Chapter 5

Systems

CHALLENGE 5

5.1 System of first degree equations with two variables

5.2 Algebraic solving of a two-variable first degree system

EVALUATION 5

CHALLENGE 5

1. At a bookstore, Sylvie pays \$13.60 for 4 notebooks and 3 pens whereas Katherine pays \$9.90 for 3 notebooks and 2 pens. How much will Raphaëlle pay for 2 notebooks and 4 pens? ***\$9.80***

2. At a department store, they sell 50 ml and 100 ml perfume bottles. Seventy bottles give a total of 5 litres of perfume. If each 50 ml bottle is sold for \$45 and each 100 ml bottle is sold for \$72, what is the total revenue from the sale of all 70 bottles? ***\$3960***

3. To raise money for the fight against juvenile diabetes, a school organizes a walk during three afternoons. Let x represent the amount raised by each student and y represent the amount raised by each teacher.

The 1st afternoon, 110 students and 25 teachers walked and raised a total amount of \$2760.
The 2nd afternoon, 90 students and 12 teachers walked and raised a total amount of \$1920.
How many teachers walked on the 3rd afternoon, if 140 students walked and a total of \$2960 was raised?

$110x + 25y = 2760$	$x = 16,\ y = 40$
$90x + 12y = 1920$	$(2960 - 140 \times 16) \div 40$

18 teachers walked on the 3rd afternoon.

4. A jar contains red, green and yellow marbles. Phil, Eric and Ellie each draw 8 marbles. The number of points awarded for each yellow marble drawn is 10 points.
Phil drew 4 red marbles, 2 green marbles and 2 yellow marbles for a total of 42 points.
Eric drew 2 red marbles, 5 green marbles and 1 yellow marble for a total of 33 points.
Who is the winner of this game if Ellie drew 3 red marbles, 3 green marbles and 2 yellow marbles?

x: number of points per red ball	$4x + 2y + 20 = 42$ $\quad x = 4,\ y = 3$
y: number of points per green ball	$2x + 5y + 10 = 33$

Ellie has 41 points. Phil is the winner.

5. In a school, there are 85 students in secondary 4. There are three times as many students in the Technical and science math option as there are in the Science option. If there are 21 students in the Cultural – social and technical option, how many students are in the Technical and science option?

x: number of students in TS	$x + y + 21 = 85;\ x = 48$
y: number of students in SN	$x = 3y$

There are 48 students in Technical and science option.

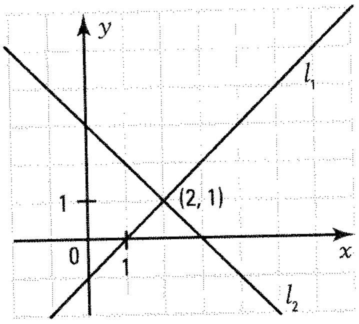
5.1 System of first degree equations with two variables

ACTIVITY 1 Graphic representation of a system

a) Given the line l_1 with the equation: $x - y - 1 = 0$.
This equation is a first degree equation with two variables. The ordered pair (4, 3) is a solution to this equation.

1. Find another ordered pair that is a solution to the equation: $x - y - 1 = 0$. ***(1, 0)***
2. How many ordered pairs are solutions to this equation? ***An infinite number of ordered pairs***
3. Complete: The graphic representation of the set of all solutions to the equation: $x - y - 1 = 0$ is ***the set of all points on the line l_1.***
4. Represent, in the Cartesian plane, the set of all solutions to the equation: $x - y - 1 = 0$.

b) In the same Cartesian plane, draw the line l_2 representing the set of all solutions to the equation: $x + y - 3 = 0$.



c) 1. Explain how, using the graph on the right, to find the common solution to the equations $x - y - 1 = 0$ and $x + y - 3 = 0$.
Find the point of intersection of the two lines.
2. What is this common solution? ***(2, 1)***
3. Verify that this is the common solution.
2 - 1 - 1 = 0 and 2 + 1 - 3 = 0

SYSTEM OF FIRST DEGREE EQUATIONS WITH TWO VARIABLES

- A system of first degree equations with two variables is any system that can be written in the form:

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

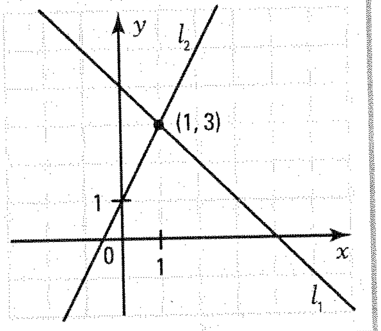
where x and y represent the variables and $a_1, b_1, c_1, a_2, b_2, c_2$ are real constants.

