

1 Example of an appropriate solution

Slope of left ray

$$\frac{7 - 4}{0 - (-2)} = \frac{3}{2}$$

∴ equation of left ray

$$y = \frac{3}{2}x + 7$$

Slope of right ray

$$\frac{-3}{2}$$

Equation of right ray

$$y = \frac{-3}{2}x + b$$

$$5 = \frac{-3}{2}(4) + b$$

$$5 = \frac{-12}{2} + b$$

$$5 = -6 + b$$

$$11 = b$$

$$y = \frac{-3}{2}x + 11$$

Vertex of absolute value function is the intersection of both rays

$$\frac{3}{2}x + 7 = \frac{-3}{2}x + 11$$

$$\frac{6}{2}x = 4$$

$$x = \frac{4}{3}$$

$$y = \frac{3}{2}\left(\frac{4}{3}\right) + 7$$

$$y = \frac{12}{6} + 7$$

$$y = 9$$

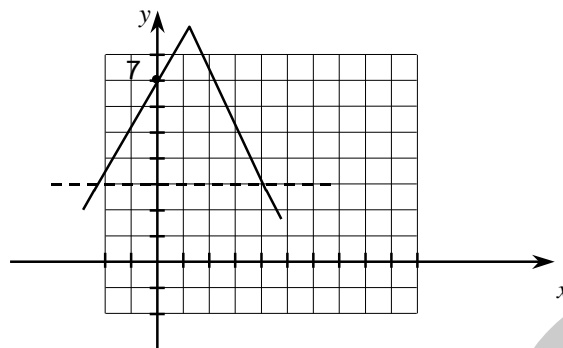
Equation of absolute value function

$$y = a|x - h| + k$$

$$y = \frac{-3}{2}\left|x - \frac{4}{3}\right| + 9$$

The vertex of the birdhouse is $\left(\frac{4}{3}, 9\right)$, which yields the altitude of the triangle to be $9 - 3 = 6$ units.

The base is the intersection of the absolute value function and the constant function



$$\frac{-3}{2} \left| x - \frac{4}{3} \right| + 9 = 3$$

$$\frac{-3}{2} \left| x - \frac{4}{3} \right| = -6$$

$$\left| x - \frac{4}{3} \right| = 4$$

x

$$\therefore x - \frac{4}{3} = 4 \quad \text{or} \quad x - \frac{4}{3} = -4$$

$$x = \frac{16}{3}$$

$$x = \frac{-8}{3}$$

and base has length $\frac{16}{3} - \frac{-8}{3} = 8$ units

$$\begin{aligned} \therefore \text{Area of front face} &= \frac{1}{2} (6)(8) \\ &= 24u^2 \end{aligned}$$

Answer: The area of the shaded triangular section of the birdhouse is $24u^2$.

Note: Students who have determined the rule of the absolute value function have shown they have a partial understanding of the problem.

2 The number of hours elapsed is 5.

3 Example of an appropriate method

$$V(t) = -25|t - 8| + 65$$

$$V(t) \geq 35$$

$$-25|t - 8| + 65 \geq 35$$

$$-25|t - 8| \geq 35 - 65$$

$$|t - 8| \leq \frac{-30}{-25}$$

$$|t - 8| \leq 1.2$$

If $t - 8 \geq 0$, then $|t - 8| = t - 8$;

$$t - 8 \leq 1.2$$

$$t \leq 9.2$$

if $t - 8 < 0$, then $|t - 8| = -(t - 8)$

$$-(t - 8) \leq 1.2$$

$$-t + 8 \leq 1.2$$

$$-t \leq -6.8$$

$$t \geq 6.8$$

$$9.2 - 6.8 = 2.4$$

Answer The police officer should be on duty for 2.4 hours.
(Accept 2 hours and 24 minutes)

