $9w^2 - (x^2 + 4xy + 4y^2)$

$9w^2 - (x + 2y)^2$

$(9w - (x + 2y))(9w + (x + 2y))$

(7) $4x^2 - 9y^2 + 12y - 4$

$(2x + 4y)1$

$4x^2 + 12y - 9y^2 - 4$

$(2x - 3y)^2 - 4$

$4x^2 - (9y^2 - 12y + 4)$

$4x^2 - (3y - 2)^2$

$(2x - (3y - 2))(2x + 3y - 2)$

(8) $(x+y)^2 - (x+y)$

$(x+y)(x+y-1)$

(9) $(x+3)^2 - 5(x+3)$

$(x+3)(x+3-5)$

$(x+3)(x-2)$

(10) $1 - x^2 - y^2 - 2xy$

$1 - (x^2 + 2xy + y^2)$

$1 - (x+y)^2 = (1-x-y)(1+x+y)$

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Section 14

① $x-4=0$
$x \neq 4$

② $x \neq 0$

③ $4x \neq 0$
$x \neq 0$

④ $x \neq 0$

⑤ $(x-1)(x+1) \neq 0$
$x-1 \neq 0$ $\quad x+1 \neq 0$
$x \neq 1$ $\quad x \neq -1$

⑥ $(x-3)(x-2) \neq 0$
$x-3 \neq 0$ $\quad x-2 \neq 0$
$x \neq 3$ $\quad x \neq 2$

⑦ $x(4x^2-9) \neq 0$
$x \neq 0$ $\quad 4x^2-9 \neq 0$
$(2x-3)(2x+3) \neq 0$
$2x-3 \neq 0$ $\quad 2x+3 \neq 0$
$x \neq 3/2$ $\quad x \neq -3/2$

(8) $2x^3 - x^2 - 6x$

$x(2x^2 - x - 6)$

$x(2x+3)(x-2) = 0$

$x \neq 0$ $\quad 2x = -3$ $\quad x \neq 2$

$\frac{2x}{2} = \frac{-3}{2}$

$x \neq -3/2$

(9) $x^4 - 16 \neq 0$

$(x^2-4)(x^2+4) = 0$

$x^2 - 4 \neq 0$ $\quad x^2 + 4 \neq 0$

$(x-2)(x+2) \neq 0$ $\quad x^2 \neq -4$

$x - 2 \neq 0$ $\quad x + 2 \neq 0$

$x \neq 2$ $\quad x \neq -2$

(10) $4x^4 + 5x^3 - 6x^2 \neq 0$

$x^2(4x^2 + 5x - 6) \neq 0$

$x^2(4x+3)(x-2) \neq 0$

$x^2 \neq 0$ $\quad 4x + 3 \neq 0$ $\quad x - 2 \neq 0$

$x \neq 0$ $\quad x \neq -3/4$ $\quad x \neq 2$

FACTORING TEST REVIEW

Name: Solutions

Math 306

Factor the following polynomials completely.