Solving a system: Graphical method

- To solve graphically a system of first degree equations with two variables, you need to represent, in the same Cartesian plane, the solution set of each equation and determine the set S of all ordered pairs (x, y) that verify simultaneously both equations.

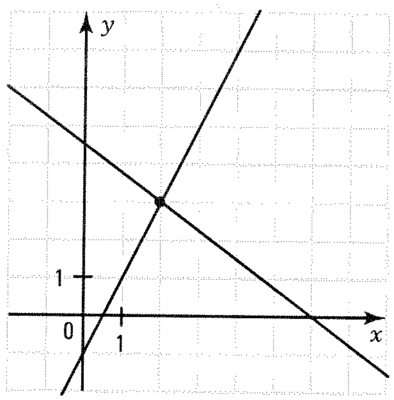
Ex.: The system $\begin{cases} x + y = 4 \\ 2x - y = -1 \end{cases}$

is represented in the Cartesian plane on the right by two lines l_1 and l_2 with equations l_1: $x + y = 4$ and l_2: $2x - y = -1$.
The point (1, 3) is common to both intersecting lines l_1 and l_2.
The point (1, 3) therefore simultaneously verifies both equations of the system.
The solution set of the system is therefore: $S = \{(1, 3)\}$.



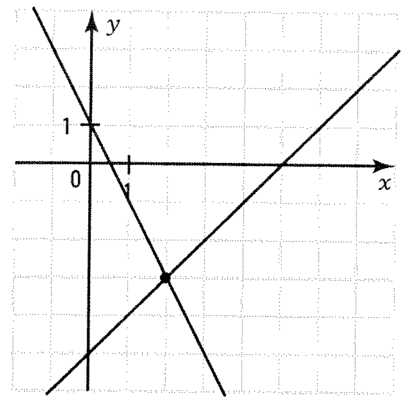
1. Solve the following systems graphically.

a) $\begin{cases} 3x + 4y = 18 \\ -2x + y = -1 \end{cases}$



S = {(2, 3)}

b) $\begin{cases} y = -2x + 1 \\ y = x - 5 \end{cases}$



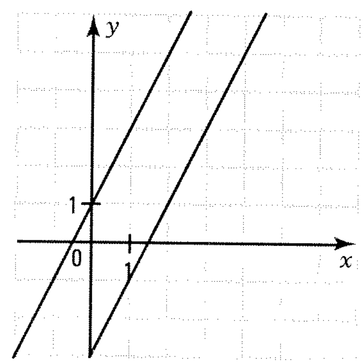
S = {(2, –3)}

ACTIVITY 2 Particular systems

a) Consider the system $\begin{cases} 2x - y = 3 \\ 4x - 2y = -2 \end{cases}$ of the form $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$.

1. Solve this system graphically. *S = Ø*
2. Justify your answer. *The lines are parallel and distinct.*
3. Identify the coefficients:

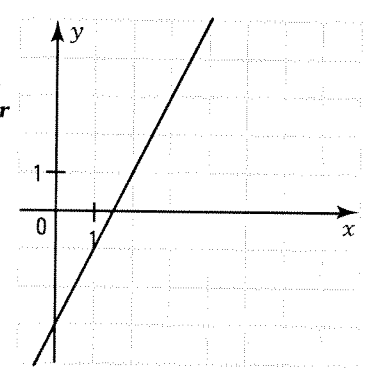
   $a_1 =$ **2** $a_2 =$ **4** $b_1 =$ **-1** $b_2 =$ **-2** $c_1 =$ **3** $c_2 =$ **-2**
4. Verify that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. $\frac{2}{4} = \frac{-1}{-2} \neq \frac{3}{-2}$



b) Consider the system $\begin{cases} 2x - y = 3 \\ 4x - 2y = 6 \end{cases}$ of the form $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$.