5 Example of an appropriate method

$$y = a|x - b| + c$$

$$b = (2 + 8) \div 2 = 5$$

$$a = -\left(\frac{\Delta y}{\Delta x}\right) = \frac{(-4 - 0)}{(0 - 2)} = -2$$

$$(c - (-4)) \div (5 - 0) = 2$$

$$c = 6$$

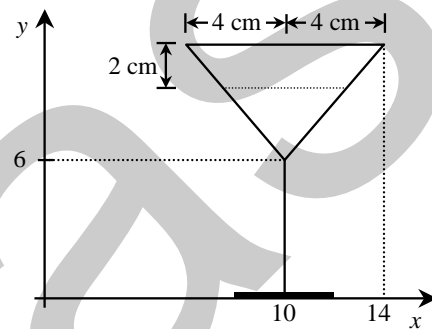
Answer The rule of correspondence is $y = -2|x - 5| + 6$.

6 Example of an appropriate method

Height of the cocktail glass

If $x = 10 + 4 = 14$, then $f(14) = \frac{5}{4}|14 - 10| + 6 = 11$.

The height of the cocktail glass is 11 cm.



Point on the glass where the gold leaf strip is applied

$$11 - 2 = 9 \text{ cm}$$

Diameter of the glass at the point where the gold leaf strip is applied

$$f(x) = 9 \text{ therefore } 9 = \frac{5}{4}|x - 10| + 6$$

$$3 = \frac{5}{4}|x - 10|$$

$$2.4 = |x - 10|$$

$$x - 10 = 2.4 \quad \text{and} \quad |x - 10| = -2.4$$

$$x = 12.4 \quad \quad \quad x = 7.6$$

The diameter of the glass at the point where the gold leaf strip is applied:

$$12.4 - 7.6 = 4.8$$

Answer: The diameter of the glass at the point where the gold leaf strip will be applied is **4.8** cm.

Note: Students who used an appropriate method in order to determine the point on the glass where the gold leaf strip is applied have shown that they have a partial understanding of the problem.

7 Example of an appropriate method

The equation of the absolute value function is of the form

$$f(x) = a|x - h| + k$$

Water quantity at the beginning of the task: (0, 60)

Minimum quantity of water: (35, 45) = (h, k)

Value of parameter a using the ordered pair (0, 60)

$$f(x) = a|x - 35| + 45$$

$$60 = a|0 - 35| + 45$$

$$a = \frac{3}{7}$$

Function rule

$$f(x) = \frac{3}{7}|x - 35| + 45$$

Value of x where $f(x) < 53$

$$\frac{3}{7}|x - 35| + 45 < 53$$

$$|x - 35| < 18.\bar{6}$$

$$\therefore x - 35 < 18.\bar{6}$$

$$x < 53.\bar{6}$$

$$\therefore -x + 35 < 18.\bar{6}$$

$$-x < -16.\bar{3}$$

$$x > 16.\bar{3}$$

Duration where $f(x) < 53$

$$53.\bar{6} - 16.\bar{3} \approx 37.\bar{3}$$

Answer: The quantity of water in the aquarium is less than 53 litres for 37.3 seconds.

Note Accept 37 seconds as a final answer.

8 The solution set of this inequality is [-1, 7].

Note: Accept any equivalent notation for this solution set.

9 Example of an appropriate method

Equation of the function

$$P(t) = a|t - h| + k$$

$$P(t) = a|t - 5| + 6$$

$$4 = a|0 - 5| + 6$$

$$-2 = 5a$$

$$a = \frac{-2}{5}$$

$$P(t) = \frac{-2}{5}|t - 5| + 6$$

Solving

$$4.4 \leq \frac{-2}{5}|t - 5| + 6$$

$$-1.6 \leq \frac{-2}{5}|t - 5|$$

$$4 \geq |t - 5|$$

$$4 \geq t - 5 \quad 4 \geq -t + 5$$

$$9 \geq t \quad 1 \leq t$$

Number of months

$$9 - 1 = 8 \text{ months}$$

Answer The price of one kilogram of green grapes was at least \$4.40 for 8 months.