1- GCF and Grouping

a) $ax + ay$

$a(x+y)$

b) $x^4 + 3x^3 - 6x^2$

$x^2(x^2 + 3x - 6)$

d) $10at^4 + 65a^2t^2 - 30a^3t$

$5at(2t^3 + 13at - 6a^2)$

e) $14b^4c^2 - 21b^3c^3 + 49b^2c^4$

$7b^2c^2(2b^2 - 3bc + 7c^2)$

f) $a(x+y) - a$

$a[(x+y) - 1]$

$a(x+y-1)$

g) $2\pi r^2 + 2\pi rh$

$2\pi r(r^2 + h)$

h) $(x-y) + (x-y)(x+y)$

$(x-y)[1 + (x+y)]$

$(x-y)(1+x+y)$

i) $8(a-2) + (a-2)(a-8)$

$(a-2)[8 + (a-8)]$

$(a-2)(a)$

$a(a-2)$

j) $12d^2c + 6d^2 - 4c^2 - 2c$

$6d^2(2c+1) - 2c(2c+1)$

$(2c+1)(6d^2 - 2c)$

$2(3d^2 - c)(2c+1)$

k) $4x^2 + 16x - 3x - 12$

$4x(x+4) - 3(x+4)$

$(x+4)(4x-3)$

l) $3d^2(4c+2) - c(4c+2)$

$(4c+2)(3d^2 - c)$

$2(2c+1)(3d^2 - c)$

m) $6(2a-5) + (5b+4)(2a-5)$

$(2a-5)[6 + (5b+4)]$

$(2a-5)(5b+10)$

$5(b+2)(2a-5)$

n) $(x-3)(2a+3) - (x-3)(a+7)$

$(x-3)[(2a+3) - (a+7)]$

$(x-3)[2a+3-a-7]$

$(x-3)(a-4)$

o) $8x(y+3) - (4y+x)(y+3)$

$(y+3)[8x - (4y+x)]$

$(y+3)[8x - 4y - x]$

$(y+3)(7x - 4y)$

p) $2x^3 + 6xy - 10x - x^2 - 3y + 5$

$2x(x^2 + 3y - 5) + 1(x^2 + 3y - 5)$

$(x^2+3y-5)(2x+1)$

q) $a(x+y) - b(x+y) + c(x+y)$

$(x+y)(a-b+c)$

2- Simple TRINOMIALS: x^2+bx+c

a) x^2+7x+6

$(x+6)(x+1)$

b) x^2-5x-6

$(x-6)(x+1)$

c) $-24-5y+y^2$

$y^2-5y-24$

$(y+3)(y-8)$

d) $x^2+13xy+12y^2$

$(x+12y)(x+y)$

e) $m^2-21mn+110n^2$

$(m-11n)(m-10n)$

f) $-40y^2-3xy+x^2$

$x^2-3xy-40y^2$

$(x-8y)(x+5y)$

g) x^6+19x^3+70

$(x^3+14)(x^3+5)$

3- GCF and Simple TRINOMIALS x^2+bx+c

a) $2p^2-6p-36$

$2(p^2-3p-18)$

$2(p-6)(p+3)$

b) x^3+16x^2-36x

$x(x^2+16x-36)$

$x(x+18)(x-2)$

c) $2m^2+14mn+12n^2$

$2(m^2+7mn+6n^2)$

$2(m+6n)(m+1n)$

d) $12m^3+48m+48m^2$

$12m(m^2+4+4m)$

$12m(m^2+4m+4)$

$12m(m+2)(m+2)$

$12m(m+2)^2$

e) $2x^3y^2+30y^2x^2+72y^2x$

$2y^2x(x^2+15x+36)$

$2y^2x(x+3)(x+12)$

f) $-x^2+21x-68$

$-1(x^2-21x+68)$

$-1(x-17)(x-4)$

4- Complex TRINOMIALS: ax^2+bx+c

a) $2m^2+5m+2$

$(2m+1)(m+2)$

b) $2y^2-y-3$

$(2y-3)(y+1)$

c) $6x^2-7x+2$

$(3x-2)(2x-1)$

d) $9a^2-6a+1$

$(3a-1)(3a-1)$

$(3a-1)^2$

e) $6x^2-25x-9$

$(2x-9)(3x+1)$

f) $100x^2-60x+9$

$(10x-3)(10x-3)$

$(10x-3)^2$

5- PERFECT SQUARES (some may also be found in other sections)

a) $16x^2 - 16x + 4$

$4(4x^2 - 4x + 1)$

$4(2x-1)^2$

b) $5x^2 - 40x + 80$

$5(x^2 - 8x + 16)$

$5(x-4)^2$

c) $9x^2 + 30xy + 25y^2$

$(3x+5y)(3x+5y)$

$(3x+5y)^2$

6- DIFFERENCE OF SQUARES

a) $121x^6y^2 - 144x^2y^2$

$(11x^3y - 12xy)(11x^3y + 12xy)$

b) $28a^3 - 7a$

$7a(4a^2 - 1)$

$7a(2a-1)(2a+1)$

c) $36a^2 - (a+b)^2$

$[6a - (a+b)][6a + (a+b)]$

$(6a - a - b)(6a + a + b)$

$(5a - b)(7a + b)$

d) $81x^4 - 16y^8$

$(9x^2 - 4y^4)(9x^2 + 4y^4)$

$(3x - 2y^2)(3x + 2y^2)(9x^2 + 4y^4)$

e) $(3x+2)^2 - (x+1)^2$

$[(3x+2) - (x+1)][(3x+2) + (x+1)]$

$[3x + 2 - x - 1][3x + 2 + x + 1]$

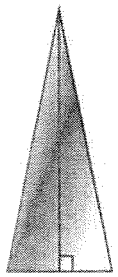
$(2x+1)(4x+3)$

f) $\frac{x^2}{196} - 169y^4$

$\left(\frac{x^2}{14} - 13y^2\right)\left(\frac{x^2}{14} + 13y^2\right)$

7- APPLICATIONS

A) The isosceles triangle illustrated below has an area of $(1.5x^2 - 2x - 2)$ cm². Knowing that it's perimeter is $(11x - 16)$ cm, what algebraic expression best represents the length of the "slanted" congruent sides of this triangle? (Reminder: $Area = \frac{Base \cdot Height}{2}$)



$A = \frac{B \cdot h}{2}$

$2A = B \cdot h$

$3x^2 - 4x - 4 = B \cdot h$

$(3x+2)(x-2)$

Height · Base

$(Perim - Base) \div 2 = side$

$\frac{11x - 16 - (x-2)}{2}$

$\frac{10x - 14}{2}$

$5x - 7$ cm

B) Two different sized rectangles share the same width.
One rectangle has an area of $x^2 + 10x + 24$ m² and the other has an area of $x^2 + 13x + 36$ m².
What is the length of the bigger rectangle?

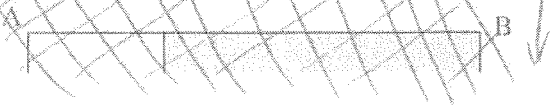
$A = x^2 + 10x + 24$	$A = x^2 + 13x + 36$
$(x+6)(x+4)$	$(x+9)(x+4)$

? ←

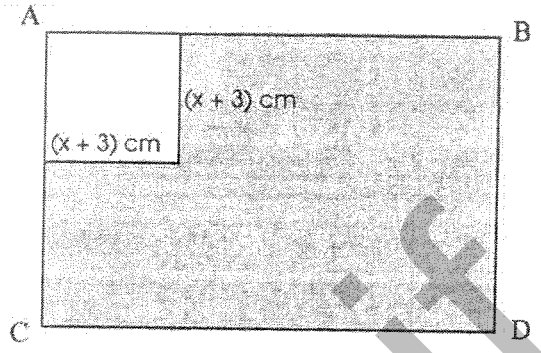
① The common dimension for both □'s is the height. Therefore, $(x+4)$ is the height

② Length of bigger triangle is $(x+9)$ m

C) Show that the area of the shaded region is equal to $3(x+12)(x-8)$ cm².