1. Represent this system graphically. What do you notice?
   The lines are parallel and coincident.
2. How many solutions does this system have? *An infinite number*
3. Where are the solutions to this system located?
   On the line 2x – y = 3
4. Identify the coefficients:

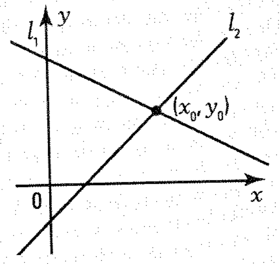
   $a_1 =$ **2** $a_2 =$ **4** $b_1 =$ **-1** $b_2 =$ **-2** $c_1 =$ **3** $c_2 =$ **6**
5. Verify that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. $\frac{2}{4} = \frac{-1}{-2} = \frac{3}{6}$



SYSTEM OF FIRST DEGREE EQUATIONS WITH TWO VARIABLES

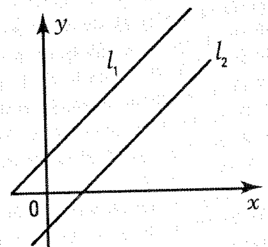
When solving the system $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$, we distinguish the following 3 cases:

- 1st case: $\boxed{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$ $\boxed{S = \{(x_0, y_0)\}}$



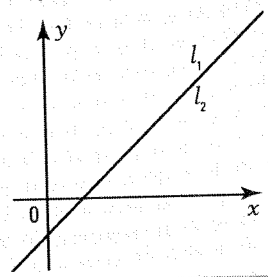
The lines l_1 and l_2 are intersecting.
The system has one unique solution. The system is called compatible.

- 2nd case: $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$ $\boxed{S = \varnothing}$



The lines l_1 and l_2 are parallel and distinct.
The system has no solution. The system is called incompatible.

- 3rd case: $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$ $\boxed{S = \{(x, y) \mid a_1x + b_1y = c_1\}}$



The lines l_1 and l_2 are coincident.
The system has an infinite number of solutions. The system is called indeterminate.

2. Indicate, for each case, the number of solutions to the system. Justify your answer.

a) $\begin{cases} -2x + 3y = 6 \\ 2x - y = 2 \end{cases}$

One solution

$\frac{-2}{2} \neq \frac{3}{-1}$

b) $\begin{cases} 2x - 5y = 12 \\ 4x - 10y = 24 \end{cases}$

An infinite number of solutions

$\frac{2}{4} = \frac{-5}{-10} = \frac{12}{24}$

c) $\begin{cases} 2x - 5y = 12 \\ 4x - 10y = 10 \end{cases}$

No solution

$\frac{2}{4} = \frac{-5}{-10} \neq \frac{12}{10}$

ACTIVITY 3 Solving a problem using a system of equations

To finance their graduation activities, the senior students of a high school have decided to sell t-shirts and long sleeve shirts. The table below indicates their profit according to the number of shirts sold.

Establish a procedure for calculating their profit if the students sell 120 t-shirts and 300 long sleeve shirts.

Number of shirts sold		Profit
t-shirts	long sleeve	
150	200	$1000
100	400	$1200

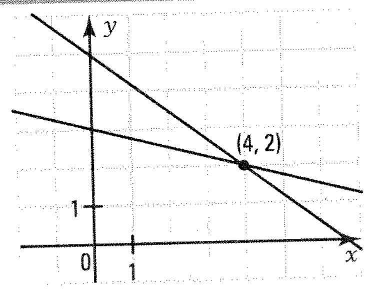
1° Let x represent the profit per t-shirt sold and y represent the profit per long sleeve shirt sold.

2° Set up a system of two equations with two variables.

