ANALYTIC GEOMETRY

1 Example of an appropriate method

Slope of segment QR

$$\frac{56 - 20}{36 - 54} = \frac{36}{-18}$$
$$= -2$$

Slope of segment PQ

The product of the slopes of two perpendicular segments is equal to -1.

Slope of segment PQ × slope of segment QR = -1

Slope of segment PQ × -2 = -1

Slope of segment PQ: $\frac{1}{2}$

Coordinate of point P

y-intercept of segment PQ

$$y = \frac{1}{2}x + b$$
$$56 = \frac{1}{2}(36) + b$$
$$56 = 18 + b$$
$$38 = b$$

Coordinates of point P: P(0, 38)

Coordinates of point E

Since the diagonals of a square bisect each other, point E is the midpoint of segment PR.

$$E\left(\frac{0+54}{2}, \frac{38+20}{2}\right)$$
$$E(27, 29)$$

Answer: The coordinates of point E are E(**27, 29**).

Note: Students who use an appropriate method in order to determine the coordinates of point P have shown that they have a partial understanding of the problem.

2

Example of an appropriate method

Length of segment CD

The length of segment CD is equal to the distance between vertex C and segment AD.

$$\text{m } \overline{\text{CD}} = \frac{|3(18) + 4(14) - 60|}{\sqrt{3^2 + 4^2}}$$

$$= \frac{50}{5}$$

$$= 10$$

Length of segment CD

10 cm

Length of segment AD

Perimeter of the rectangle

$$2(\text{m } \overline{\text{AD}} + \text{m } \overline{\text{CD}}) = 52 \text{ cm}$$

$$2(\text{m } \overline{\text{AD}} + 10 \text{ cm}) = 52 \text{ cm}$$

$$\text{m } \overline{\text{AD}} = 16 \text{ cm}$$

Length of segment AD

16 cm

Answer: The length of segment AD is **16** cm.

Note: Students who use an appropriate method in order to determine the length of segment CD have shown that they have a partial understanding of the problem.

3 Example of an appropriate method

Length of segment BC

$$m\ \overline{AB} = 10 - 0 = 10$$

$$m\ \overline{BC} = m\ \overline{AB} = 10$$

Length of segment BD

$$m\ \overline{BD} = \sqrt{(30 - 10)^2 + (7 - 22)^2} = \sqrt{625} = 25$$

Location of point C

$$\frac{m\ \overline{BC}}{m\ \overline{BD}} = \frac{10}{25} = \frac{2}{5}$$

Point C is located $\frac{2}{5}$ of the way along segment BD.

Coordinates of point C

$$C\left(10 + \frac{2}{5}(30 - 10),\ 22 + \frac{2}{5}(7 - 22)\right)$$

C(18, 16)

Answer: The coordinates of point C are C**(18, 16)**.

Note: Students who use an appropriate method in order to determine the length of segment BC **and** the length of segment BD have shown that they have a partial understanding of the problem.

4 Example of an appropriate method

Coordinates of C

Since Δ ABC $\cong$ Δ DEC and $\overline{AB}$ // $\overline{ED}$, $\overline{AC}$ and $\overline{DC}$ are congruent, corresponding segments.

Point C is therefore the midpoint of segment AD.

$$C\left(\frac{0+40}{2}, \frac{42+2}{2}\right)$$

C(20, 22)

Equation associated with segment AB

y-intercept: 42

Slope: $\frac{42-10}{0-16} = \frac{32}{-16} = -2$

$y = -2x + 42$

$2x + y - 42 = 0$

Length of altitude CH

The length of altitude CH is equal to the distance between point C and segment AB.