10 Example of an appropriate method

The situation can be represented by an absolute value function of the form $f(x) = a|x - h| + k$

where x represents the number of months elapsed and $f(x)$ represents the value of the share, in \$.

Share's initial value: (0, 25)

Share's value May 31st: (5, 10) = (h, k)

Value of parameter a , using ordered pair (0, 25)

$$f(x) = a|x - 5| + 10$$

$$25 = a|0 - 5| + 10$$

$$a = 3$$

Rule of the function

$$f(x) = 3|x - 5| + 10$$

Value of $f(x)$ on December 31st

$$f(x) = a|12 - 5| + 10$$

$$f(x) = 31$$

Answer On December 31st 1999, the share's value was \$31.

11 Example of an appropriate method

Solving the inequality

$$4 - |3x - 5| > 0$$

$$|3x - 5| < 4$$

$$3x - 5 < 4 \quad \text{and} \quad 3x - 5 > -4$$

$$3x < 9 \quad 3x > 1$$

$$x < 3 \quad x > \frac{1}{3}$$

Answer The solution set is $\left] \frac{1}{3}, 3 \right[$.

Accept the same solution set expressed with different notation.

12 The solution set is $] -3, 4[$.
Accept any equivalent notation.

13 Example of an appropriate method

Start-up time

$$-3|x - 6| + 36 = 21$$

$$-3|x - 6| = -15$$

$$|x - 6| = 5$$

$$x - 6 = 5 \quad \text{or} \quad x - 6 = -5$$

$$x = 11 \quad \text{or} \quad x = 1$$

Since the start-up time is located to the left of the vertex on the graph, we must use the value of x that is less than 6.

The air conditioning system starts up 1 hour after sunrise.

Shutdown time

$$-3|x - 6| + 36 = 20$$

$$-3|x - 6| = -16$$

$$|x - 6| = \frac{16}{3}$$

$$x - 6 = \frac{16}{3} \quad \text{or} \quad x - 6 = -\frac{16}{3}$$

$$x = \frac{34}{3} \quad \text{or} \quad x = \frac{2}{3}$$

Since the shutdown time is located to the right of the vertex on the graph, we must use the value of x that is greater than 6.

The air conditioning system stops $\frac{34}{3}$ hours after sunrise.

Time during which air conditioning system is in operation

$$\frac{34}{3} - 6 = 10\frac{2}{3} \text{ hours}$$

Answer On that day, the air conditioning system was in operation for 10 hours and 20 minutes or approximately 10.3 hours.

Note Students may also use a graphic method.

14 The rule is $V(t) = 2|t - 20| + 10$.

15 D

16 A

17 Work : (example)

General equation of the absolute value function

$$t(x) = a|x - h| + k$$

Determining the value of a knowing one point and the minimum

$$-1 = a|0 - 8| + (-5) \text{ thus } a = 0.5$$

Equation of function t

$$t(x) = 0.5|x - 8| - 5$$

Calculating the length of time

$$0.5|x - 8| - 5 \leq -3$$

$$\text{thus } x \geq 4 \text{ and } x \leq 12$$

$$\text{therefore the length of time is } 12 - 4 = 8$$

Result : The length of time the temperature was less than or equal to -3°C was 8 hours.

Note : Part of the solution could consist of a graph.

18 A

19 A

20 B

21 The area of the triangle is 9 units.

22 B

23 B

24 D

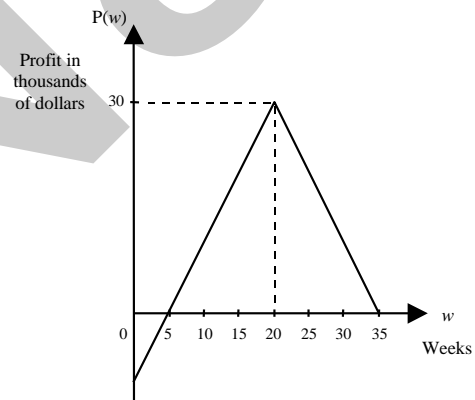
25 Work : (example)

Vertex : (20, 30)