$\begin{cases} 150x + 200y = 1000 \\ 100x + 400y = 1200 \end{cases}$ or $\begin{cases} 3x + 4y = 20 \\ x + 4y = 12 \end{cases}$

3° Solve the system. S = {(4, 2)}

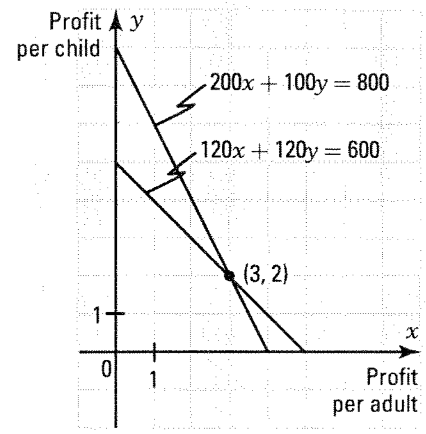
4° Answer the question.
Profit per t-shirt: $4; Profit per long sleeve shirt: $2.
Total profit: 120 × 4 + 300 × 2 = $1080.



SOLVING A PROBLEM USING A SYSTEM OF EQUATIONS

Problem: The profit made by a travel agency organizing tours depends on the number of adults and children taking the tour.

Number of adults	Number of children	Profit ($)
200	100	800
120	120	600



We want to calculate the agency's profit for a tour with 150 adults and 60 children.

Procedure:

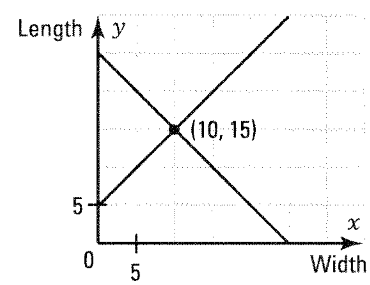
1. **We define the variables.**
   x: profit (in $) per adult.
   y: profit (in $) per child.
2. **We set up the system.**
   $$\begin{cases} 200x + 100y = 800 \\ 120x + 120y = 600 \end{cases}$$
3. **We solve the system** to determine the values of x and y.
   S = {(3, 2)}. The agency makes a profit of $3 per adult and $2 per child.
4. **We answer the question.**
   The profit made from a tour with 150 adults and 120 children is equal to $690.

3. A rectangular field has a perimeter of 50 hm.
The length of the field is 5 hm greater than the width. What is the area of the field?

x: width; y: length

$$\begin{cases} 2x + 2y = 50 \\ y = x + 5 \end{cases} \Rightarrow S = \{(10, 15)\}$$

Area = 10 × 15 = 150 hm²

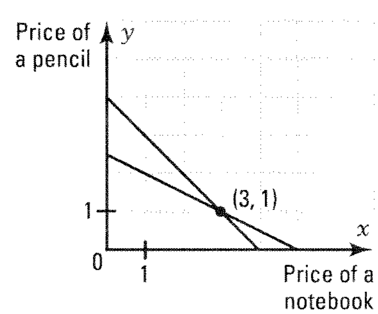


4. Karen buys 2 notebooks and 4 pencils for $10 whereas Valerie buys 2 notebooks and 2 pencils for $8. How much will David pay for 3 notebooks and 4 pencils?

x: price of a notebook; y: price of a pencil

$$\begin{cases} 2x + 4y = 10 \\ 2x + 2y = 8 \end{cases} \Rightarrow S = \{(3, 1)\}$$

David will pay $13.



5.2 Algebraic solving of a two-variable first degree system

ACTIVITY 1 Solving by addition

Consider the system $\begin{cases} 3x + 2y = 7 & (1) \\ 2x - 3y = -4 & (2) \end{cases}$ ← The coefficients of x are not opposite.

a) By multiplying both sides of the 1st equation by 2 and both sides of the 2nd equation by –3, we obtain the following system: $\begin{cases} 6x + 4y = 14 & (3) \\ -6x + 9y = 12 & (4) \end{cases}$ ← The coefficients of x are opposite.

Explain why the resulting system is equivalent to the initial system and therefore has the same solution set.

The equations (1) and (3) and the equations (2) and (4) are equivalent.

b) Add the same sides of the equations of the second system to determine the value of the variable y.

