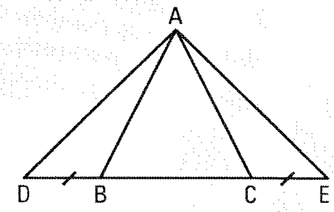
8. Triangle ABC is isosceles with main vertex A and the segments BD and CE are congruent. Justify the steps proving that triangle ADE is isosceles.



Hypothesis: – ΔABC isosceles.
– $\overline{BD} \cong \overline{CE}$.

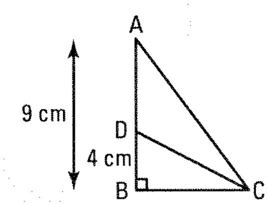
Consider triangles ABD and ACE.

Statement	Justification
1. $\angle ABC \cong \angle ACB$	***The angles at the base of an isosceles triangle are congruent.***
2. $\angle ABD \cong \angle ACE$	***The supplementary angles to two congruent angles are also congruent.***
3. $\overline{BD} \cong \overline{CE}$	***By hypothesis.***
4. $\overline{AB} \cong \overline{AC}$	***Triangle ABC is isosceles with main vertex A.***
5. $\Delta ABD \sim \Delta ACE$	***Theorem of congruence SAS.***
6. $\overline{AD} \cong \overline{AE}$	***Corresponding elements are congruent.***
7. ΔADE is isosceles.	***Sides AD and AE are congruent (definition of an isosceles triangle).***

9. In the figure on the right, triangles ABC and CBD are similar.

Find the perimeter of triangle ABC, to the nearest hundredth.

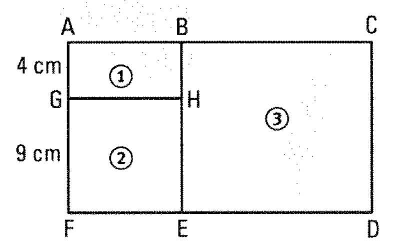
25.82 cm



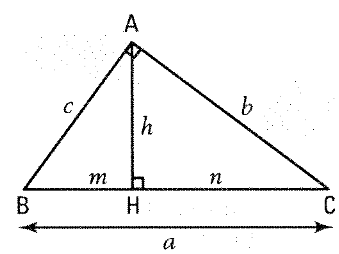
10. In the figure on the right, rectangles ①, ② and ③ are similar.

Determine the perimeter of rectangle ACDF.

77 cm



11. ABC is a right triangle in A and $\overline{AH}$ is the altitude from vertex A. Calculate the missing measures.



a) $b = 1.8$; $c = 2.4$ ***h = 1.44; m = 1.92; n = 1.08; a = 3***

b) $h = 10.8$; $b = 18$ ***c = 13.5; m = 8.1; n = 14.4; a = 22.5***

c) $h = 1.8$; $n = 2.4$ ***a = 3.75; b = 3; c = 2.25; m = 1.35***

d) $m = 1.8$; $n = 3.2$ ***h = 2.4; a = 5; b = 4; c = 3***

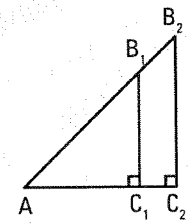
e) $a = 2.5$; $m = 0.9$ ***c = 1.5; h = 1.2; b = 2; n = 1,6***

Chapter 7

Trigonometry

CHALLENGE 7

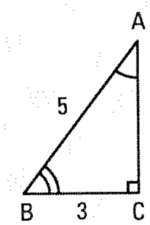
1. Consider the triangles AB_1C_1 and AB_2C_2.



Complete.

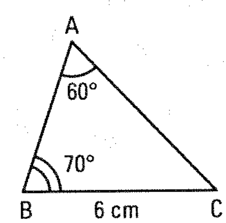
a) $\frac{m\overline{B_1C_1}}{m\overline{AB_1}} = \frac{m\overline{B_2C_2}}{\boldsymbol{m\overline{AB_2}}}$ b) $\frac{m\overline{AC_1}}{m\overline{AB_1}} = \frac{m\overline{AC_2}}{\boldsymbol{m\overline{AB_2}}}$ c) $\frac{m\overline{B_1C_1}}{m\overline{AC_1}} = \frac{m\overline{B_2C_2}}{\boldsymbol{m\overline{AC_2}}}$

2. Calculate.



a) m ∠A ***36.9°*** b) m ∠B ***53.1°***

3. Consider triangle ABC on the right.



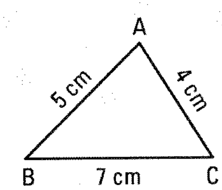
a) Calculate its perimeter to the nearest tenth.

$m\overline{AC}$ = 6.5 cm; $m\overline{AB}$ = 5.3 cm; Perimeter = 17.8 cm

b) Calculate its area to the nearest hundredth.

Area = 14.9 cm²

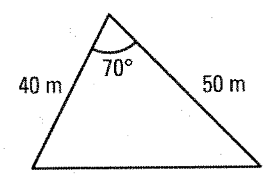
4. Calculate the area of triangle ABC to the nearest tenth.



$p = 8$; Area = $\sqrt{8 \times 3 \times 1 \times 4} \approx 9.8$ cm²

5. Consider the triangular field on the right. The field is sold for \$25 per square metre.

We want to surround this field with a fence. If the price of the fence costs \$12 per metre, calculate, to the nearest dollar, the total cost of the field with the fence.



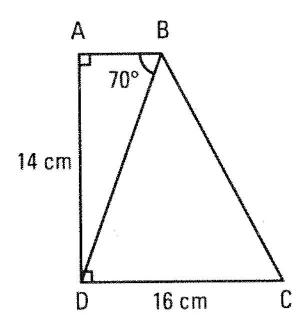
Perimeter ≈ 142 m

Area = $\sqrt{71 \times 31 \times 19 \times 21} \approx 937$ m²

Total cost ≈ \$25 129

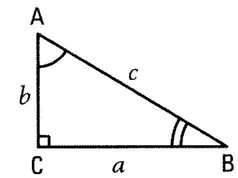
6. What is the area of the trapezoid ABCD on the right?