$$\frac{|2(20)+1(22)-42|}{\sqrt{2^2+1^2}}=\frac{20}{\sqrt{5}}\approx 8.944$$

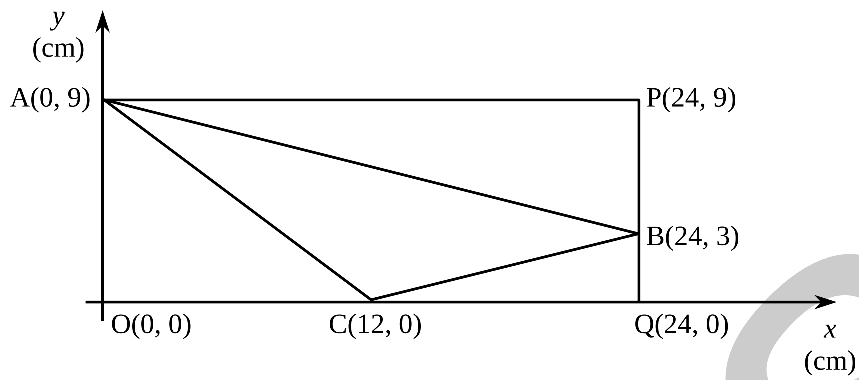
Answer: To the nearest tenth, the length of altitude CH is **8.9** units.

Note: Do not penalize students who did not round off the final answer or who made a mistake in rounding it off.

Students who use an appropriate method in order to determine the coordinates of C **or** the equation associated with segment AB have shown that they have a partial understanding of the problem.

Written information is considered unclear if students do not indicate why point C is the midpoint of segment AD.

5 Example of an appropriate method



Area of triangle ABC

Given rectangle APQO

Area of rectangle APQO

$24 \times 9 = 216 \text{ cm}^2$

Area of triangle APB

$$\frac{m\ \overline{AP} \times m\ \overline{PB}}{2} = \frac{24 \times 6}{2} = 72 \text{ cm}^2$$

Area of triangle AOC

$$\frac{m\ \overline{AO} \times m\ \overline{OC}}{2} = \frac{9 \times 12}{2} = 54 \text{ cm}^2$$

Area of triangle BQC

$$\frac{m\ \overline{BQ} \times m\ \overline{CQ}}{2} = \frac{3 \times 12}{2} = 18 \text{ cm}^2$$

Area of triangle ABC

$216 - 72 - 54 - 18 = 72\ \text{cm}^2$

Scale factor

$$k^2 = \frac{\text{Area of triangle A'B'C'}}{\text{Area of triangle ABC}} = \frac{200}{72} = \frac{25}{9}$$

$$k = \frac{5}{3}$$

Answer: The scale factor that must be used in this case is $\frac{5}{3}$ or $1.\overline{6}$.

Note: Students who use an appropriate method in order to determine the area of triangle ABC have shown that they have a partial understanding of the problem.

There are different ways of finding the area of triangle ABC (e.g. law of cosines, the distance between a point and a line).

6

Example of an appropriate method

The teams never pass each other.

Length of segment RB

$$\sqrt{(92-20)^2+(95-15)^2} = \sqrt{11\ 584} = 107.6289...$$

Distance travelled by the red team

$$\frac{3}{8} \text{ of } \mathrm{m}\ \overline{\mathrm{RB}} = \frac{3}{8} \text{ of } 107.6289 = 40.3608...$$

Distance travelled by the blue team

$$\frac{1}{4} \text{ of } \mathrm{m}\ \overline{\mathrm{RB}} = \frac{1}{4} \text{ of } 107.6289 = 26.9072...$$

Distance between the two teams

107.6289 – 40.3608 – 26.9072 = 40.3609

Answer: Rounded to the nearest metre, the distance between the two teams by the end of this activity is 40 m.

Note: Students who use an appropriate method in order to determine the distance travelled by either team have shown that they have a partial understanding of the problem.

Do not penalize students who did not round off their final answer or who made a mistake in rounding it off.

The following are the steps associated with another appropriate method:

– Coordinates of the red team's position: (47, 45)

– Coordinates of the blue team's position: (74, 75)

– Distance between their positions: 40.3608…

In this case, students who use an appropriate method in order to determine the values of the coordinates of either team's position have shown that they have a partial understanding of the problem.

7 Example of an appropriate method

Equation of the line passing through B and C

Slope: $\frac{9-33}{56-8} = \frac{-24}{48} = -0.5$

$y = -0.5x + b$

$33 = -0.5(8) + b$

$37 = b$

$y = -0.5x + 37$

Measure of altitude AH

$\text{m}\ \overline{\text{AH}}$ = distance between A and the line passing through B and C

$$\text{m}\ \overline{\text{AH}} = \frac{\left|-0.5 \times 56 - 54 + 37\right|}{\sqrt{(-0.5)^2 + 1}} = 40.2492...$$

Answer The measure of altitude AH to the nearest tenth is 40.2 units.