$13y = 26 \Rightarrow y = 2$

c) What is the value of the variable x? ***$x = 1$***

d) What is the solution set of the system? ***$S = \{(1, 2)\}$***

ALGEBRAIC SOLVING OF A SYSTEM: ADDITION METHOD

The addition method for solving a system (also called elimination method) is illustrated in the following example.

Given the system $\begin{cases} 2x + 5y = -4 \\ 3x - 2y = 13 \end{cases}$.

$$\begin{matrix} \times 3 \\ \times -2 \end{matrix} \begin{cases} 2x + 5y = -4 \\ 3x - 2y = 13 \end{cases}$$

- We multiply the sides of each equation by a non zero real number in order to get opposite coefficients for the variable x (or the variable y).

$$\begin{cases} 6x + 15y = -12 \\ -6x + 4y = -26 \end{cases}$$

- We add the same sides of the equations of the resulting system to obtain an equation in only one variable.

$$19y = -38$$

- We determine the value of this variable.

$$y = -2$$

- We substitute this value into one of the system's equations to deduce the value of the other variable.

$$2x + 5(-2) = -4$$
$$x = 3$$

- We establish the solution set S of the system.

$$S = \{(3, -2)\}$$

1. Solve the following systems by addition.

a) $\begin{cases} 3x + 5y = 9 \\ 2x + y = -1 \end{cases}$ ***$S = \{(-2, 3)\}$***

b) $\begin{cases} 5x + 3y = -3 \\ 3x + 2y = -1 \end{cases}$ ***$S = \{(-3, 4)\}$***

c) $\begin{cases} -3x + 10y = 2 \\ x - 5y = 1 \end{cases}$ ***$S = \{(-4, -1)\}$***

d) $\begin{cases} x+6y=6 \\ x-4y=1 \end{cases}$

S = $\left\{\left(3, \frac{1}{2}\right)\right\}$

e) $\begin{cases} 4x+y=-1 \\ 8x+3y=0 \end{cases}$

S = $\left\{\left(-\frac{3}{4}, 2\right)\right\}$

f) $\begin{cases} 3x+2y=-1 \\ 6x-4y=10 \end{cases}$

S = $\left\{\left(\frac{2}{3}, -\frac{3}{2}\right)\right\}$

2. In each of the following situations,

1. identify the variables.
2. translate the situation into a system of two first degree equations with two variables.
3. solve the system by addition and give a complete answer.

a) The sum of two numbers is equal to 20 and their difference is equal to 4. What is the product of these two numbers?

1. ***x*: first number** / ***y*: second number**
2. $\begin{cases} x+y=20 \\ x-y=4 \end{cases}$
3. **S = {(12, 8)}** — **The product is equal to 96.**

b) The perimeter of a rectangular yard is equal to 60 m. If we double its length and triple its width, the perimeter is then equal to 144 m. What is the area of the initial yard?

1. ***x*: length of yard** / ***y*: width of yard**
2. $\begin{cases} 2x+2y=60 \\ 4x+6y=144 \end{cases}$
3. **S = {(18, 12)}** — **The area is equal to 216 m².**

c) Julia buys two sweaters and three pairs of pants for \$220 in a store. Evelyn buys three sweaters and two pairs of pants in the same store for \$230.

How much will Sandra pay for four sweaters and two pairs of pants in this same store?

1. ***x*: price of a sweater** / ***y*: price of a pair of pants**
2. $\begin{cases} 2x+3y=220 \\ 3x+2y=230 \end{cases}$
3. **S = {(50, 40)}** — **Sacha will pay \$280.**

d) Raphael buys a certain number of 50¢ and 10¢ stamps. If he pays \$6.30 for a total of 19 stamps, how many of each type of stamp did he buy?