$m\overline{AB}$ = 5.1 cm; Area of trapezoid = 147.7 cm²



7.1 Trigonometric ratios

ACTIVITY 1 Right triangles



Consider triangle ABC on the right.
Let a, b and c represent the respective measures of the sides BC, AC and AB.

a) Write the Pythagorean Theorem associated with triangle ABC. ***$a^2 + b^2 = c^2$***

b) 1. Which angles are the acute angles? ***∠A and ∠B***

2. Explain why the acute angles are complementary.

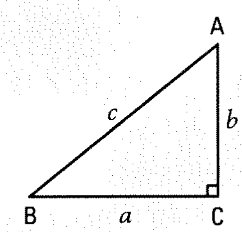
m ∠A + m ∠B + m ∠C = 180° (sum of interior angles of a triangle).

Since m ∠C = 90° then m ∠A + m ∠B = 90°

c) In this triangle, side AB is the hypotenuse, side AC is the opposite side to angle B whereas side BC is the adjacent side to angle B. What is

1. the opposite side to angle A? ***$\overline{BC}$*** 2. the adjacent side to angle A? ***$\overline{AC}$***

RIGHT TRIANGLES



- In any right triangle, the sum of the squares of the sides of the right angle is equal to the square of the hypotenuse.

  We have: $a^2 + b^2 = c^2$

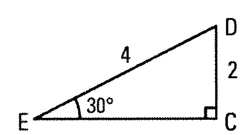
  This relation is called the Pythagorean Theorem.

- The acute angles of a right triangle are complementary.

  $m\angle A + m\angle B = 90°$

- The opposite side AB to the right angle is the hypotenuse.
  Side AC is the opposite side to angle B or the adjacent side to angle A.
  Side BC is the adjacent sid to angle B or the opposite side to angle A.

1. Consider the given right triangle.



a) Find m ∠D. ***60°***

b) Find m$\overline{EC}$. ***$\sqrt{12}$ or $2\sqrt{3}$***

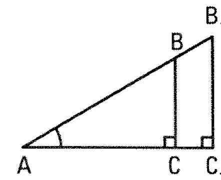
c) Identify

1. the hypotenuse; ***$\overline{ED}$***
2. the opposite side to ∠D; ***$\overline{EC}$***
3. the adjacent side to ∠D; ***$\overline{DC}$***
4. the opposite side to ∠E; ***$\overline{DC}$***
5. the adjacent side to ∠E. ***$\overline{EC}$***

ACTIVITY 2 Trigonometric ratios

a) Which theorem of similarity enables you to justify that triangles ABC and AB_1C_1 on the right are similar?

The theorem of similarity AA. Angle A is common to both triangles and angles BCA and B_1C_1A are congruent. The two triangles ABC and AB_1C_1 have two congruent corresponding angles and are therefore similar.

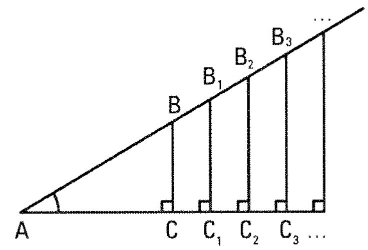


b) Complete: $\frac{m\overline{BC}}{m\overline{B_1C_1}} = \frac{m\overline{AB}}{\boldsymbol{m\overline{AB_1}}}$.

c) By switching the middle terms of the preceding proportion, we get $\frac{m\overline{BC}}{m\overline{AB}} = \frac{m\overline{B_1C_1}}{\boldsymbol{m\overline{AB_1}}}$.

d) 1. What can be said of the right triangles on the right?

They are all similar.



2. Complete: $\frac{m\overline{BC}}{m\overline{AB}} = \frac{m\overline{B_1C_1}}{\boldsymbol{m\overline{AB_1}}} = \frac{m\overline{B_2C_2}}{\boldsymbol{m\overline{AB_2}}} = \frac{m\overline{B_3C_3}}{\boldsymbol{m\overline{AB_3}}} = \ldots$

Therefore, for each triangle, we observe that the ratio of the measure of the opposite side to angle A over the measure of the hypotenuse remains the same. Such a ratio is called sine A.

e) 1. Since triangles ABC and AB_1C_1 are similar, complete: $\frac{m\overline{AC}}{m\overline{AB}} = \frac{m\overline{AC_1}}{\boldsymbol{m\overline{AB_1}}}$.

2. Generalize: $\frac{m\overline{AC}}{m\overline{AB}} = \frac{m\overline{AC_1}}{\boldsymbol{m\overline{AB_1}}} = \frac{m\overline{AC_2}}{\boldsymbol{m\overline{AB_2}}} = \frac{m\overline{AC_3}}{\boldsymbol{m\overline{AB_3}}} = \ldots$

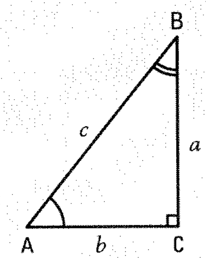
Therefore, for each triangle, we observe that the ratio of the measure of the adjacent side to angle A over the measure of the hypotenuse remains the same. Such a ratio is called cosine A.

f) 1. Since triangles ABC and $A_1B_1C_1$ are similar, complete: $\frac{m\overline{BC}}{m\overline{AC}} = \frac{m\overline{B_1C_1}}{\boldsymbol{m\overline{AC_1}}}$.

2. Generalize: $\frac{m\overline{BC}}{m\overline{AC}} = \frac{m\overline{B_1C_1}}{\boldsymbol{m\overline{AC_1}}} = \frac{m\overline{B_2C_2}}{\boldsymbol{m\overline{AC_2}}} = \frac{m\overline{B_3C_3}}{\boldsymbol{m\overline{AC_3}}} = \ldots$

Therefore, for each triangle, we observe that the ratio of the measure of the opposite side to angle A over the measure of the adjacent side to angle A remains the same. Such a ratio is called tangent A.