Zeros : (5, 0) and (35, 0)

Value of a

The form of the equation is $P(w) = a|5 - 20| + 30$



$$\begin{aligned}\text{Therefore } P(5) &= a|5 - 20| + 30 \\ 0 &= a|-15| + 30 \\ -30 &= 15a \\ a &= -2\end{aligned}$$

$$\text{Equation : } P(w) = -2|5 - 20| + 30$$

Profit made in the 29th week

$$\begin{aligned}P(29) &= -2|29 - 20| + 30 \\ &= -2 \times 9 + 30\end{aligned}$$

$$= -18 + 30$$

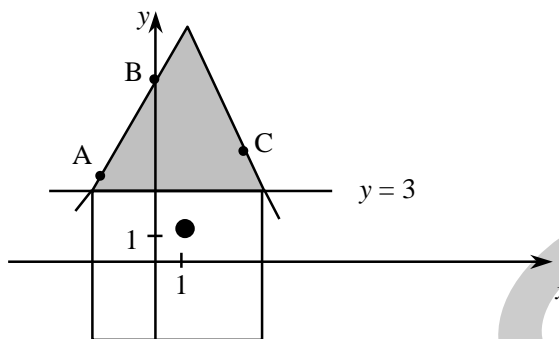
$$= 12$$

Result : In the 29th week the dealer realized a profit of \$12 000.

26 the minimum distance between the two planes is 100 m.

- 1 Ethan's diagram, not drawn to scale, shows the front of a birdhouse. The base of the roof corresponds to the line $y = 3$.

The sides of the roof form an absolute value function that passes through the points $A(-2, 4)$, $B(0, 7)$ and $C(4, 5)$.



What is the area of the shaded triangular section of the front of the birdhouse?

- 2 On a given day, the market value, $V(t)$, of *Bio Tech* stock shares fluctuated in relation to the time elapsed in hours, t , from the opening of the day's trading session, according to an absolute value function.

At the opening of trading, *Bio Tech* stock was worth \$6. Three hours later, it reached its maximum value of \$9.

How many hours had elapsed from the time the share first reached \$8 until it decreased to \$5?

- 3 Each morning, the traffic on a certain highway increases, reaches a peak, and then decreases again.

This situation is represented mathematically by the function:

$$V(t) = -25 | t - 8 | + 65$$

$V(t)$ = the number of vehicles passing per minute

t = the time, in hours, since midnight

To ensure a smooth and safe flow of traffic, a police officer has been assigned to monitor this section of highway when the volume of traffic is at least 35 vehicles per minute.

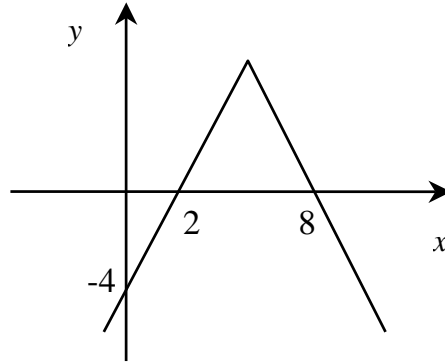
For how many hours should the police officer be on duty to ensure the safe flow of traffic?

- 5 During one month this spring the value of a share in a travel company behaved according to an absolute value function.

Its initial value was \$4 below its average price.

On Day 2 and Day 8 it was exactly the same as its average price.

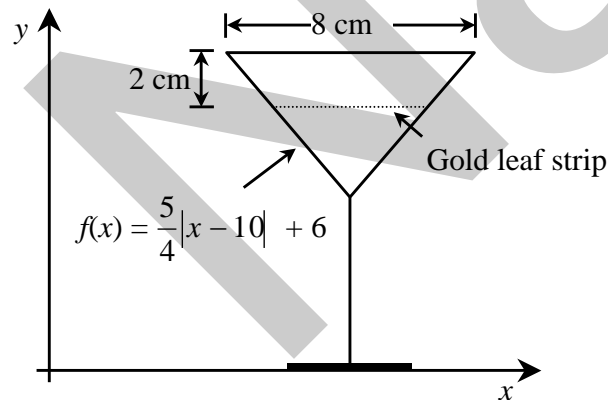
This situation can be represented by the graph below.



