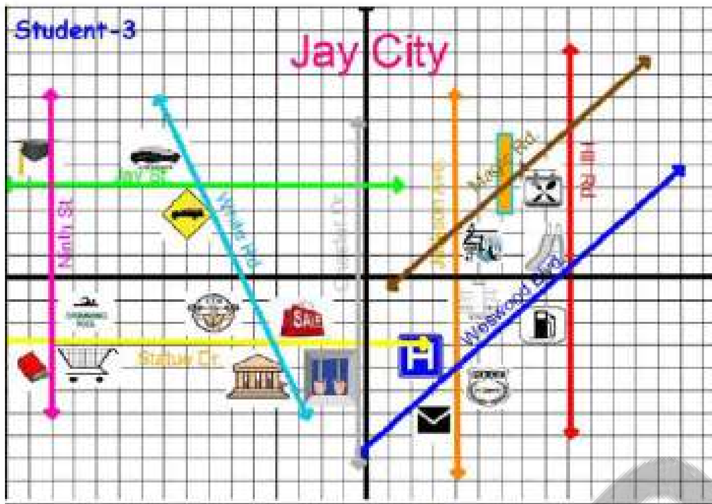
Chapter 2: System of Equations



A system of equations is a set of 2 equations or more.

Example: $\begin{cases} y = 2x + 9 \\ y = -x - 3 \end{cases}$ POI

To SOLVE a system means to find the point of intersection (POI). We need to find the POINT where the 2 lines meet.

There are 4 methods of solving equations:

1. Graphing Method
2. Comparison Method
3. Substitution Method
4. Elimination Method *

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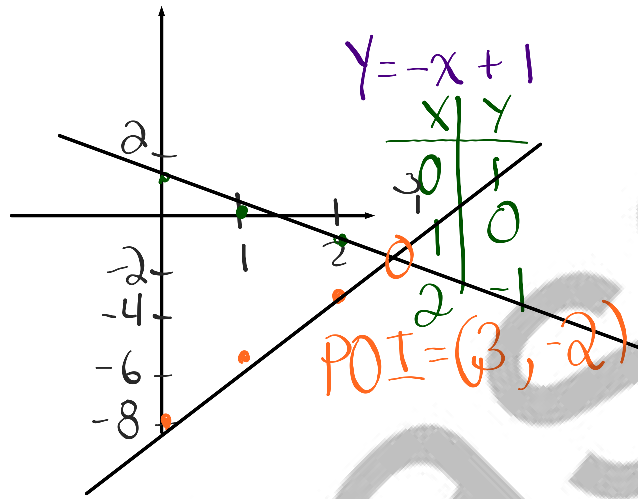
Let's study the GRAPHING METHOD

Find the POI for $\begin{cases} \boxed{y = 2x - 8} \\ \boxed{y = -x + 1} \end{cases}$

Solution

$Y = 2x - 8$

x	y
0	-8
1	-6
2	-4



$Y = -x + 1$

x	y
0	1
1	0
2	-1

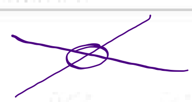
POI = (3, -2)

Let's study the COMPARISON METHOD

Format: $\begin{cases} y = ax + b \\ y = ax + b \end{cases}$

Both equations are in functional form

Example 1: Solve $\begin{cases} y = \boxed{2x - 8} \\ y = \boxed{-x + 1} \end{cases}$



Solution: Make the 2 equations equal to each other.

$2x - 8 = -\cancel{x} + 1$

$+x \qquad \cancel{+x}$

$3x - \cancel{8} = 1$

$\cancel{+8} \qquad +8$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

To find y

plug in x=3

in ANY

equation.

$Y = 2(3) - 8$

$Y = -2$

POI = (3, -2)

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Example 2: Solve $\begin{cases} y = \boxed{-\frac{2}{5}x + 10} \\ y = \boxed{-\frac{1}{2}x - 6} \end{cases}$

<u>Solution</u>

$10\left(-\frac{2}{5}x + 10 = -\frac{1}{2}x - 6\right)$

LCM of 5 and 2

or do $5 \times 2 = 10$

$-4x + 100 = -5x - 60$

$-4x + 5x = -60 - 100$

$x = -160$

$y = -\frac{2}{5}(-160) + 10$

$y = 74$

POI $= (-160, 74)$

$\quad\quad\quad\; x \quad\; y$

Let's study the **SUBSTITUTION METHOD**

Format: $\begin{cases} \boxed{y = ax + b} \to \text{functional} \\ \boxed{ax + by = c} \to \text{general} \end{cases}$

Example 1: Find POI for $\begin{cases} y = \boxed{2x + 5} \\ -3x - 2y = 25 \end{cases}$ Replace here

Distribute

Solution: Replace the functional equation into the other one.