TRIGONOMETRIC RATIOS IN A RIGHT TRIANGLE



- The sine of an acute angle is equal to the ratio of the measure of the opposite side to that angle over the measure of the hypotenuse.
  The sine of angle A is written sin A.

$$\sin A = \frac{\text{measure of opposite side}}{\text{measure of hypotenuse}} = \frac{a}{c}$$

- The cosine of an acute angle is equal to the ratio of the measure of the adjacent side to that angle over the measure of the hypotenuse.
  The cosine of angle A is written cos A.

$$\cos A = \frac{\text{measure of adjacent side}}{\text{measure of hypotenuse}} = \frac{b}{c}$$

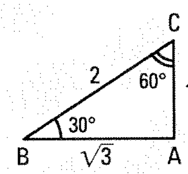
- The tangent of an acute angle is equal to the ratio of the measure of the opposite side to that angle over the measure of the adjacent side to that angle.
  The tangent of angle A is written tan A.

$$\tan A = \frac{\text{measure of opposite side}}{\text{measure of adjacent side}} = \frac{a}{b}$$

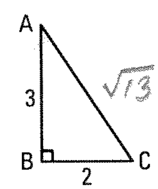
- When writing a trigonometric ratio, we can write the measure of the angle when it is known. Thus, the sine of angle B measuring 30° is written sin 30°.

Ex.: $\sin 30° = \frac{1}{2}$; $\cos 30° = \frac{\sqrt{3}}{2}$; $\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60° = \frac{\sqrt{3}}{2}$; $\cos 60° = \frac{1}{2}$; $\tan 60° = \sqrt{3}$



2. Using the triangle on the right, determine

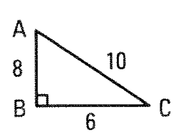


a) sin A. $\frac{2}{\sqrt{13}}$ b) cos A. $\frac{3}{\sqrt{13}}$ c) tan A. $\frac{2}{3}$

d) sin C. $\frac{3}{\sqrt{13}}$ e) cos C. $\frac{2}{\sqrt{13}}$ f) tan C. $\frac{3}{2}$

3. In each of the following cases, determine the value of the sine, cosine and tangent of angle A.

a)

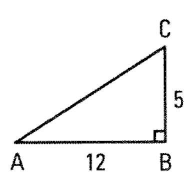


sin A = 0.6

cos A = 0.8

tan A = $\frac{3}{4}$

b)

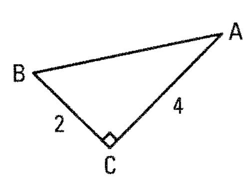


sin A = $\frac{5}{13}$

cos A = $\frac{12}{13}$

tan A = $\frac{5}{12}$

c)

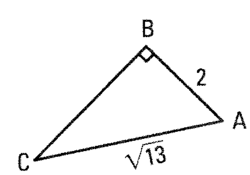


sin A = $\frac{2}{\sqrt{20}}$

cos A = $\frac{4}{\sqrt{20}}$

tan A = $\frac{1}{2}$

d)



sin A = $\frac{3}{\sqrt{13}}$

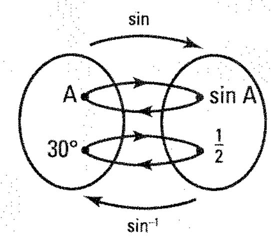
cos A = $\frac{2}{\sqrt{13}}$

tan A = $\frac{3}{2}$

CALCULATOR

- The key [sin] on the calculator enables you to calculate the value of sin A knowing the measure of angle A.
- The key [$\sin^{-1}$] on the calculator enables you to calculate the measure of angle A knowing sin A.

Thus, $\sin 30° = \frac{1}{2}$ and $\sin^{-1}\left(\frac{1}{2}\right) = 30°$.



4. a) Using a calculator, complete the following table.

b) When m ∠A increases from 0° to 90°,

1. sin A increases or decreases? ***sin A increases.***
2. cos A increases or decreases? ***cos A decreases.***
3. tan A increases or decreases? ***tan A increases.***

c) Verify that when two angles are complementary, the sine of one is equal to the cosine of the other.

d) Verify that $\tan A = \frac{\sin A}{\cos A}$.

m ∠A	sin A	cos A	tan A
0°	*0*	*1*	*0*
20°	*0.3420*	*0.9848*	*0.3640*
30°	*0.5*	*0.8660*	*0.5774*
45°	*0.7071*	*0.7071*	*1*
60°	*0.8660*	*0.5*	*1.7321*
80°	*0.9848*	*0.3420*	*5.6713*
90°	*1*	*0*	

5. a) Using a calculator, complete the following table (round m ∠A to the nearest unit).

b) Verify your answers from question 4b).

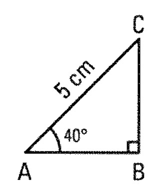
c) Verify question 4 c).

d) Verify question 4 d).

m ∠A	sin A	cos A	tan A
10°	0.1736	*0.9848*	*0.1763*
50°	*0.7660*	0.6428	*1.1918*
75°	*0.9659*	*0.2588*	3.7321
80°	0.9848	*0.1736*	*5.6713*
89°	*0.9998*	0.0175	*57.2900*

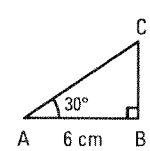
ACTIVITY 3 Find the missing sides

a) Triangle ABC is a right triangle. Angle A measures 40° and the hypotenuse measures 5 cm.



1. Calculate $m\overline{BC}$. $\sin 40° = \frac{m\overline{BC}}{5} \Rightarrow m\overline{BC} = 5 \sin 40° = 3.21$ cm
2. Calculate $m\overline{AB}$. $\cos 40° = \frac{m\overline{AB}}{5} \Rightarrow m\overline{AB} = 5 \cos 40° = 3.83$ cm
3. Verify your results using the Pythagorean Theorem. $(3.21)^2 + (3.83)^2 \simeq 5^2$

b) ABC is a right triangle. Angle A measures 30° and its adjacent side measures 6 cm.

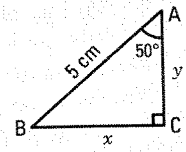


1. Calculate $m\overline{BC}$. $\tan 30° = \frac{m\overline{BC}}{6} \Rightarrow m\overline{BC} = 6 \tan 30° = 3.46$ cm
2. Calculate $m\overline{AC}$. $\cos 30° = \frac{6}{m\overline{AC}} \Rightarrow m\overline{AC} = \frac{6}{\cos 30°} = 6.93$ cm
3. Verify your results using the Pythagorean Theorem. $6^2 + (3.46)^2 \simeq (6.93)^2$

FINDING MISSING SIDES USING TRIGONOMETRIC RATIOS

In a right triangle,

- finding the measure x of side BC opposite to the known angle A, knowing also the measure of the hypotenuse, requires the use of sin A.