What is the rule of correspondence that represents this situation?

- 6 A glassblower wants to apply a gold leaf strip around a cocktail glass. The gold leaf strip will be applied 2 cm from the rim of the glass.

The side view of this glass is represented in the following Cartesian plane. The scale of the graph is in centimetres.



The rule $f(x) = \frac{5}{4}|x - 10| + 6$ is associated with the top part of the glass.

The maximum diameter of the glass is 8 cm.

What is the diameter of the glass at the point where the gold leaf strip will be applied?

- 7 Sonia is replacing some of the water in her aquarium without removing the fish.

When she started working, the aquarium contained 60 litres. After 35 seconds, 45 litres of water remained in the aquarium. This is the minimum quantity of water needed to ensure that the fish will not be harmed. Then, using a pump, Sonia filled the aquarium to its maximum capacity of 70 litres.

Sonia noted that the relation between the time elapsed, in seconds, and the quantity of water in the aquarium, in litres, corresponds to an absolute value function.

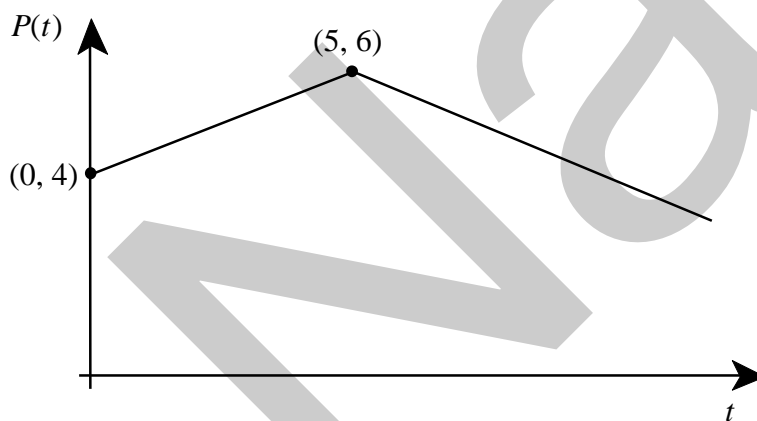
Throughout this process, for how many seconds was the quantity of water in the aquarium less than 53 litres?

Show all your work.

- 8 Given the following inequality: $2|3x - 9| \leq 24$ where $x \in \mathbb{R}$.

What is the solution set of this inequality?

- 9 A recent study showed that the price of a kilogram of green grapes at the "Fine Fruit Market" varied according to the rule of the absolute value function shown below,



where t represents the number of months elapsed since the beginning of the study $t \in [0, 12]$ and $P(t)$ represents the price of one kilogram of green grapes in dollars.

For how many months of the study was the price of one kilogram of green grapes at least \$4.40?

Show all your work.

- 10 Jenny and Eric analyzed the changes in the value of Future Telecom's shares in 1999. On January 1st 1999, the initial value of a share was \$25. On May 31st, the share reached its minimum value of \$10. Since then, the value of shares has been on the increase.

They noticed that the relation between the elapsed time, in months, and the value of a share, in dollars, was an absolute value function.

What was the value of a share on December 31st, 1999?

MS. Nassif

- 11 What is the solution set of the following inequality within \mathfrak{R} ?

$$4 - |3x - 5| > 0$$

- 12 Solve the given inequality, in \mathfrak{R} .