1. ***x*: number of 50¢ stamps** / ***y*: number of 10¢ stamps**
2. $\begin{cases} 50x+10y=630 \\ x+y=19 \end{cases}$
3. **S = {(11, 8)}** — **Raphael bought eleven 50¢ stamps and eight 10¢ stamps.**

e) Caroline is working this summer at a grocery store and at a pharmacy. The first week, she earned \$138 working 12 h at the grocery store and 8 h at the pharmacy. The second week, she earned \$142 working 8 h at the grocery store and 12 h at the pharmacy. How much will she earn the third week if she works 10 h at the grocery store and 14 h at the pharmacy? **\$170**

f) A jar contains a total of 20 red, black and blue marbles. There are two more black marbles than red marbles and four more blue marbles than black marbles. How many marbles of each color are there? **4 red, 6 black and 10 blue**

g) In a warehouse, there is a total of 32 boxes. There are small 240 dm³ boxes and large 320 dm³ boxes. If the total volume occupied by these boxes is 9280 dm³, how many small boxes are there? **12 small boxes**

ACTIVITY 2 Solving by substitution

Consider the system $\begin{cases} -x+y=1 & (1) \\ 2x+3y=13 & (2) \end{cases}$.

a) Express y as a function of x using the equation (1). **$y = x + 1$**

b) Replace the variable y in equation (2) by the expression in x obtained in a). **$2x + 3(x + 1) = 13$**

c) Solve the resulting equation obtained in b) to determine the value of x. ***x = 2***

d) Deduce the value of the other variable y. ***y = 3***

e) What is the solution set of the system? ***S = {(2, 3)}***

ALGEBRAIC SOLVING OF A SYSTEM: SUBSTITUTION METHOD

The substitution method for solving a system is illustrated in the following example.

Given the system $\begin{cases} 3x + 4y = -6 \\ 2x + y = 1 \end{cases}$

- We isolate one of the variables using one of the system's equations. $y = -2x + 1$
- In the other equation, we substitute the isolated variable by the obtained expression. $3x + 4(-2x + 1) = -6$
- We solve the equation $3x - 8x + 4 = -6$, $x = 2$
- Then, we substitute this resulting value into one of the system's equations and deduce the value of the other variable. $3(2) + 4y = -6$, $y = -3$
- We establish the solution set S of the system. $S = \{(2, -3)\}$

3. Solve the following systems by substitution.

a) $\begin{cases} 2x - 3y = -7 \\ y = 2x - 3 \end{cases}$ ***S = {(4, 5)}***

b) $\begin{cases} y = 2x - 5 \\ 2x - 5y = 9 \end{cases}$ ***S = {(2, −1)}***

c) $\begin{cases} x = 3y + 1 \\ 2x - 5y = 3 \end{cases}$ ***S = {(4, 1)}***

4. In each of the following situations,

1. identify the variables.
2. translate the situation into a system of two first degree equations with two variables.
3. solve the system by substitution and give a complete answer.

a) The length of a rectangular plot of land measures 10 m more than twice its width. The plot has a perimeter of 110 m. How much does this plot cost if it is sold for $50 per square metre?

1. ***x: width of yard***
   y: length of yard
2. $\begin{cases} y = 2x + 10 \\ 2x + 2y = 110 \end{cases}$
3. ***S = {(15, 40)}***
   $30 000

b) A father is 10 years older than three times his son's age.
The sum of their ages is 58. What is the difference between the father's age and his son's?

1. ***x: son's age***
   y: father's age
2. $\begin{cases} y = 3x + 10 \\ x + y = 58 \end{cases}$
3. ***S = {(12, 46)}***
   The father is 34 years older than is son

c) Nathalie earns an hourly wage of $7.50 from her employer whereas Eric earns an hourly wage of $6.50 at his job.
Over the course of a weekend, Nathalie worked 4 h less than Eric. Together, they earned a total of $194. How much would they have earned if Nathalie had worked the same number of hours as Eric?

1. ***x: Eric's number of hours***
   y: Nathalie's number of hours
2. $\begin{cases} y = x - 4 \\ 6.5x + 7.5y = 194 \end{cases}$
3. ***S = {(16, 12)}***
   They would have earned a total of $224.

d) In a class of 30 students, there are six more boys than girls. What percentage of this class are girls?