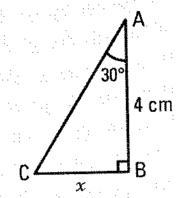


$\sin 50° = \frac{x}{5} \Rightarrow x = 5 \sin 50° \approx 3.83$ cm

- finding the measure y of side AC adjacent to the known angle A, knowing also the measure of the hypotenuse, requires the use of cos A.

$\cos 50° = \frac{y}{5} \Rightarrow y = 5 \cos 50° \approx 3.21$ cm

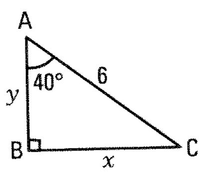
- finding the measure x of side BC opposite to the known angle A, knowing also the measure of the adjacent side to angle A, requires the use of tan A.



$\tan 30° = \frac{x}{4} \Rightarrow x = 4 \tan 30° \approx 2.31$ cm

6. Calculate in each case x and y. (Round each answer to the nearest tenth).

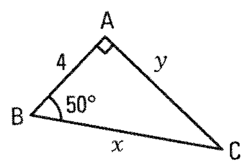
a)



$x = 6 \sin 40° = 3.9$

$y = 6 \cos 40° = 4.6$

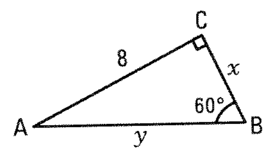
b)



$x = \frac{4}{\cos 50°} = 6.2$

$y = 4 \tan 50° = 4.8$

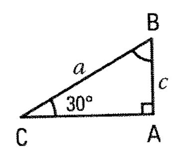
c)



$x = \frac{8}{\tan 60°} = 4.6$

$x = \frac{8}{\sin 60°} = 9.2$

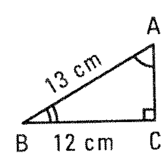
7. Explain why, in a right triangle, the measure of the side opposite to a 30° angle is half the measure of the hypotenuse.



$\sin 30° = \frac{c}{a}$; $\sin 30° = \frac{1}{2} \Rightarrow \frac{c}{a} = \frac{1}{2} \Rightarrow c = \frac{1}{2}a$

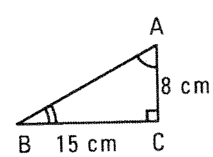
ACTIVITY 4 Finding missing angles

a) Consider the given right triangle ABC. The opposite side to angle A measures 12 cm and the hypotenuse measures 13 cm.



1. Calculate m ∠A. $\sin A = \frac{12}{13} \Rightarrow m\angle A = \sin^{-1}\left(\frac{12}{13}\right) \approx 67.4°$
2. Calculate m ∠B. $\cos B = \frac{12}{13} \Rightarrow m\angle B = \cos^{-1}\left(\frac{12}{13}\right) \approx 22.6°$
3. Verify that angles A and B are complementary. $67.4° + 22.6° = 90°$

b) Consider the given right triangle ABC with the sides of the right angle measuring 8 cm and 15 cm.



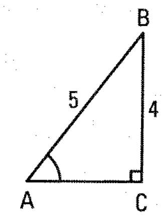
1. Calculate m ∠A. $\tan A = \frac{15}{8} \Rightarrow m\angle A = \tan^{-1}\left(\frac{15}{8}\right) = 61.9°$
2. Calculate m ∠B. $\tan B = \frac{8}{15} \Rightarrow m\angle B = \tan^{-1}\left(\frac{8}{15}\right) = 28.1°$
3. Verify that angles A and B are complementary. $61.9° + 28.1° = 90°$

FINDING MISSING ANGLES USING TRIGONOMETRIC RATIOS

In a right triangle,

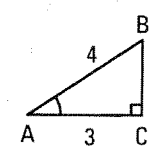
- finding the acute angle A when its opposite side and the hypotenuse are known requires the use of sin A.

  $\sin A = \frac{4}{5} \Rightarrow m\angle A = \sin^{-1}\left(\frac{4}{5}\right) = 53.1°$



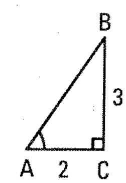
- finding the acute angle A when its adjacent side and the hypotenuse are known requires the use of cos A.

  $\cos A = \frac{3}{4} \Rightarrow m\angle A = \cos^{-1}\left(\frac{3}{4}\right) = 41.4°$



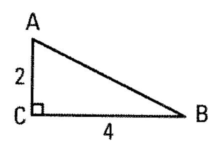
- finding the acute angle A when its opposite side and adjacent side are known requires the use of tan A.

  $\tan A = \frac{3}{2} \Rightarrow m\angle A = \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$



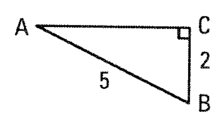
8. Find the measures of angles A and B. (Round each answer to the nearest unit.)

a)



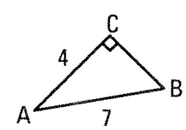
m ∠A = 63°; m ∠B = 27°

b)



m ∠A = 24°; m ∠B = 66°

c)

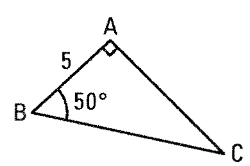


m ∠A = 55°; m ∠B = 35°

Solving a triangle consists of determining the measure of all its sides and angles.

9. Solve the following triangles. (Round your answers to the nearest tenth.)

a)

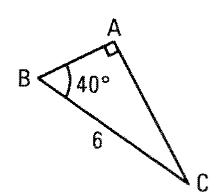


m ∠C = 40°

$m\overline{AC}$ = 6

$m\overline{BC}$ = 7.8

b)

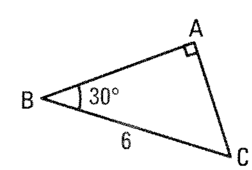


m ∠C = 50°

$m\overline{AB}$ = 4.6

$m\overline{AC}$ = 3.9

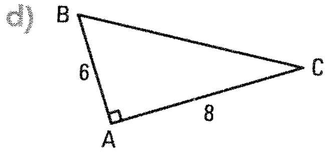
c)



m ∠C = 60°

$m\overline{AB}$ = 5.2

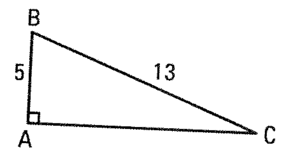
$m\overline{AC}$ = 3

d) 