$$5 - 3|2x - 1| > -16$$

- 13 One very hot summer day, the outside temperature is described by the following rule:

$$T(x) = -3|x - 6| + 36$$

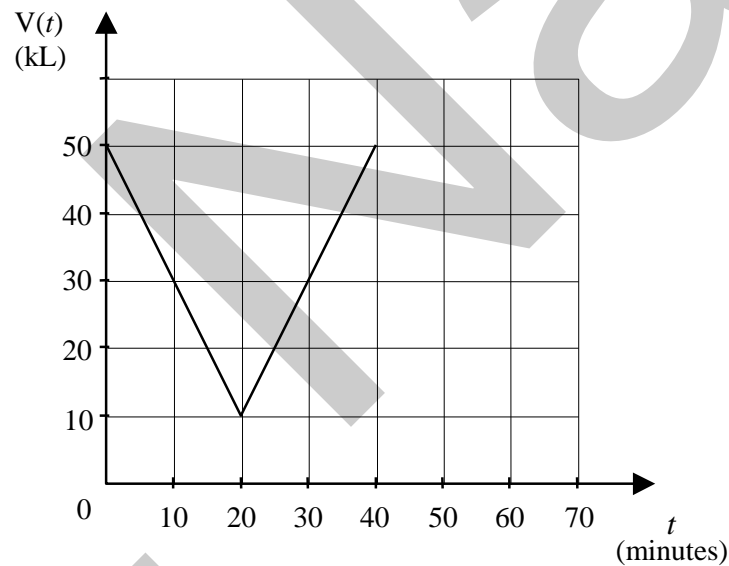
where x represents the number of hours that have elapsed since sunrise
 $T(x)$ represents the temperature in $^{\circ}\text{C}$

To make the temperature inside a shopping centre more comfortable, an air conditioning system starts up when the outside temperature reaches 21°C and stops when the outside temperature drops to 20°C .

How long was the air conditioning system in operation on that day?

- 14 Water levels in a reservoir are monitored by a control system that allows a constant flow.

According to the given figure, the maximum possible quantity of water is 50 kilolitres and the minimum is 10 kilolitres. Emptying and refilling the reservoir takes 40 minutes.



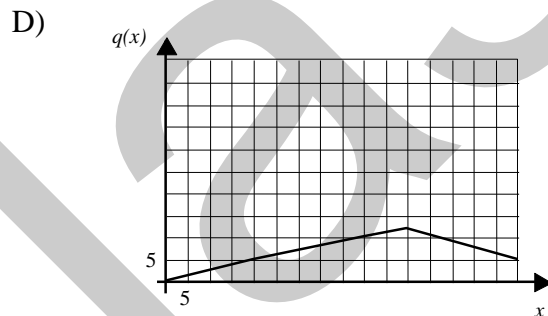
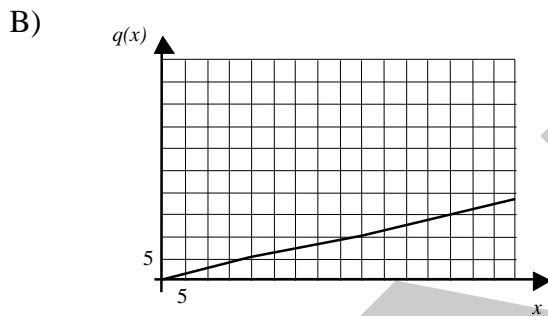
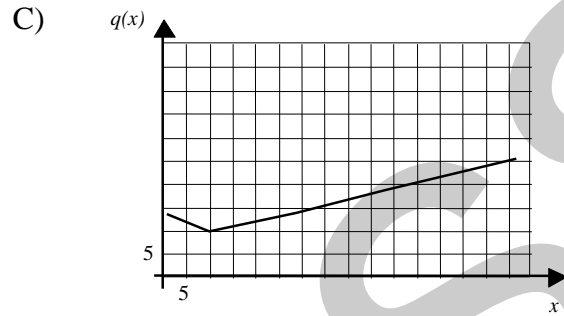
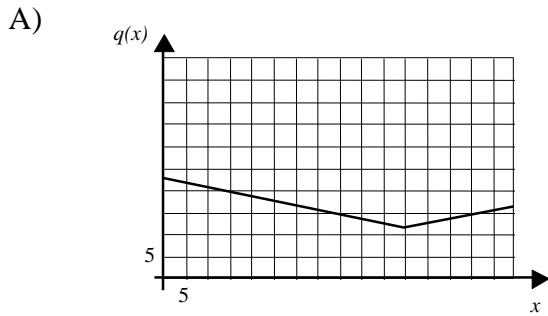
What is the rule of correspondence that represents this situation for $t \in [0, 40]$?