1. ***x: number of girls***
y: number of boys

2. $\begin{cases} x + y = 30 \\ y = x + 6 \end{cases}$

3. ***S = {(12, 18)}***
The girls represent 40% of this class.

e) There are 52 boats in a marina: sailboats and speedboats. There are 3 times as many sailboats as speedboats. How many sailboats and speedboats are there?
39 sailboats and 13 speedboats

f) The length of a rectangle measures 5 times its width. If the perimeter of this rectangle is 144 cm, what is its area? ***720 cm²***

ACTIVITY 3 Solving by comparison

Consider the system $\begin{cases} 3x + y = 5 & (1) \\ -2x + y = -5 & (2) \end{cases}$

a) Isolate y in each of the system's equations. What equivalent system is obtained? $\begin{cases} y = -3x + 5 \\ y = 2x - 5 \end{cases}$

b) What equation with the variable x can we deduce by comparing the two equations of the system obtained in a)? ***$-3x + 5 = 2x - 5$***

c) Solve this last equation to determine the value of the variable x. ***$x = 2$***

d) Deduce the value of the variable y. ***$y = -1$***

e) What is the solution set of the system? ***S = {(2, −1)}***

ALGEBRAIC SOLVING OF A SYSTEM: COMPARISON METHOD

The comparison method for solving a system is illustrated in the following example.

Given the system $\begin{cases} -2x + y = 1 \\ 3x + 2y = 9 \end{cases}$

- We isolate the same variable in each equation. $\begin{cases} y = 2x + 1 \\ y = -\frac{3}{2}x + \frac{9}{2} \end{cases}$
- We deduce by transitivity an equation in only one variable. $2x + 1 = -\frac{3}{2}x + \frac{9}{2}$
- We solve the resulting equation. $4x + 2 = -3x + 9$, $x = 1$
- Then, we substitute this value into one of the system's equations and deduce the value of the other variable. $y = 2 \times 1 + 1$, $y = 3$
- We establish the solution set S of the system. $S = \{(1, 3)\}$

5. Solve the following systems by comparison.

a) $\begin{cases} y = 2x + 9 \\ y = -3x - 1 \end{cases}$ ***S = {(−2, 5)}***

b) $\begin{cases} x = 2y + 7 \\ x = -4y - 5 \end{cases}$ ***S = {(3, −2)}***

c) $\begin{cases} y = \frac{3}{4}x + \frac{1}{2} \\ y = \frac{2}{3}x - 1 \end{cases}$ ***S = {(−18, −13)}***

6. In each of the following situations,

1. identify the variables.
2. translate the situation into a system of two first degree equations with two variables.
3. solve the system by comparison and give a complete answer.

a) A school principal has the choice of two transportation companies to organize a field trip for the students.
The first company charges a base amount of \$120 plus \$1.50 per student. The second company charges a base amount of \$80 plus \$2 per student. How many students must come for the transportation costs to be the same for both companies?

1. ***x*: number of students**
   ***y*: total cost**
2. $\begin{cases} y = 1.5x + 120 \\ y = 2x + 80 \end{cases}$
3. ***S* = {(80, 240)}**
   For 80 students, they both charge \$240.

b) Joseph and Nathan are car salesmen for two different dealerships. Joseph receives a weekly base salary of \$350 and a 0.5% commission on his sales. Nathan receives a base salary of \$100 and a 1% commission on his sales. What must be the amount of sales for Joseph and Nathan to receive the same weekly salary?

1. ***x*: amount of sales (\$)**
   ***y*: salary (\$)**
2. $\begin{cases} y = 0.005x + 350 \\ y = 0.01x + 100 \end{cases}$
3. ***S* = {(50 000, 600)}**
   For \$50 000 in sales, Joseph and Nathan both receive a salary of \$600.

c) A line l_1 has a slope of $\frac{3}{2}$ and a y-intercept of −3. A line l_2, perpendicular to l_1, has a y-intercept of 10. What is the point of intersection of these two lines?

1. ***x*: *x*-coordinate of the intersection point**
   ***y*: *y*-coordinate of the intersection point**
2. $\begin{cases} y = \frac{3}{2}x - 3 \\ y = -\frac{2x}{3} + 10 \end{cases}$
3. ***S* = {(6, 6)}**
   The intersection point is (6, 6).

d) Caroline receives a weekly base salary of \$120 plus a \$10 commission for every item sold. Her friend Jessica receives a weekly base salary of \$150 and an \$8 commission for every item sold. How many items must they each sell to earn the same weekly salary?