$m\overline{BC} = 10$

$m\angle B = 53.1°$

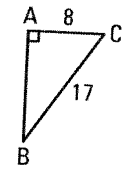
$m\angle C = 36.9°$

e) 

$m\overline{AC} = 12$

$m\angle B = 67.4°$

$m\angle C = 22.6°$

f) 

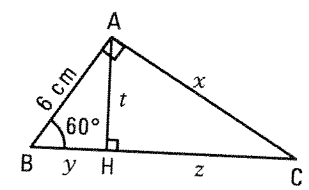
$m\overline{AB} = 15$

$m\angle B = 28.1°$

$m\angle C = 61.9°$

10. In the triangle ABC on the right, the altitude AH is drawn. Determine the value of the unknowns x, y, z, t.

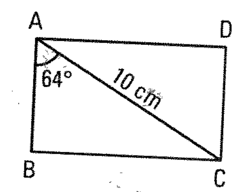
x = 10.4 cm; y = 3.0 cm; z = 9.0 cm; t = 5.2 cm



11. Calculate the area of rectangle ABCD on the right. (Round your answer to the nearest tenth.)

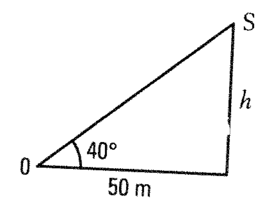
$m\overline{AB}$ = 10 cos 64° = 4.38 cm; $m\overline{BC}$ = 10 sin 64° = 8.99 cm

Area ≈ 39.4 cm²



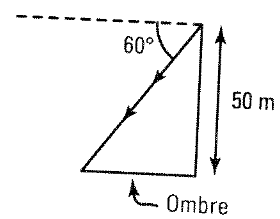
12. An observer 0 is located 50 m away from the base of a building and is looking up to the top S of the building with a 40° angle of elevation. Determine the height h of the building.

h = 50 tan 40° = 42 m



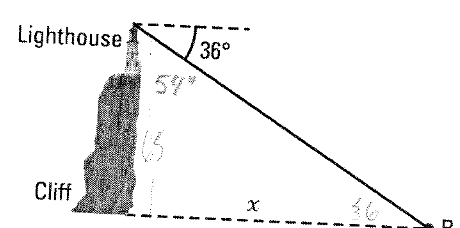
13. What is the length of the shadow cast by a 50 m high building when the sun's rays make a 60° angle of depression?

Length of the shadow = 50 tan 30° = 28.9 m



14. From the top of a 20 m high lighthouse, a boat is seen with a 36° angle of depression. If the lighthouse is located at the top of a 45 m cliff, calculate the distance x separating the boat and the base of the cliff.

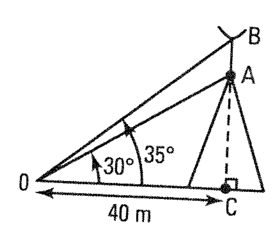
x = 65 tan 54° = 89.5 m



15. A parabolic antenna is located at the top of a tower. An observer located at 0, and 40 m away from the centre of the tower's base, uses a clinometer to measure the angles of elevation to the top of the tower (A) and to the top of the antenna (B). These angles measure 30° and 35° respectively. Calculate the height of the antenna.

$m\overline{AC}$ = 40 tan 30°; $m\overline{BC}$ = 40 tan 35°

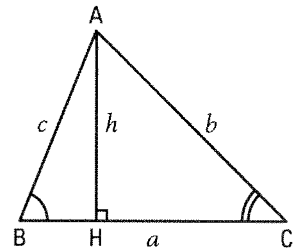
$m\overline{AB}$ = $m\overline{BC}$ − $m\overline{AC}$ = 4.9 m



7.2 Sine law

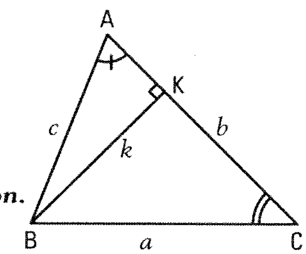
ACTIVITY 1 Sine law

a) In the right triangle ABC, let a, b and c represent the measures of sides BC, AC and AB. The altitude AH is drawn and represented by h. Explain why



1. $h = c \sin B$. ***In the $\triangle ABH$, we have: $\sin B = \frac{h}{c}$.***
2. $h = b \sin C$. ***In the $\triangle ACH$, we have: $\sin C = \frac{h}{b}$.***
3. $c \sin B = b \sin C$. ***From equations 1 and 2, we deduce the 3rd equation.***
4. $\frac{b}{\sin B} = \frac{c}{\sin C}$. ***Property of cross-products in a proportion.***

b) In the same triangle ABC, the altitude BK is drawn and represented by k. Explain why

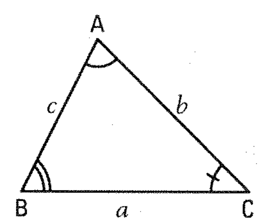


1. $k = c \sin A$. ***In the $\triangle ABK$, on a: $\sin A = \frac{k}{c}$.***
2. $k = a \sin C$. ***In the $\triangle BCK$, on a: $\sin C = \frac{k}{a}$.***
3. $c \sin A = a \sin C$. ***From equations 1 and 2, we deduce the 3rd equation.***
4. $\frac{a}{\sin A} = \frac{c}{\sin C}$. ***Property of cross-products in a proportion.***

c) Compare the three ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$ and $\frac{c}{\sin C}$. What can be deduced?

$\boldsymbol{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$

SINE LAW

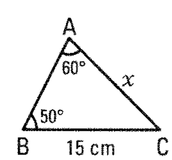


- The sides in a triangle are directly proportional to the sine of the opposite angles to these sides.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- The sine law can be used to find the measure of a missing side or angle.

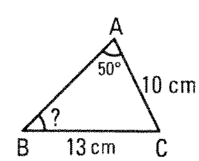
1st case: Finding a side when we know two angles and a side.



We calculate the measure x of AC.

$$\frac{x}{\sin 50°} = \frac{15}{\sin 60°} \Rightarrow x = \frac{15 \sin 50°}{\sin 60°} \approx 13.27 \text{ cm}$$

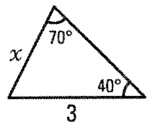
2nd case: Finding an angle when we know two sides and the opposite angle to one of these two sides.