- 18 During an 80 day period in the summer of 1992, a municipality kept track of fluctuations in its drinking water supply. The variations are expressed by the equation

$$q(x) = \left| \frac{1}{5}x - 11 \right| + 12$$

where $q(x)$ corresponds to the supply in millions of litres and x is the number of days since the beginning of the study.

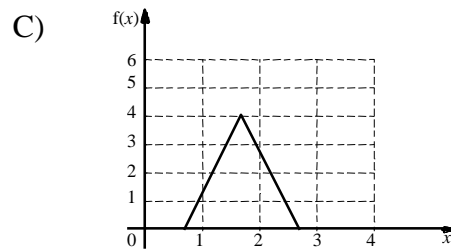
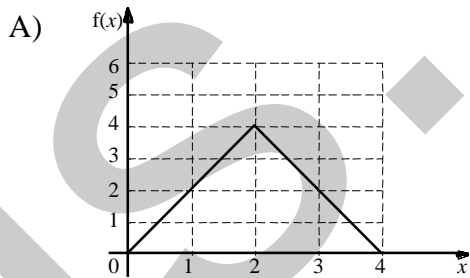
Which graph could represent function q ?



- 19 The shape of a roof on a house can be described by the following function:

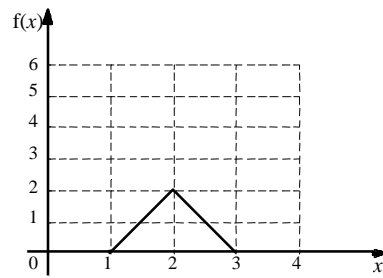
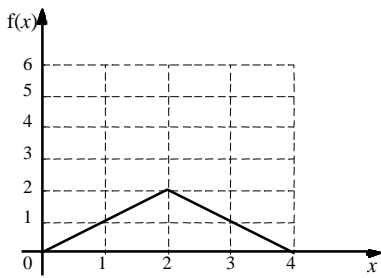
$$f(x) = -|x - 4| + 4$$

Which graph represents this function for $f(x) \geq 0$?



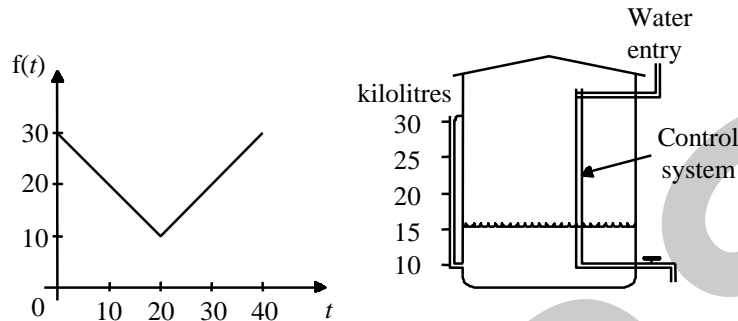
B)

D)



- 20 The water level in a reservoir is controlled by a system which allows for a constant flow of water.

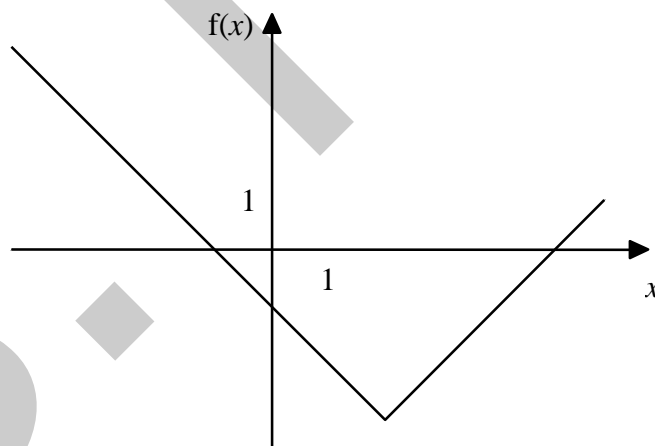
According to the diagrams below, the maximum water level corresponds to 30 kilolitres and the minimum level, 10 kilolitres. The process of emptying and refilling the reservoir takes 20 minutes.