Note Do not penalize students who did not round off their final answer or who made a mistake in rounding it off.

Students who correctly or incorrectly determine the equation of the line passing through B and C have shown that they have a partial understanding of the problem.

The following are the steps in another appropriate method:

- m $\overline{AC}$
- m $\overline{AB}$
- m $\overline{BC}$
- m $\angle$ACB using the law of cosines
- m $\overline{AH}$ using the sine ratio

In this case, students who correctly or incorrectly determine the measure of angle ACB have shown that they have a partial understanding of the problem.

8

Example of an appropriate solution

Scale factor

$$m\ \overline{CR} = \sqrt{(110-80)^2 + (10-82)^2} = 78$$

$$m\ \overline{CB} = \sqrt{(110-65)^2 + (10-118)^2} = 117$$

Scale factor = $\frac{m\ \overline{CR}}{m\ \overline{CB}} = \frac{78}{117} = \frac{2}{3}$

Point P is located $\frac{2}{3}$ of the way along segment CA starting from point C.

Let (*x*, *y*) be the coordinates of point P.

$$\frac{110-x}{110-20} = \frac{2}{3} \qquad \frac{10-y}{10-58} = \frac{2}{3}$$

$$\frac{110-x}{90} = \frac{2}{3} \qquad \frac{10-y}{-48} = \frac{2}{3}$$

$110 - x = 60$ $\qquad$ $10 - y = -32$

$x = 50$ $\qquad$ $y = 42$

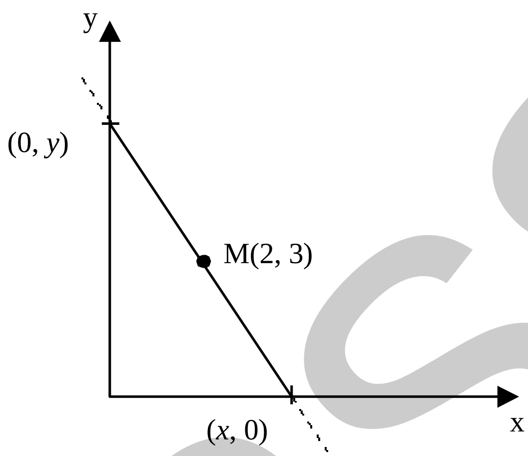
Answer The coordinates of point P are (50, 42).

9 B

10 B

11 Example of an appropriate solution

Given the straight line D going through points $(x_1, 0)$, $(0, y_1)$ and M(2, 3).



Area of triangle is 12 square units.

Therefore $\frac{x_1 \times y_1}{2} = 12$ where $x_1 = \frac{24}{y_1}$

Equation of line D

$y = ax + b$ where $b = y_1$

Substituting point (2, 3)

$3 = 2a + b$ or $3 = 2a + y_1$

Value of y_1 by slope

$$\frac{y_1 - 3}{0 - 2} = \frac{3 - 0}{2 - x_1}$$

$$y_1 - 3 = \frac{-6}{2 - x_1} \quad \text{where} \quad x_1 = \frac{24}{y_1}$$

where $y_1 - 3 = \frac{-6y_1}{2y_1 - 24}$

$$2y_1^2 - 24y_1 + 72 = 0$$

$$y_1^2 - 12y_1 + 36 = 0$$

$(y_1 - 6)(y_1 - 6) = 0$

therefore y_1 = 6 = b

The slope of the straight line with (0, 6) and (2, 3)

$$a = \frac{6-3}{0-2} = \frac{-3}{2}$$

The equation of line D is $y = \frac{-3}{2}x + 6$

Final answer The equation of the straight line through point M is $y = \frac{-3}{2}x + 6$.