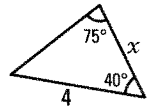
We calculate the measure of angle B.

$$\frac{10}{\sin B} = \frac{13}{\sin 50°} \Rightarrow \sin B = \frac{10 \sin 50°}{13} = 0.5893 \Rightarrow m\,\angle B \approx 36°$$

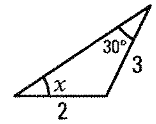
1. Use the sine law to calculate the value of x. (Round to the nearest tenth.)

a) 

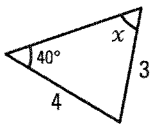
***x* = 2.1**

b) 

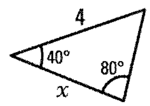
***x* = 3.8**

c) 

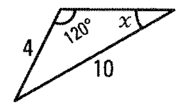
***x* = 48.6°**

d) 

***x* = 59°**

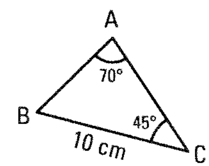
e) 

***x* = 3.5**

f) 

***x* = 20.3°**

2. Solve the following triangle. (Round each measure to the nearest tenth.)

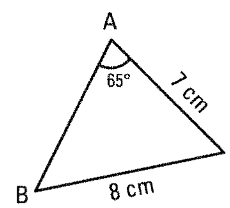


***m* ∠*B* = 65°**

***m*$\overline{AB}$ = 7.5 cm**

***m*$\overline{AC}$ = 9.6 cm**

3. Solve the following triangle. (Round each measure to the nearest tenth.)

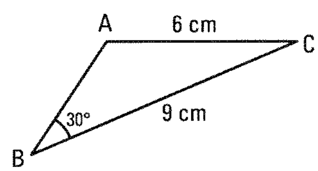


***m* ∠*B* = 52.5°**

***m* ∠*C* = 62.5°**

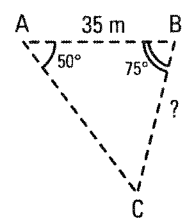
***m*$\overline{AB}$ = 7.8 cm**

4. Angle A in the triangle on the right is obtuse. Use the sine law to determine the measure of angle A.



sin *A* = 0.75; *m* ∠*A* = 180° – 48.6° = 131.4°

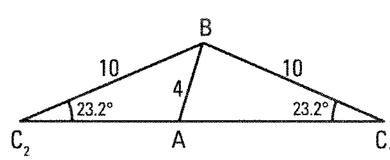
5. Three boats are near each other. What distance separates boats B and C if the distance, to the nearest metre, separating boats A and B is 35 m?



$\frac{\sin 55°}{35} = \frac{\sin 50°}{x} \Rightarrow x = \frac{35 \sin 50°}{\sin 55°} = 32.7$ **m**

The distance separating B and C is 33 m.

6. There exists two triangles where m ∠C = 23.2°, m$\overline{AB}$ = 4 and m$\overline{BC}$ = 10. These two triangles ABC_1 and ABC_2 are represented on the right.



a) Solve both triangles.

$m\overline{AC_1} = 9.9$; $m\overline{AC_2} = 8.5$

$m\angle A_1 = 80°$, $m\angle B_1 = 76.8°$

$m\angle A_2 = 100°$, $m\angle B_2 = 56.8°$

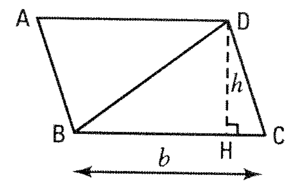
b) Explain why the angles BAC_1 and BAC_2 have the same sine.

They are supplementary angles.

7.3 Area of a triangle

ACTIVITY 1 General formula

On the right, parallelogram ABCD and its altitude DH are represented. Let b represent the parallelogram's base and h the height.



a) What is the area of the parallelogram? ***b × h***

b) Explain why triangles ABD and BCD are congruent.

m ∠ADB = m ∠DBC (alternate-interior angles)

m ∠ABD = m ∠BDC (alternate-interior angles)

$\overline{BD}$ is a common side to both triangles.

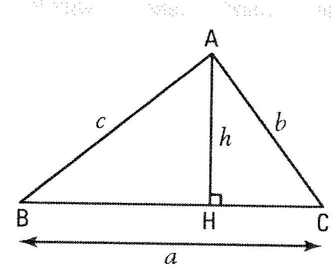
△ABD ≅ △CDB (Theorem of congruence ASA)

c) 1. Establish the formula that can be used to calculate the area of triangle BCD. ***Area = $\frac{b \times h}{2}$***

2. Calculate the area of triangle BCD if $b = 6$ cm and $h = 4$ cm. ***A = 12 cm²***

ACTIVITY 2 Trigonometric formula

Triangle ABC and its height AH are represented on the right. Let a, b and c represent the measure of the sides and h represent the height.



a) Justify the steps that establish the formula for calculating the area of triangle ABC.

1. Area $= \frac{a \cdot h}{2}$ ***General formula (activity 1).***

2. Area $= \frac{a \cdot c \sin B}{2}$ or Area $= \frac{a \cdot b \sin C}{2}$ ***h = c sin B or h = b sin C***

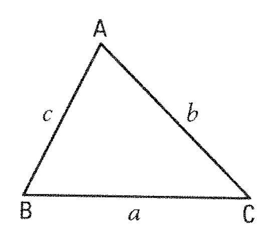
b) Calculate the area of triangle ABC if

1. $a = 6$ cm, $c = 4$ cm and m ∠B = 30°. ***Area = $\frac{6 \times 4 \times \sin 30°}{2}$ = 6 cm²***

2. $a = 8$ cm, $b = 3$ cm and m ∠C = 60°. ***Area = $\frac{8 \times 3 \times \sin 60°}{2}$ = $6\sqrt{3}$ cm²***

ACTIVITY 3 Hero's formula

When you are given the measures of all three sides a, b and c of a triangle, Hero's formula enables you to calculate the area of the triangle.

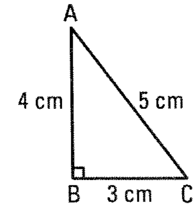


$A = \sqrt{p(p-a)(p-b)(p-c)}$ where p represents half the perimeter of the triangle.