$-3x - 2(2x + 5) = 25$

$-3x - 4x - 10 = 25$

$\frac{-7x}{-7} = \frac{35}{-7}$

$x = -5$

To find y, plug in $x = -5$ in any equation

$y = 2(-5) + 5$

$y = -5$

POI $= (-5, -5)$

$\quad\quad\quad x \quad y$

Example 2: Find POI for $\begin{cases} x = \boxed{2y - 8} \\ 2x + 5y = 56 \end{cases}$

Distribute

Solution:

$2(2y-8) + 5y = 56$

$4y - 16 + 5y = 56$

$\frac{9y}{9} = \frac{72}{9}$

$y = 8$

$x = 2(8) - 8$

$x = 8$

POI = $(\underset{x}{8}, \underset{y}{8})$

Example 3: Find the POI $\begin{cases} y = \boxed{\frac{2}{3}x + 8} \\ 3y - 4x = 20 \end{cases}$

Dist

Solution:

$3\left(\frac{2}{3}x + 8\right) - 4x = 20$

$2x + 24 - 4x = 20$

$-2x = -4$

$x = 2$

Find y

$y = \frac{2}{3}(2) + 8$

$y = 9.3$

$(2, 9.3)$

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Let's study the ELIMINATION METHOD

Format: $\begin{cases} ax + by = c \\ ax + by = c \end{cases}$ → GENERAL

Example 1: Solve $\begin{cases} 2x + 3y = 60 \\ 2x + 4y = -80 \end{cases}$

Solution:

$2x + 3y = 60$

$+ \; -(2x + 4y = -80)$

Distribute to all terms.

$\cancel{2x} + 3y = 60$

$+ \; \cancel{-2x} - 4y = 80$

$-1y = 140$

$\frac{-1y}{-1} = \frac{140}{-1}$

$\boxed{y = -140}$

To find x $\quad 2x + 3(-140) = 60$

$2x - 420 = 60$

$2x = 480$

$\boxed{x = 240}$

$POI = (240, -140)$

Example 2: Solve $\begin{cases} 2x + 6y = 80 \\ 1x + y = 10 \end{cases}$

Solution: Need the same # but opposite signs.

$+ \; 2x + 6y = 80$

$-2(1x + y = 10)$

$\cancel{2x} + 6y = 80$

$+ \; \cancel{-2x} - 2y = -20$

$4y = 60$

$\frac{4y}{4} = \frac{60}{4}$

$\boxed{y = 15}$

To find x $\quad x + y = 10$

$x + 15 = 10$

$\boxed{x = -5}$

$POI = (-5, 15)$

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Example 3: Find POI for $\begin{cases} -2x - 3y = 60 \\ 3x + 2y = 100 \end{cases}$

Solution: careful

$3(-2x - 3y = 60) \quad -6x - 9y = 180$

$2(3x + 2y = 100) \quad + \; 6x + 4y = 200$

$-5y = 380$

$\frac{-5y}{-5} = \frac{380}{-5}$

$\boxed{y = -76}$

To find x:

$-2x - 3(-76) = 60$

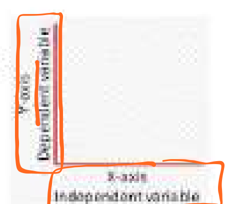
$-2x + 228 = 60$

$\boxed{x = 84}$

POI = (84, -76)

Independent vs. Dependent Variables

The independent (x) variable is the "thing" that can stand alone. If we increase or decrease this factor, it will impact the dependent variable.



Usually: $\begin{cases} time\ (\#\ of\ min, hours) \\ \#\ of\ students, adults \\ \#\ of\ cars \\ amount\ of\ products \end{cases}$

Amount

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The dependent (y) variable relies on and feels the effect of the independent variable. It changes depending on how much of the other variable you have.

Usually: $\begin{cases} Cost \\ Price \end{cases}$ \$

ASK YOURSELF: Which variable depends on the other?

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Examples: Name the variables.

1. The amount of time you study vs. the grade on your test.
   indep. dependent
2. dep
   Your muscle weight vs. the amount of time training at gym.
   indep
3. The amount of hockey tickets vs. the total cost.
   indep dep
4. dep
   The time it takes to drain a bath tub vs. the number of liters the bath tub contains.
   indep

We will study 4 types of word problems.

TYPE 1:

Both equations are

'a' is the amount per $\begin{cases} hour \\ day \\ student \end{cases}$

'b' is the amount paid $\boxed{\text{once}}$

$\begin{cases} a\ membership\ to\ a\ gym \\ bonus\ at\ a\ job \end{cases}$

Solve by COMPARISON

TYPE 2:

Both equations are $\begin{cases} ax + by = c \\ ax + by = c \end{cases}$ total

'c' is usually the total of the left side.

Solve by elimination

TYPE 3:

Equations are $\begin{cases} ax + by = c \\ x + y = \# \end{cases}$ Total

Solve by substitution

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TYPE 4:

Two coordinates are given $\begin{cases}(x, y)\\(x, y)\end{cases}$

Solve by finding the equation of a line

→ Find 'a' by $a = \boxed{\frac{y_2 - y_1}{x_2 - x_1}}$

→ Find 'b' by plugging in a point

Special Types of Lines

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Oct 15-5:11 PM