12 Work : (example)

Ratio $\frac{2}{3}$

Distance on the *x* axis :

$$(11-7)\times\frac{2}{3}=4\times\frac{2}{3}=\frac{8}{3} \text{ or } 2\frac{2}{3}$$

Distance on the *y* axis :

$$\left(5-\frac{3}{2}\right)\times\frac{2}{3}=\frac{7}{2}\times\frac{2}{3}=\frac{7}{3} \text{ or } 2\frac{1}{3}$$

Translation from point (7,5) :

$$t:(7,5)\mapsto\left(7+2\frac{2}{3},\ 5-2\frac{1}{3}\right)$$

Coordinates of the meeting point :

$$7+2\frac{2}{3} = 9\frac{2}{3}$$

$$5-2\frac{1}{3} = 2\frac{2}{3}$$

$$= \left(9\frac{2}{3},\ 2\frac{2}{3}\right)$$

Result The coordinates of the meeting point are : $\left(9\frac{2}{3}, 2\frac{2}{3}\right)$

Name : ___________________________________

Group : ___________________________________

Date : ___________________________________

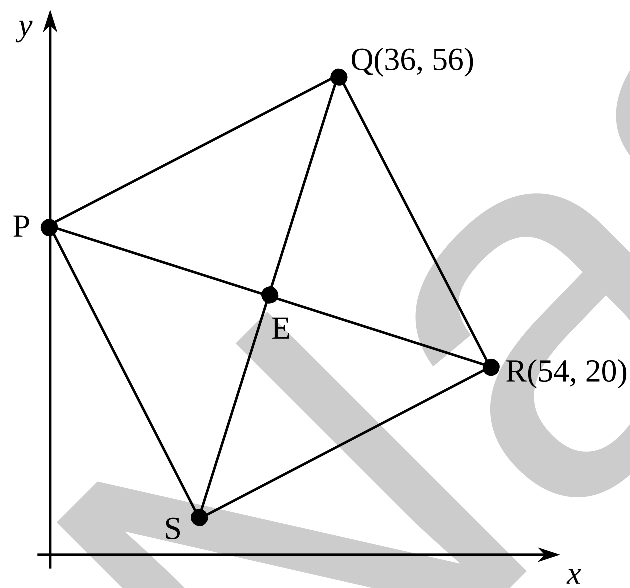
568436 - Mathematics

Question Booklet

1 Consider square PQRS in the Cartesian plane below.

Vertex P is located on the *y*-axis.

Diagonals PR and QS intersect at E.

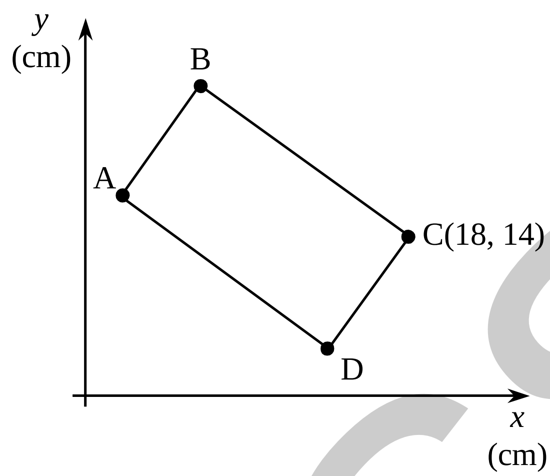


What are the coordinates of point E?

Show all your work.

2

The perimeter of rectangle ABCD represented in the Cartesian plane on the right is 52 cm.

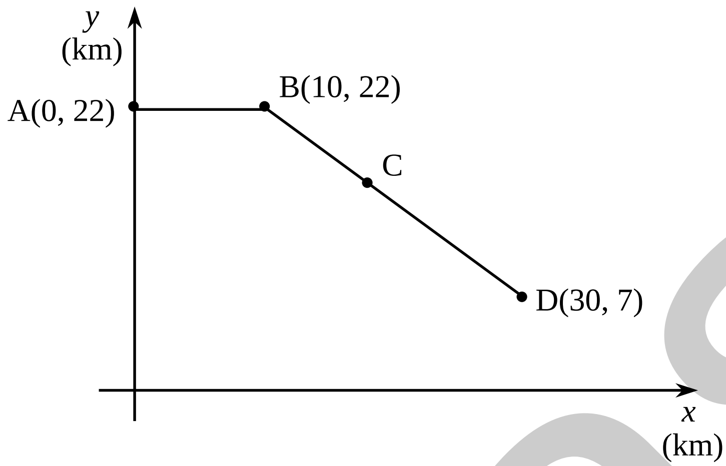


The equation associated with segment AD is $3x + 4y - 60 = 0$.

What is the length of segment AD?

Show all your work.

3 In the following Cartesian plane, line segment AB and BD represent two streets along which rally participants must travel. Points A, B, C and D represent checkpoints set up for the rally.