$p = \frac{1}{2}(a + b + c)$

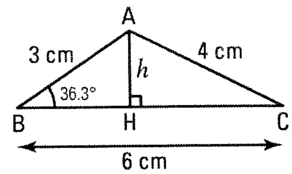
a) Calculate the area of the given right triangle using

1. the general formula $A = \frac{3 \times 4}{2} = 6\ cm^2$
2. Hero's formula. $p = 6\ cm;\ A = \sqrt{6 \cdot (6-4) \cdot (6-3) \cdot (6-5)}$
   $A = \sqrt{36} = 6\ cm^2$



b) Calculate, to the nearest hundredth, the area of the triangle on the right using

1. the trigonometric formula (activity 2).
   $A = \frac{ac \sin B}{2} = \frac{6 \cdot 3 \sin 36.3°}{2} = 5.33\ cm^2$
2. Hero's formula. $A = \sqrt{6.5 \times 3.5 \times 0.5 \times 2.5} = \sqrt{28.4375} = 5.33\ cm^2$

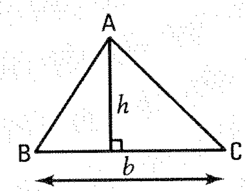


AREA OF A TRIANGLE

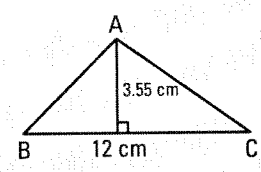
- **General formula**

Given the measure of the base b and the height h relative to that base, the area A of the triangle is given by:

$$A = \frac{b \times h}{2}$$



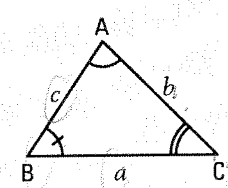
Ex.: $A = \frac{12 \times 3.55}{2} = 21.3\ cm^2$



- **Trigonometric formula**

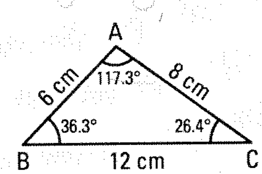
In a triangle, given the measure of an angle and its two sides,

$$A = \frac{ac \sin B}{2} \quad \text{or} \quad A = \frac{ab \sin C}{2} \quad \text{or} \quad A = \frac{bc \sin A}{2}$$



Ex.: $A = \frac{12 \times 6 \times \sin 36.3°}{2}$ or $A = \frac{12 \times 8 \times \sin 26.4°}{2}$ or $A = \frac{6 \times 8 \times \sin 117.3°}{2}$

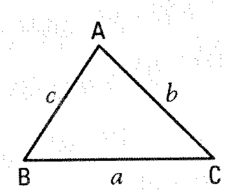
$A = 21{,}3\ cm^2$



- **Hero's formula**

In a triangle, given the measures of all three sides,

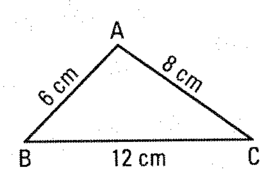
$$A = \sqrt{p(p-a)(p-b)(p-c)}$$



where p represents half of the triangle's perimeter. $p = \frac{a+b+c}{2}$.

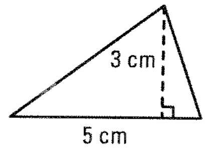
Ex.: $p = \frac{1}{2}(6 + 12 + 8) = 13$ cm

$A = \sqrt{13 \times 7 \times 1 \times 5} = \sqrt{455} \approx 21.3\ cm^2$



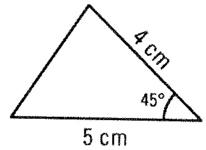
1. Calculate the area of the following triangles using an appropriate formula.

a)



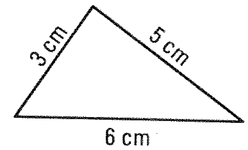
A = 7.5 cm²

b)



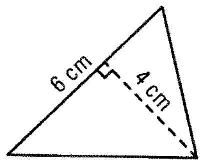
A = 7.07 cm²

c)



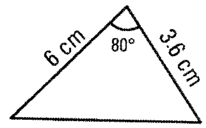
A = 7.48 cm²

d)



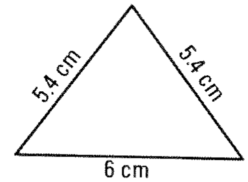
A = 12 cm²

e)



A = 10.64 cm²

f)

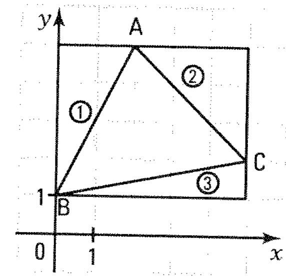


A = 13.47 cm²

2. Calculate the area of triangle ABC with vertices A(2, 5), B(0, 1)and C(5, 2).

Area of rectangle = 20 u^2; A_1 = 4 u^2; A_2 = 4.5 u^2; A_3 = 2.5 u^2

Area △ABC = 20 − (4 + 4.5 + 2.5) = 9 u^2

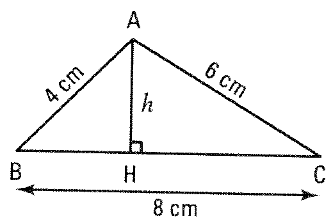


3. Calculate, to the nearest unit, the length of the altitude AH of triangle ABC on the right.

Area = $\frac{8 \times h}{2}$ = 4 h

Area = $\sqrt{9 \times 5 \times 1 \times 3}$ = 11.62 cm²

h = 3 cm

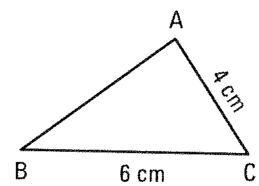


4. The area of the triangle on the right is equal to 11.6 cm². What is, to the nearest degree, the measure of angle C?

Area = $\frac{6 \times 4 \times \sin C}{2}$ = 12 sin C

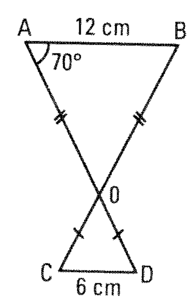
12 sin C = 11.6 ⇒ m ∠C = 75.2°

Angle C measures 75°.

