# Chapter 3 – Analytic Geometry

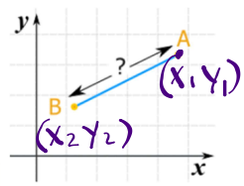
## Sub-topics:

- ➔ Distance
- ➔ Midpoint
- ➔ Division Point



### Distance between 2 points

**The distance or length between 2 points $A$ and $B$ can be found by:**




$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**The result is a #.**

**Note: You can label $(x_1, y_1)$ either A or B.**

**Examples:**

1. **Find the distance between** $E(-10,50)$ **and** $F(-20,-70)$.

$X_1\ Y_1 \qquad X_2\ Y_2$

$$d(E,F)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(-20--10)^2+(-70-50)^2}$$

$$\sqrt{(-10)^2+(-120)^2}$$

$$\sqrt{100+14400}$$

$$\sqrt{14500}$$

$$120.42 \text{ units}$$

2. **Find the length between** $M(15,27)$ **and** $N(-14,20)$.

distance

$X_1\ Y_1 \qquad X_2\ Y_2$

$$d(M,N)=\sqrt{(-14-15)^2+(20-27)^2}$$

$$=\sqrt{(-29)^2+(-7)^2}$$

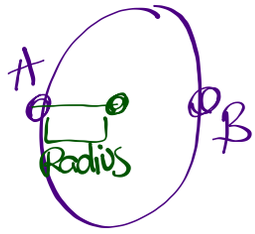
$$=\sqrt{841+49}$$

$$=29.83 \text{ units}$$

**3. Find the radius of a circle if the diameter's endpoints are $A(15, 20)$ and $B(20, 40)$.**

distance

$x_1\ y_1$ $x_2\ y_2$




$d(A,B) = \sqrt{(20-15)^2+(40-20)^2}$

$= 20.62$ units

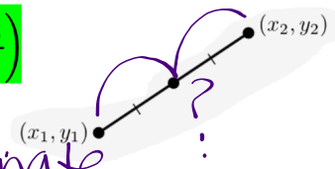
$\text{Radius} = \frac{20.62}{2} = 10.31$ units



**Midpoint of a segment**

**To find the coordinates** halfway **between a segment with endpoints $A$ and $B$, we use:**

$$M(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$




**The result is a** coordinate **.**

**Examples:**

1. **Find the midpoint between** $E(-3, 10)$ **and** $F(-6, 20)$**.**



(1)

2. **Find the coordinates of the point halfway between** $M(10,20)$ **and** $N(-40,60)$.

$M(x,y) = \left(\frac{10-40}{2}, \frac{20+60}{2}\right)$

$(15, 40)$

2

**3. Find the center of a circle if the diameter's endpoints are** $A(-10, 40)$ **and** $B(-50, -80)$.

$x_1\ y_1 \qquad x_2\ y_2$

$$M(x,y) = \left(\frac{-10 + -50}{2}, \frac{40 + -80}{2}\right)$$

$$(-30, -20)$$

**(Optional)**

**Finding an endpoint given midpoint**

**(Midpoint Backwards)**

**Example 1:** $M(6,3)$ **is the midpoint of segment** $\overline{AB}$ **with point** $A(2,0)$. **Find the coordinates of** $B$.

**Example 2: Find the endpoint**

**A)** $M(6,4)$ **and** $A(2,4)$

**B)** $M(6,1)$ **and** $T(10,4)$

## Division Point

**Convert the fractions to a ratio**

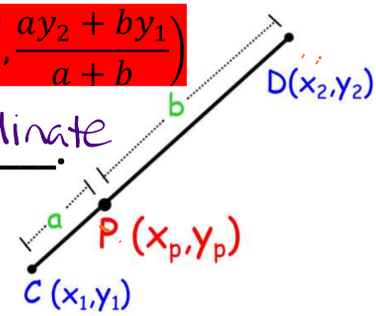
| | |
|---|---|
| $\frac{2}{5}$ | $2:3$ |
| $\frac{3}{4}$ | $3:1$ |
| $\frac{5}{7}$ | $5:2$ |

**Note: A ratio may already be given as $a:b$.**

**The point of division is the point $P(x_p, y_p)$ that divides a segment with endpoints $C$ and $D$ in a specific ratio $a:b$.**

$$P(x_p, y_p) = \left(\frac{ax_2 + bx_1}{a+b}, \frac{ay_2 + by_1}{a+b}\right)$$

**The result is a** _____.



**Examples:** $x_1\ y_1$ $x_2\ y_2$

**1)** $A(10,-20)$ $B(30,20)$

$3:2$ *from A*

$a=3$ $b=2$


$$P=\left(\frac{ax_2+bx_1}{a+b},\frac{ay_2+by_1}{a+b}\right)$$

$$=\left(\frac{3(30)+2(10)}{3+2},\frac{3(20)+2(-20)}{3+2}\right)$$

$$\left(\frac{110}{5},\frac{20}{5}\right)$$

$$(22,4)$$

$x_2\ y_2$ $x_1\ y_1$

**2)** $C(-6,10)$ $D(-14,20)$

$\frac{3}{4}$ *from D*

$a=3$

$b=1$

$$P=\left(\frac{3(-6)+1(-14)}{3+1},\frac{3(10)+1(20)}{3+1}\right)$$

$$=\left(\frac{-32}{4},\frac{50}{4}\right)$$

$$(-8,12.5)$$