The distance between checkpoints A and B is equal to the distance between checkpoints B and C.

What are the coordinates of point C?

Show all your work.

4

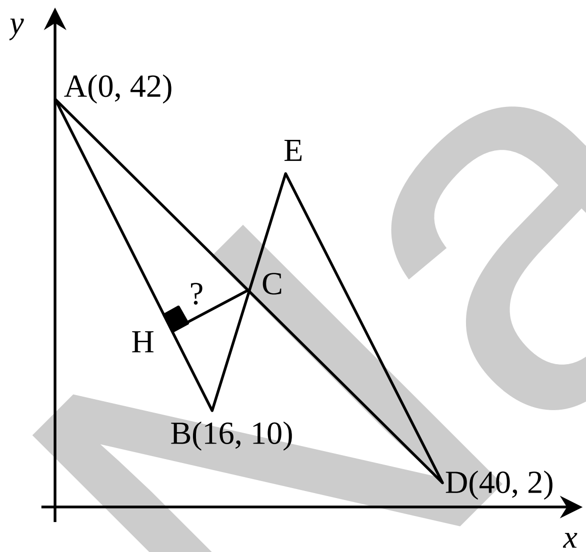
In the Cartesian plane below:

$\overline{AB} \parallel \overline{ED}$.

Line segments AD and BE intersect at C.

Triangles ABC and DEC are congruent.

Segment CH is an altitude of triangle ABC.

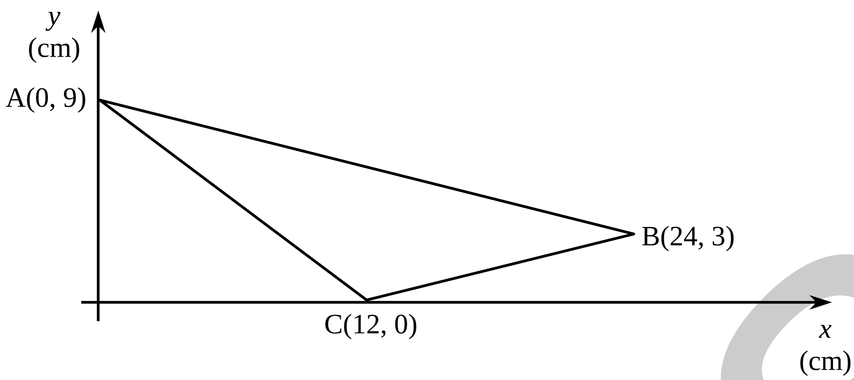


What is the length of altitude CH to the nearest tenth?

Show all your work.

5

The coordinates of the vertices of triangle ABC are given in the Cartesian plane below.

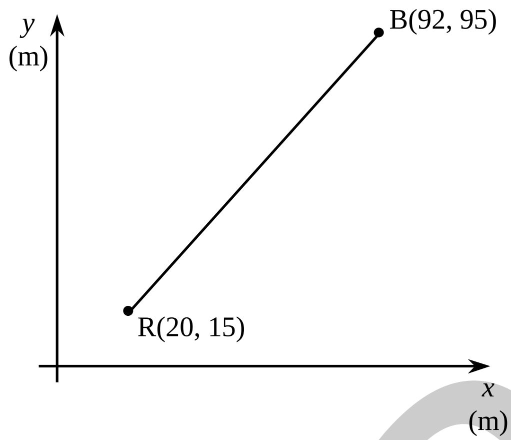


Applying a similarity transformation to triangle ABC produces triangle A'B'C'. The area of triangle A'B'C' is 200 cm^2.

What scale factor must be used in this case?

Show all your work.

6 In a cadet camp, two teams are assigned the task of finding an object hidden along a linear path. When the activity begins, the red team sets off from point R of the path, and the blue team sets off from point B. Path RB is represented in the following Cartesian plane. The scale of the graph is in metres.