C) Show that the area of the **shaded region is equal to 3(x + 12)(x - 8) cm².**



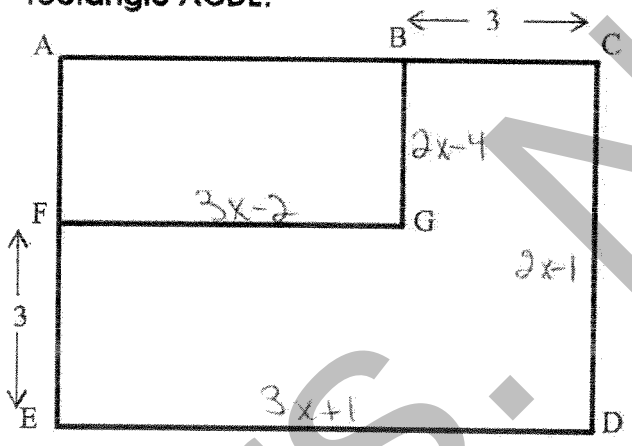
Area of rectangle ABCD = $(4x^2 + 18x - 279)$ cm²

① $A_{square} = (x+3)(x+3)$
$= (x^2+6x+9)\ cm^2$

② $A_{shaded} = A_{rect.} - A_{square}$
$= 4x^2+18x-279-(x^2+6x+9)$
$= 4x^2+18x-279-x^2-6x-9$
$= 3x^2+12x-288\ cm^2$

Factored answer
$3(x^2+4x-96)$
$3(x+12)(x-8)\ cm^2$

D) Given the diagram below. The polynomial **$6x^2 - x - 1$** represents the **area of the rectangle ACDE.**



$m\overline{BC} = 3\ units$

$m\overline{EF} = 3\ units$

What is the polynomial that represents the **area of rectangle ABGF?**

① Dimensions of Rect. ACDE
$6x^2 - x - 1$
$(3x+1)(2x-1)$
Base · Height

② Dimensions of Rect. ABGF
$3x+1-3 = 3x-2$ units (Base)
$2x-1-3 = 2x-4$ units (Height)

③ Area = $(3x-2)(2x-4)$
$= 6x^2-12x-4x+8$
$= 6x^2-16x+8\ units^2$

Kuta Software - Infinite Algebra 2

Name_____

Date_____ Period____

Rational Expressions

State the excluded values for each.

1) $\frac{60x^3}{12x}$

$\{0\}$

2) $\frac{70v^2}{100v}$

$\{0\}$

3) $\frac{m+7}{m^2+4m-21}$

$\{-7, 3\}$

4) $\frac{n^2+6n+5}{n+1}$

$\{-1\}$

5) $\frac{35x-35}{25x-40}$

$\left\{\frac{8}{5}\right\}$

6) $\frac{-n^2+16n-63}{n^2-2n-35}$

$\{-5, 7\}$

Simplify each and state the excluded values.