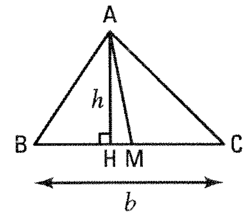


5. Triangles A0B and D0C represented on the right are similar.

What is the sum of the areas of the two triangles? *123.6 cm²*

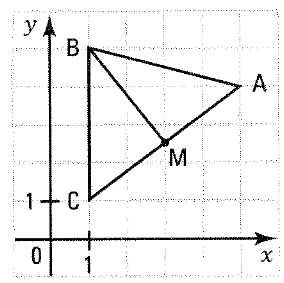


6. Consider the triangle ABC and the median AM from vertex A. Let b and h respectively represent the measure of the base and the height relative to this base.
Justify the steps proving that the median of a triangle separates the triangle into two triangles of equal area.
Hypothesis: $\overline{AM}$ is the median.



Statement	Justification
1. $m\overline{BM} = m\overline{MC} = \frac{b}{2}$	***The median AM divides segment BC in 2 congruent segments $\overline{MB}$ and $\overline{MC}$ (Definition of median).***
2. Area $\Delta ABC = \frac{bh}{2}$	***Area formula of triangle ABC with base b and height h.***
3. Area $\Delta ABM = \frac{\frac{b}{2} \cdot h}{2} = \frac{bh}{4}$	***Area formula of triangle ABM with base $\frac{b}{2}$ and height h.***
4. Are $\Delta AMC = \frac{\frac{b}{2} \cdot h}{2} = \frac{bh}{4}$	***Area formula of triangle AMC with base $\frac{b}{2}$ and height h.***
5. Area ΔABM = Area ΔAMC	***See steps 3 and 4. Equality is transitive.***

7. Consider triangle ABC on the right with vertices A(5, 4), B(1, 5) and C(1, 1).

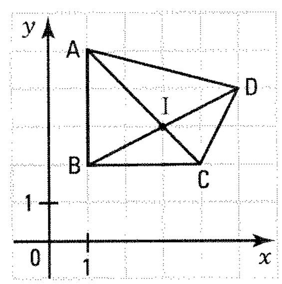


a) What is the area of triangle ABC? ***8 u^2***

b) Let M represent the mid-point of side AC. What is the area of triangle ABM?

4 u^2

8. Consider the quadrilateral ABCD with vertices A(1, 5), B(1, 2), C(4, 2) and D(5, 4).



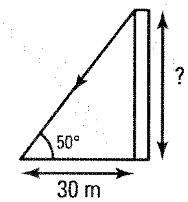
a) Determine the area of quadrilateral ABCD.
Area $\Delta ABD = 6\ u^2$; Area $\Delta BCD = 3\ u^2$; Area ABCD = $6\ u^2 + 3\ u^2 = 9\ u^2$

b) Let I represent the mid-point of the diagonal BD. Determine the area of triangles AIB, BIC, CID and DIA.
Area $\Delta AIB = \frac{1}{2}$ (Area ΔABD) = $3\ u^2$; Area $\Delta BIC = \frac{1}{2}$(Area ΔBCD) = $1.5\ u^2$

Area ΔCID = Area ΔBIC = $1,5\ u^2$; Area ΔDI = Area ΔAIB = $3\ u^2$

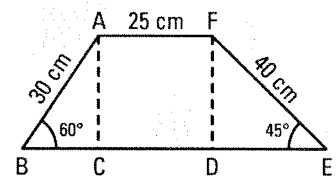
Evaluation 7

1. The shadow cast by a building measures 30 m when the sun's rays hit the ground at an angle of 50°. What is, to the nearest metre, the height of the building?



36 m

2. Calculate, to the nearest unit, the area of the trapezoid on the right.



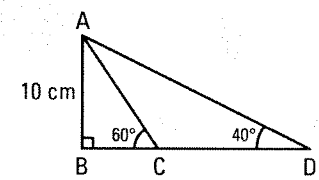
m$\overline{BC}$ = 30 cos 60° = 15 cm; m$\overline{DE}$ = 40 cos 45° = 28.28 cm

m$\overline{AC}$ = 30 sin 60° = 25.98 cm

A = (68.28 + 25) × 25.98 ÷ 2 = 1211.71 cm^2

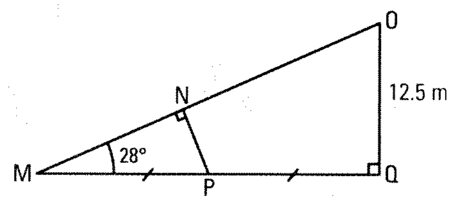
The area is equal to 1212 cm^2.

3. In the figure on the right, we have:
m$\overline{AB}$ = 10 cm; m ∠ACB = 60°; m ∠ADC = 40°.
Calculate, to the nearest tenth, the measure of segment CD.



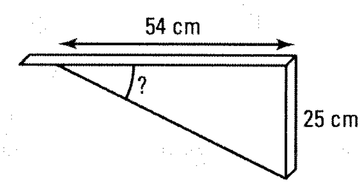
m$\overline{BD}$ = $\frac{10}{\tan 40^\circ}$ = 11.92 cm; m$\overline{BC}$ = $\frac{10}{\tan 60^\circ}$ = 5.77 cm; m$\overline{CD}$ = 6.2 cm

4. In the figure on the right, triangles MNP and M0Q are right triangles.
If m$\overline{MP}$ = m$\overline{PQ}$, determine the area of triangle MNP.



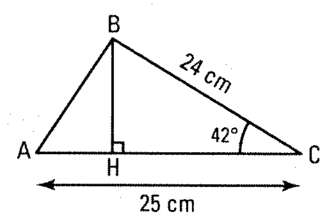
28.65 m^2

5. A bookshelf is held against a wall by a support in the shape of a right triangle.
What is, to the nearest degree, the measure of one of its acute angles?



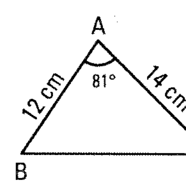
25°

6. In triangle ABC on the right, $\overline{BH}$ is the altitude from B.
What is, to the nearest cm^2, the area of triangle ABH?



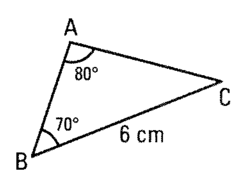
57 cm^2

7. Determine, to the nearest cm^2, the area of triangle ABC.



83 cm^2

8. Solve the following triangle. (Round your answer to the nearest tenth.)



m$\overline{AC}$ = 5.7 cm; m$\overline{AB}$ = 3 cm; m ∠C = 30°