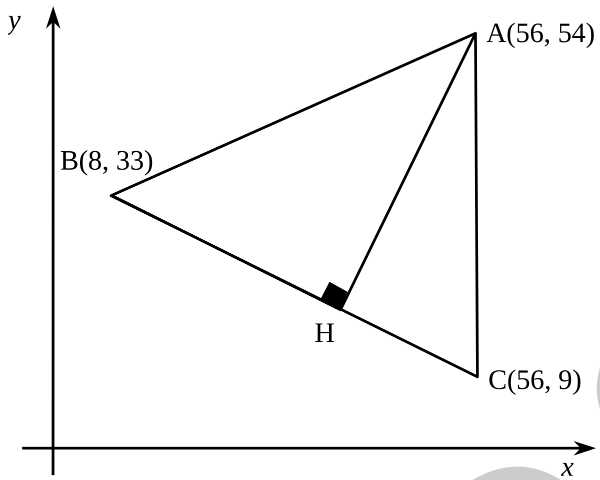


By the end of the activity, the red team's position divides path RB in the ratio 3:5. At the same time, the blue team is located one quarter of the distance from point B to point R.

Rounded to the nearest metre, what is the distance between the two teams by the end of this activity?

Show all your work.

7 Points A(56, 54), B(8, 33) and C(56, 9) are the vertices of a triangle. Segment AH is an altitude of this triangle.

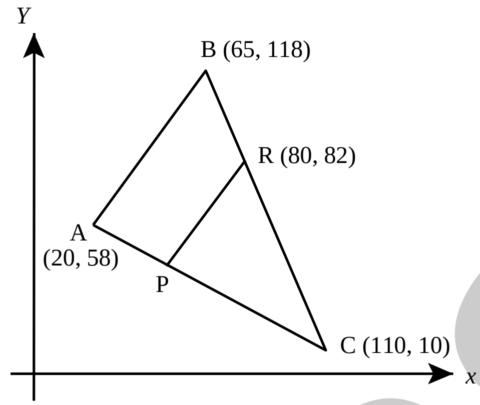


What is the measure of altitude AH to the nearest tenth?

Show all your work.

8

In the Cartesian plane on the right, a dilatation with centre C is applied to triangle ABC in order to produce triangle PRC.



What are the coordinates of point P?

Show all your work.

9 The equations of two parallel lines are as follows:

$$2x - 5y - 10 = 0$$

$$2x - 5y + 4 = 0$$

Rounded to the nearest tenth, what is the distance between these two lines?

A) 2.5 units

B) 2.6 units

C) 2.7 units

D) 2.8 units

10 The coordinates of the vertices of a triangle are (1, 2), (5, 5) and (-2, 6).

In which interval does the area of this triangle fall?

A) [10, 12[

B) [12, 14[

C) [16, 18[

D) [24, 26[

11 In a right triangle, one side of the right angle is located on the *x*-axis; the other side of the right angle is located on the *y*-axis.

The hypotenuse of this triangle is formed by a segment of one line passing through the point M(2, 3).

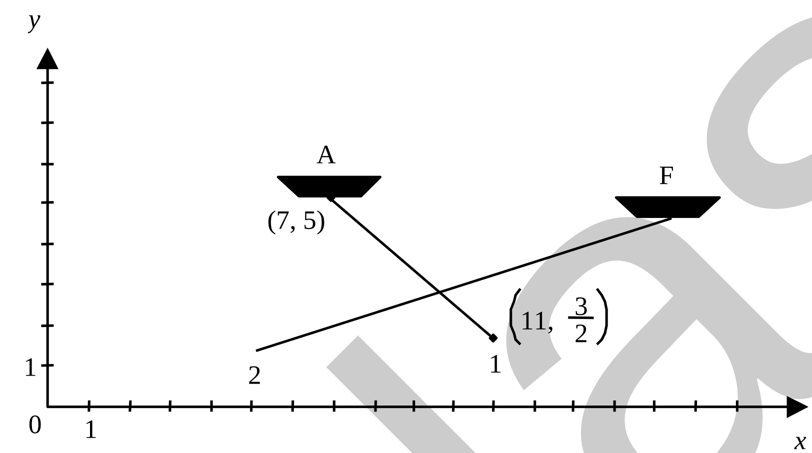
The area of this right triangle is 12 square units.

What is the equation of the line passing through the point M?

Show all the work needed to solve the problem.

12 Martin (M) was returning to wharf 1, having spent the day fishing. When he was $\frac{2}{3}$ of the way back, he met Jason (J) and they talked about their respective catches.

A graphic representation of the situation is shown below.



What are the coordinates of the meeting point of these two fishermen?

Show your work.