7) $\frac{p+4}{p^2+6p+8}$

$\frac{1}{p+2}$; $\{-2, -4\}$

8) $\frac{9}{15a-15}$

$\frac{3}{5(a-1)}$; $\{1\}$

9) $\frac{2a^2+10a}{3a^2+15a}$

$\frac{2}{3}$; $\{0, -5\}$

10) $\frac{p^2-3p-10}{p^2+p-2}$

$\frac{p-5}{p-1}$; $\{-2, 1\}$

11) $\frac{x^2+x-6}{x^2+8x+15}$

$\frac{x-2}{x+5}$; $\{-3, -5\}$

12) $\frac{a^2+5a+4}{a^2+9a+20}$

$\frac{a+1}{a+5}$; $\{-4, -5\}$

13) $\frac{x^2-2x-15}{x^2-6x+5}$

$\frac{x+3}{x-1}$; $\{1, 5\}$

14) $\frac{10x-6}{10x-6}$

1; $\left\{\frac{3}{5}\right\}$

15) $\frac{(v-7)(v+8)}{(v+8)(v-10)} \div \frac{1}{v-10}$

$v-7$; $\{-8, 10\}$

16) $\frac{n+3}{n+2} \div \frac{(n-1)(n+3)}{(n-1)^2}$

$\frac{n-1}{n+2}$; $\{-2, 1, -3\}$

17) $\frac{x+3}{4} \cdot \frac{3(x-6)}{3(x+3)}$

$\frac{x-6}{4}$; $\{-3\}$

18) $\frac{x-8}{(x+6)(x-8)} \cdot \frac{4x(x+10)}{x+10}$

$\frac{4x}{x+6}$; $\{-6, 8, -10\}$

19) $\frac{2b^2-12b}{b+5} \div \frac{b-6}{b+5}$

$2b$; $\{-5, 6\}$

20) $\frac{1}{n+9} \div \frac{6-n}{3n-18}$

$-\frac{3}{n+9}$; $\{-9, 6\}$

21) $\frac{28-7b}{b-4} \cdot \frac{1}{b+10}$

$\frac{7}{b+10}$; $\{4, -10\}$

22) $\frac{2}{v^2-12v+27} \cdot \frac{v^2-12v+27}{3}$

$\frac{2}{3}$; $\{3, 9\}$

23) $\frac{1}{5p^2} \div \frac{9p-36}{5p^3-35p^2}$

$\frac{p-7}{9(p-4)}$; $\{0, 7, 4\}$

24) $\frac{8-7x-x^2}{x+8} \cdot \frac{x+5}{9x-9}$

$\frac{(x+5)}{9}$; $\{-8, 1\}$

25) $\frac{x^2-16}{9-x} \cdot \frac{x^2+x-90}{x^2+14x+40}$

$-(x-4)$; $\{9, -4, -10\}$

26) $\frac{10x^2-20x}{40x^3-80x^2} \cdot \frac{16x^3+80x^2}{6x+30}$

$\frac{2x}{3}$; $\{0, 2, -5\}$

p. 3

Kuta Software - Infinite Algebra 2

Name_____

Adding/Subtracting Rational Expressions

Date_____ Period____

Simplify each expression.

1) $\frac{u-v}{8v} + \frac{6u-3v}{8v}$

$\frac{7u-4v}{8v}$

2) $\frac{m-3n}{6m^3n} - \frac{m+3n}{6m^3n}$

$-\frac{1}{m^3}$

3) $\frac{5}{a^2+3a+2} + \frac{5a+1}{a^2+3a+2}$

$\frac{6+5a}{a^2+3a+2}$

4) $\frac{5}{10n^2+16n+6} + \frac{n-6}{10n^2+16n+6}$

$\frac{-1+n}{10n^2+16n+6}$

5) $\frac{r+6}{3r-6} + \frac{r+1}{3r-6}$

$\frac{2r+7}{3r-6}$

6) $\frac{x+2}{2x^2+13x+20} - \frac{x+3}{2x^2+13x+20}$

$\frac{1}{2x^2+13x+20}$

7) $\frac{6}{x-1} - \frac{5x}{4}$

$\frac{24-5x^2+5x}{4(x-1)}$

8) $6 - \frac{x+5}{(7x-5)(x+4)}$

$\frac{42x^2+137x-125}{(7x-5)(x+4)}$

9) $\frac{3}{x+7} + \frac{4}{x-8}$

$\frac{7x+4}{(x+7)(x-8)}$

10) $\frac{3}{4v^2+4v} - \frac{7}{2}$

$\frac{3-14v^2-14v}{4v(v+1)}$

11) $\frac{7}{3} - \frac{8}{12x-8}$

$\frac{21x-20}{3(3x-2)}$

12) $\frac{5}{n+5} + \frac{4n}{2n+6}$

$\frac{15n+15+2n^2}{(n+3)(n+5)}$

13) $\frac{2x}{5x+4} + \frac{6x}{2x+3}$

$\frac{34x^2+30x}{(5x+4)(2x+3)}$

14) $\frac{2}{3x^2+12x} + \frac{8}{2x}$

$\frac{50+12x}{3x(x+4)}$

15) $\frac{7n}{n+1} + \frac{8}{n-7}$

$\frac{7n^2-41n+8}{(n+1)(n-7)}$

16) $\frac{2}{n+8} + \frac{4}{n+1}$

$\frac{6n+34}{(n+1)(n+8)}$

17) $\frac{3}{8} - \frac{3}{3x+4}$

$\frac{9x-12}{8(3x+4)}$

18) $\frac{3}{b-8} + \frac{7}{b+3}$

$\frac{10b-47}{(b+3)(b-8)}$

19) $\frac{3}{x+6} + \frac{7}{x-2}$

$\frac{10x+36}{(x+6)(x-2)}$

20) $\frac{4}{x+1} - \frac{2}{x+2}$

$\frac{2x+6}{(x+1)(x+2)}$

21) $\frac{5n+5}{5n^2+35n-40} + \frac{7n}{3n}$

$\frac{52n-53+7n^2}{3(n+8)(n-1)}$

22) $\frac{3}{n-5} + \frac{6}{3n-8}$

$\frac{15n-54}{(3n-8)(n-5)}$

23) $\frac{\frac{25}{4}}{\frac{1}{5} - \frac{4}{25}}$

$\frac{625}{4}$

24) $\frac{8}{\frac{4}{9} + \frac{16}{9}}$

$\frac{18}{5}$

25) $\frac{\frac{a}{25} - \frac{a}{5}}{-\frac{4}{25}a}$

26) $\frac{\frac{5}{4}}{\frac{5}{m} - \frac{4}{m}}$

$\frac{5m}{4}$

Critical thinking questions:

27) Simplify: $\frac{a}{b} + \frac{c}{d}$

$\frac{ad+bc}{bd}$

28) Split into a sum of two rational expressions with unlike denominators:

$\frac{2x+3}{x^2+3x+2}$

Many solutions. Ex: $\frac{1}{x+1} + \frac{1}{x+2}$

PRACTICE ANSWERS

1. $\frac{6x-4}{3}$
2. $\frac{x+5}{7}$
3. $\frac{x+17}{x}$
4. $\frac{1}{2}$
5. $\frac{1}{4}$
6. $\frac{x+6}{x+5}$
7. $\frac{x+10}{x+4}$
8. $\frac{4}{x+8}$
9. $\frac{6}{x-9}$
10. $\frac{4}{x-1}$
11. $\frac{3x+13}{x+5}$
12. $\frac{1}{x-2}$
13. $\frac{1}{x-8}$
14. $\frac{1}{x+6}$
15. $\frac{1}{x-4}$
16. $\frac{1}{x-7}$
17. $\frac{1}{x+3}$
18. $\frac{1}{x+5}$
19. $\frac{14}{3x}$
20. $\frac{30x+7}{5x^2}$
21. $\frac{21x+6}{7x^2}$
22. $\frac{24x+7}{6x^2}$
23. $\frac{12y^2-5x}{10x^3y^3}$
24. $\frac{10x+7y}{2x^2y}$
25. $\frac{2xy+x}{y}$
26. $\frac{15x-11}{12x}$
27. $\frac{35x-12}{15x^2}$
28. $\frac{8x+5}{x}$
29. $\frac{7x-67}{x-9}$
30. $\frac{1}{x+6}$
31. $\frac{1}{x+4}$
32. $\frac{1}{x+3}$
33. $\frac{2x^2-4x+5}{(x+2)(x-2)(x+3)}$
34. $\frac{13-7x}{(x+1)(x-3)(x+3)}$
35. $\frac{2x^2-2x+11}{(x+2)(x-2)(x+1)}$
36. $\frac{-2x^2+9x-8}{(x-4)(x-2)(x+2)}$
37. $\frac{2x^2+12x+13}{(x-1)(x+1)(x+4)}$
38. $\frac{-x+9}{(x+3)^2(x-3)}$
39. $\frac{2x^2-13x+7}{(x+3)(x-1)(x-3)}$
40. $\frac{2x^2+7x+10}{(x+6)(x+4)(x-4)}$
41. $\frac{10x-16}{(x-4)(x-1)(x+2)(x-2)}$
42. $\frac{21x-45}{(x-6)(x-1)(x+3)(x-3)}$
43. $\frac{4x^2-2x-14}{(x+2)(x-2)}$
44. $\frac{3x^2-3x-21}{(x+3)(x-3)}$
45. 0
46. $\frac{-x}{(x+3)(x-1)}$
47. $\frac{-3x^2-3x-4}{(x+1)(x-1)}$
48. $\frac{-14x^2-3x+3}{(2x+1)(2x-1)}$
49. $\frac{4x+26}{(x+3)(x+2)(x+1)(x-4)}$
50. $\frac{-6x+42}{(x-3)(x-2)(x+1)(x+3)}$

p3

Answers to Solving Quadratic Equations and Factoring Expressions

1. $x(x+4) = (x-1)(x+3) + 17$

$x^2 + 4x = x^2 + 2x - 3 + 17$

$x^2 + 4x = x^2 + 2x + 14$

$4x = 2x + 14$

$2x = 14$

$x = 7$

Perimeter, smaller rectangle $= 2(x-1) + 2(x+3)$

$= 2(6) + 2(10)$

$= 12 + 20$

$= 32$ cm

2. $A = 6x^2 + 7x - 20$ $\quad m \times n = -120$, $m + n = 7$, $15, -8$

$A = 6x^2 + 15x - 8x - 20$

$A = 3x(2x+5) - 4(2x+5)$

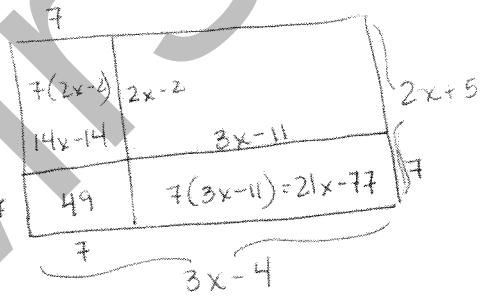
$A = (2x+5)(3x-4)$

New dimensions: $2x + 5 - 7 = 2x - 2$

$3x - 4 - 7 = 3x - 11$

Verification: $(2x-2)(3x-11) = 6x^2 - 22x - 6x + 22$

$= 6x^2 - 28x + 22$



$(6x^2 + 7x - 20) - (21x - 77) - (14x - 14) - 49$

$= 6x^2 + 7x - 20 - 21x + 77 - 14x + 14 - 49$

$= 6x^2 - 28x + 22$ ✓

3.