1. ***x*: number of items sold**
   ***y*: salary (\$)**
2. $\begin{cases} y = 10x + 120 \\ y = 8x + 150 \end{cases}$
3. ***S* = {(15, 270)**
   They must each sell 15 items.

SOLVING A SYSTEM: CHOOSING A METHOD

If a system is written in the form:

- $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$, we usually solve it by addition
- $\begin{cases} a_1x + b_1y = c_1 \\ \quad\quad y = a_2x + b_2 \end{cases}$, we usually solve it by substitution
- $\begin{cases} y = a_1x + b_1 \\ y = a_2x + b_2 \end{cases}$, we usually solve it by comparison.

7. Solve each of the following systems using the appropriate method.

a) $\begin{cases} y = x - 8 \\ y = -2x + 1 \end{cases}$ **_S_ = {(3, −5)}**

b) $\begin{cases} 3x + 2y = -2 \\ 5x + y = 6 \end{cases}$ **_S_ = {(2, −4)}**

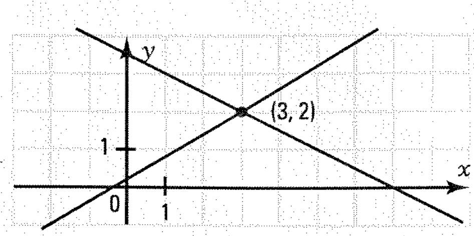
c) $\begin{cases} y = -2x + 7 \\ 5x - 2y = 4 \end{cases}$ **_S_ = {(2, 3)}**

Evaluation 5

1. Solve the following system graphically.

$$\begin{cases} 3x - 5y = -1 \\ x + 2y = 7 \end{cases}$$

***S* = {(3, 2)}**



2. Solve the following systems using an appropriate method.

a) $\begin{cases} 5x + 2y = 5 \\ -4x - 3y = 3 \end{cases}$

***S* = {(3, –5)}**

b) $\begin{cases} 8x + 3y = 5 \\ y = -3x + 1 \end{cases}$

***S* = {(–2, 7)}**

c) $\begin{cases} y = \frac{3}{4}x - \frac{1}{2} \\ y = \frac{2}{3}x + \frac{1}{5} \end{cases}$

***S* =** $\left\{\left(\frac{42}{5}, \frac{29}{5}\right)\right\}$

3. In each of the following situations,

1. identify the variables.
2. translate the situation using a system of equations.
3. answer the question in the problem.

a) In the month of June, a bicycle shop spends $3100 to buy six racing bikes and four mountain bikes. In July, the shop spends $8700 to buy twelve racing bikes and eighteen mountain bikes. How much will the shop spend in August to buy eight racing bikes and ten mountain bikes?

1. ***x*: cost of a racing bike** ***y*: cost of a mountain bike**
2. $\begin{cases} 6x + 4y = 3100 \\ 12x + 18y = 8700 \end{cases}$
3. **$5300**

b) In an office of 50 employees, there are five more men than twice the number of women. How many men and how many women are there in this office?

1. ***x*: number of women** ***y*: number of men**
2. $\begin{cases} x + y = 50 \\ y = 2x + 5 \end{cases}$
3. **35 men and 15 women**

c) The Kandev car rental company charges a basic fee of $15 per day plus 10¢ per km. The Rak car rental company charges a basic fee of $25 per day plus 5¢ per km. What distance must be traveled for the two companies to charge the same amount?

1. ***x*: number of km** ***y*: total cost**
2. $\begin{cases} y = 15 + 0.1x \\ y = 25 + 0.05x \end{cases}$
3. **200 km**

4. A group of 5 adults and 8 children must pay a total of $170 to enter an amusement park. Another group of 4 adults and 6 children must pay a total of $132 to enter the same park. How much will it cost for a group of 7 adults and 12 children to enter this park?

***x* : cost per adult**
***y* : cost per child**

$\begin{cases} 5x + 8y = 170 \\ 4x + 6y = 132 \end{cases}$

***x* = 18 and *y* = 10**
The cost of 7 adults and 12 children will be $246.