Which rule corresponds to the graph which represents this operation for $t \in [0, 40]$?

- A) $f(t) = |t + 20| + 10$ C) $f(t) = |t + 20| - 30$
 B) $f(t) = |t - 20| + 10$ D) $f(t) = |t - 20| - 30$

- 21 A real function is defined by $f(x) = |x - 2| - 3$.



What is the area of the triangle bounded by the graph of this function and the interval $[-1, 5]$ on the x -axis?

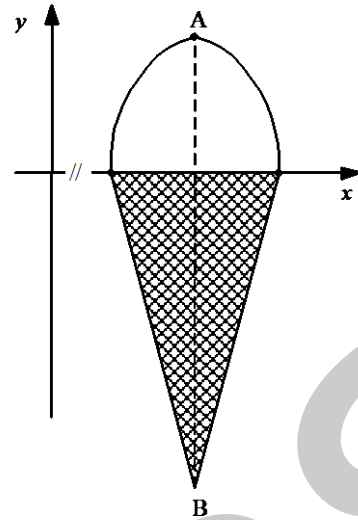
- 22 In its advertising campaign, a company uses an immense ice cream cone. Situated in a Cartesian plane, one part of the cone is a parabola and the other part is the graph of an absolute-value function.

The equation of the two functions are :

$$f(x) = -x^2 + 18x - 72 \text{ where } 6 \leq x \leq 12$$

and

$$g(x) = 4|2x - 18| - 24, \text{ where } 6 \leq x \leq 12$$



What is height AB of the ice cream cone in meters?

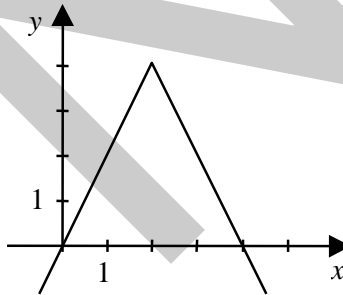
- 23 The assets A, in millions, of a company since it first opened, are given by the rule

$$A(t) = -0.5|t - 4| + 3$$

where t represents the time, in years, that has elapsed since its opening.

For how many years have the company's assets been growing?

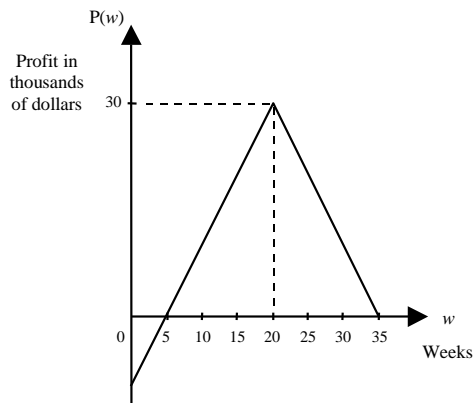
- 24 A real function is defined by the graph below :



Which of the following statements is FALSE?

- A) The zeros of the function are 0 and 4.
- B) This function is positive in the interval $[0, 4]$.
- C) The domain of this function is \mathcal{R} and the range is $-\infty, 4]$.
- D) This function is increasing on the interval $-\infty, 4]$.

- 25 An car dealer's weekly profit $P(w)$ for the last 35 weeks is graphed below.



What profit did the dealer realize in the 29th week?

Show your work.

- 26 **Two planes crisscrossed the sky during a dry run for an air show. The distance d , in metres, that separated them can be represented by the function :**

$$d(t) = |5000 - 200t| + 100$$

where the time t is expressed in seconds.

What is the minimum distance between the two planes?