	Now	Then
Jerry	$x+4$	$x+11$
Gloria	x	$x+7$

$(x+11)(x+7) = 621$

$x^2 + 18x + 77 = 621$

$x^2 + 18x = 544$

$18 \div 2 = 9$

$9^2 = 81$

$x^2 + 18x + 81 = 544 + 81$

$(x+9)^2 = 625$

$x+9 = \pm 25$

$x+9 = 25$ or $x+9 = -25$

$x = 16$ $\quad x = -34$

2014 : Jerry : 20
Gloria : 16

2015 : Jerry : 21
Gloria : 17

4. $2x(5x-3) + (7x-1)(3x+5) = 171$

$10x^2 - 6x + 21x^2 + 35x - 3x - 5 = 171$

$31x^2 + 26x - 5 = 171$

$31x^2 + 26x - 176 = 0$

$m \times n = -5456$

$m + n = 26$

$88, -62$

$31x^2 - 62x + 88x - 176 = 0$

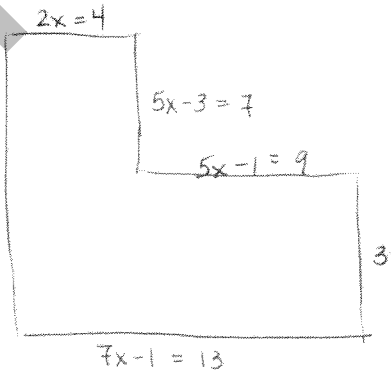
$31x(x-2) + 88(x-2) = 0$

$(x-2)(31x+88) = 0$

$x-2 = 0$ or $31x + 88 = 0$

$x = 2$ $\quad 31x = -88$

$x = -88/31$



$8x + 2 = 18$

Perimeter $= 4 + 7 + 9 + 11 + 13 + 18$

$= 62$

5. $4x^2 + 28x + 49$

$= (2x+7)^2$

Side 1: $2x+7+2 = 2x+9$

Side 2: $2x+7-5 = 2x+2$

Area $= (2x+2)(2x+9) = 4x^2 + 22x + 18$

As a product of factors: $2(x+1)(2x+9)$

6. Rectangle: Area $= (5x+1)(13x-8)$

$= 65x^2 - 27x - 8$

Square: Area $= 6 \times 6 = 36\,cm^2$

Area remaining: $162 = 65x^2 - 27x - 8 - 36$

$162 = 65x^2 - 27x - 44$

$0 = 65x^2 - 27x - 206$

$m \times n = -13\,340$

$m + n = -27$

$+103, -130$

$0 = 65x^2 - 130x + 103x - 206$

$0 = 65x(x-2) + 103(x-2)$

$0 = (x-2)(65x+103)$

$x - 2 = 0$ or $65x + 103 = 0$

$x = 2$ $65x = -103$

$x = -\frac{103}{65}$

∴ Original dimensions: $5x+1 = 11\,cm$

$13x - 8 = 18\,cm$

Solving Second Degree Equations

p4 ① $(3x+5)(2x-3) = 50$

$6x^2 - 9x + 10x - 15 = 50$

$6x^2 + 1x - 65 = 0$

$\frac{-1 \pm \sqrt{1^2 - 4(6)(-65)}}{2(6)}$ → $\frac{38.5}{12} = 3.2$

→ $\frac{-40.5}{12} = -3.37$

② $\frac{x(2x+4)}{2} = 8(2x-3)$

$\frac{2x^2+4x}{2} = 16x - 24$

$x^2 + 2x - 16x + 24 = 0$

$x^2 - 14x + 24 = 0$

$(x-12)(x-2) = 0$

~~$x = 2$~~ because horizontal sides are shorter than vertical sides

$x = 12$ ✓

$A_{rect} = 8(2x-3)$

$= 8(2(12)-3))$

$168\ cm^2$

p5 ③ $Area_{ABCD} = x(2x+4) = 240$

$2x^2 + 4x - 240 = 0$

$2(x^2 + 2x - 120) = 0$

$2(x-10)(x+12) = 0$

$x = 10$ $\quad x = -12$

$2x+4$

$= 2(10)+4$

$= 24$

$A_{square} = 24^2 = 576\ cm^2$ ✓

④ $(4x-4)(2x+3)=48$

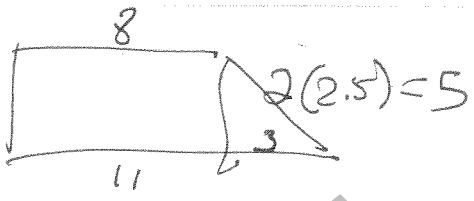
$8x^2+4x-12-48=0$

$8x^2+4x-60=0$

$4(2x^2+x-15)=0$

$(2x-5)(x+3)=0$

~~$x=\frac{5}{2}$~~ $\frac{2}{2}$ $x=-3$



$a^2+b^2=c^2$

$a^2+3^2=5^2$

$a=4$

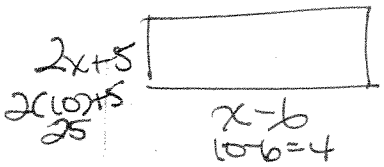
height = 4 cm

⑤ $x^2=2x^2-7x-30$

$0=x^2-7x-30$

$0=(x+3)(x-10)$

~~$x=-3$~~ $x=10$



$2x^2-7x-30$

$(2x+5)(x-6)$

$P=2(4)+2(25)$

$P=58\text{cm}$

Perimeter is 58cm.

#1 $A_{Rect\,A} - 18 = A_{Rect\,B}$

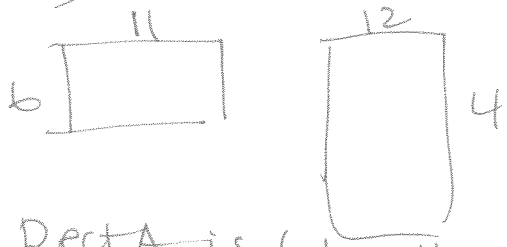
$(x+4)(2x+7) - 18 = (x+10)(x+2)$

$2x^2 + 7x + 8x + 28 - 18 = x^2 + 12x + 20$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -3$ $\quad$ $x = 2$



Rect A is 6 by 11 units

Rect B is 4 by 12 units.

#2 $Area_{ABCD} - A_{square} = A_{shaded}$

$2x(x+1) - 36 = x^2 + 6x - 4$

$2x^2 + 2x - 36 = x^2 + 6x - 4$

$x^2 - 4x - 32 = 0$

$(x+4)(x-8) = 0$

$x = -4$ $\quad$ $x = 8$

#1

$(2x+2)^2 + x^2 = 13^2$

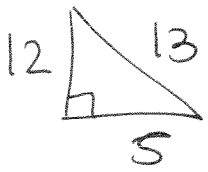
$4x^2+8x+4 + x^2 = 169$

$5x^2+8x-165=0$

$(5x+33)(x-5)=0$

$5x = -33 \quad x=5$

$\frac{5x}{5} = \frac{-33}{5}$



$P = 13+12+5$

$= 30\text{ cm}$

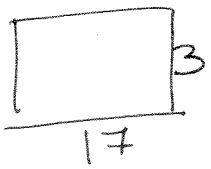
#2 $A = (x-5)(2x+1) = 51$

$2x^2-9x-5-51=0$

$2x^2-9x-56=0$

$(2x+7)(x-8)=0$

$x=-7/2 \quad x=8$



$P = 2(3)+2(17) = 40\text{ cm}$

#3 $(x-6)(2x-5) = x^2$

$2x^2-17x+30-x^2=0$

$x^2-17x+30=0$

$(x-15)(x-2)=0$

$x=15 \quad x=2$

Length of Rect would be negative

$A = 25(9)$

$= 225\text{ cm}^2$

p8

#4

$x^2 + x(x+8) = 120$

$x^2+x^2+8x-120=0$

$2x^2+8x-120=0$

$2(x^2+4x-60)=0$

$2(x-6)(x+10)=0$

$x=6 \quad x=-10$

$A_{PBCQ} = x(x+8)$

$= x^2+8x$

$A(6) = 6^2+8(6)$

$= 84\text{ cm}^2$

#5

$A_{trap} = \frac{(B+b)h}{2}$

$125 = \frac{(x+5+x)x}{2}$

$125 = \frac{2x^2+5x}{2}$

$125 = \frac{2x^2+5x}{2}$

$250 = 2x^2+5x$

$0 = 2x^2+5x-250$

$0 = (2x+25)(x-10)$

$x=-25/2 \quad x=10$

$\overline{CD} = x+5 = 10+5 = 15$

#6 $5x(3x-1) = 16 + (12x^2 - 7x + 1)$

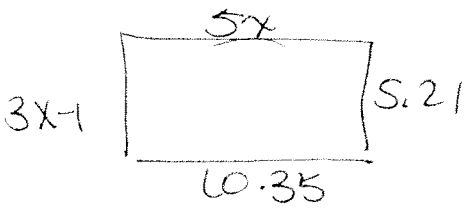
$15x^2 - 5x = 12x^2 - 7x + 17$

$3x^2 + 2x - 17 = 0$

$x = \dfrac{-2 \pm \sqrt{4 - 4(3)(-17)}}{2(3)}$

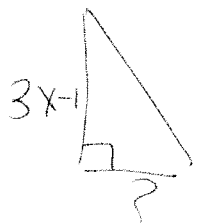
$x = \dfrac{-2 + 14.4}{6}$, $\dfrac{-2 - 14.4}{6}$

$x = 2.07$



$A = 5.21 \times 10.35$

$= 54 cm^2$



$\dfrac{b(3x-1)}{2} = 16$

$b(3x-1) = 32$

4.3-4.4 Word Problems
Algebra 2

Name: Key
Period: 1 2 3 4 5 6

1. **Area**. A town has a nature preserve with a rectangular field that measures 600 meters by 400 meters. The town wants to double the area of the field by adding ~~hand~~ land as shown. Find the new dimensions of the field.

$A_{orig} = 240000m^2 = 600m(400m)$

$A_{doubled} = 480000 = (x+600)(x+400)$

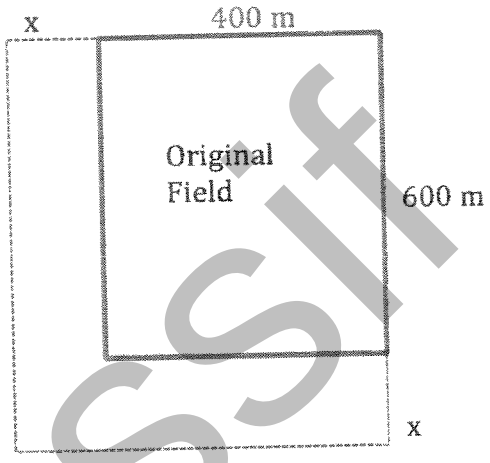
$480000 = x^2 + 1000x + 240000$

$0 = x^2 + 1000x - 240{,}000$

$0 = (x+1200)(x-200)$

$x = -1200, \boxed{200}$

Dim: $600m \times 800m$



2. **Area:** You have made a rectangular quilt that is 5 feet by 4 feet. You want to use the remaining 10 square feet of fabric to add a decorative border of uniform width to the quilt. What should the width of the quilt's border be?

$A_{Border} = A_{entire\ quilt} - A_{w/o\ Border}$

width of border = $\frac{1}{2}$ ft.

$10 = (5+2x)(4+2x) - 20$

$10 = 20 + 18x + 4x^2 - 20$

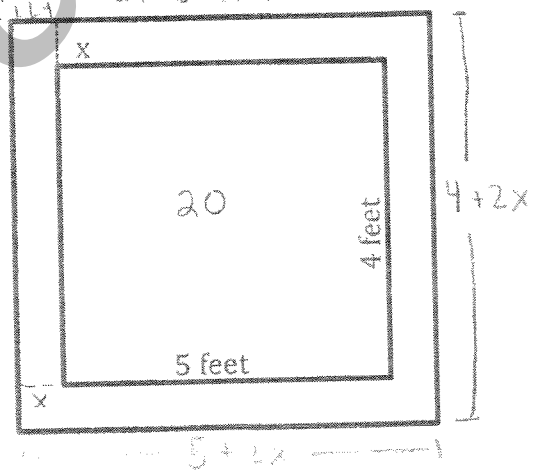
$10 = 4x^2 + 18x$

$0 = 4x^2 + 18x - 10$

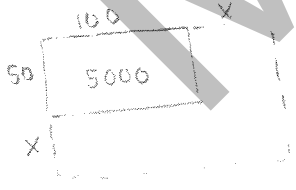
$0 = 2(2x^2 + 9x - 5)$

$0 = 2(2x-1)(x+5)$

$\boxed{x = \frac{1}{2}}, -5$



3. A city's skate park is a rectangle 100 feet long by 50 feet wide. The city wants to triple the area of the skate park by adding the same distance x to the length and width. Write and solve an equation to find the value of x. What are the new dimensions of the skate park? (When in doubt, draw it out!)



$3(5000) = (100+x)(50+x)$

$15000 = 5000 + 150x + x^2$

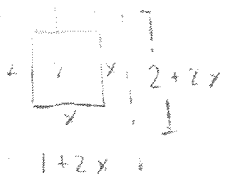
$0 = x^2 + 150x - 10000$

$0 = (x+200)(x-50)$

$x = -200, \boxed{50}$

Dim: $100ft \times 150ft$

4. You have a rectangular stained glass window that measures 2 feet by 1 foot. You have 4 square feet of glass with which to make a border of uniform width around the window. What should the width of the border be? (When in doubt, draw it out!)



$4 = (1+2x)(2+2x) - 2$

$4 = 2 + 6x + 4x^2 - 2$

$0 = 4x^2 + 6x - 4$

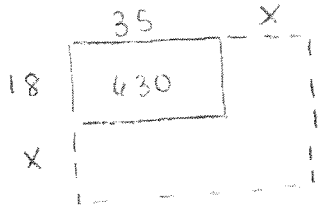
$0 = 2(2x^2 + 3x - 2)$

$0 = 2(2x - 1)(x + 2)$

$1/2, -2$

Border should be $1/2$ foot

5. A rectangular enclosure at a zoo is 35 feet long by 18 feet wide. The zoo wants to double the area of the enclosure by adding the same distance x to the length and width. Write and solve an equation to find the value of x. What are the new dimensions of the enclosure? (When in doubt, draw it out)



$1260 = (35+x)(18+x)$

$1260 = 630 + 53x + x^2$

$0 = x^2 + 53x - 630$

$0 = (x + 63)(x - 10)$

$x = -63, 10$

Dim: 28 ft x 45 ft

Solve by factoring.

6. $y = 3x^2 - 2x - 5$

$0 = (3x - 5)(x + 1)$

$\{5/3, -1\}$

7. $y = 5x^2 + 19x + 12$

$0 = (5x + 4)(x + 3)$

$\{-4/5, -3\}$

8. $y = 9x^2 + 66x + 21$

$0 = 3(3x^2 + 22x + 7)$

$0 = 3(3x + 1)(x + 7)$

$\{-1/3, -7\}$

9. $y = 16x - 2x^2$

$0 = 2x(8 - x)$

$\{0, 8\}$

10. $y = -x^2 + x + 20$

$0 = -(x^2 - x - 20)$

$0 = -(x - 5)(x + 4)$

$\{5, -4\}$

11. $y = 12x^2 + 8x - 15$

180

$-10, 18$

$12x^2 - 10x + 18x - 15$

$2x(6x - 5) + 3(6x - 5)$

$(6x - 5)(2x + 3)$

$\{5/6, -3/2\}$

FACTOR BY COMPLETING THE SQUARE

(x-1)(x+13) 1. $x^2 + 12x - 13$

$(x+3)^2$ 2. $x^2 + 6x + 9$

(x-4)(x+8) 3. $x^2 + 4x - 32$

(x-3)(x+1) 4. $x^2 - 2x - 3$

(x-5)(x+7) 5. $x^2 + 2x - 35$

2(x+1)(x+5) 6. $2x^2 + 12x + 10$

5(x+6)(x-2) 7. $5x^2 + 20x - 60$

2(x-3)(x-1) 8. $2x^2 - 8x + 6$