Chapter 3

Equations and inequalities

CHALLENGE 3

1. Given two real numbers a and b such that $a > b$.

What conditions must be put on the real number c such that

a) $ac > bc$?

$c > 0$

b) $ac < bc$?

$c < 0$

2. Consider the inequality $3x \leq 6$.

Find the solutions to this inequality when

a) x is a real number. ***S =]−∞, 2]***

b) x is an integer. ***S = {..., −1, 0, 1, 2}***

c) x is a natural number. ***S = {0, 1, 2}***

3. Consider the inequality $-2x > -6$.

Find the solutions of this inequality when

a) x is a real number. ***S =]−∞, 3[***

b) x is an integer. ***S = {..., −1, 0, 1, 2}***

c) x is a natural number. ***S = {0, 1, 2}***

4. The length of a rectangular field measures 10 m more than twice the width.

a) What is the maximum width of this field if the perimeter is less than or equal to 200 m?

30 m

b) What is the minimum width of this field if the perimeter is greater than or equal to 110 m?

15 m

c) In what interval does the width of this field belong to if its perimeter is greater than 140 m but less than 170 m?

]20, 25[

d) What is the area of the field if the cost of the fence surrounding it is \$3000 and the fence costs \$15 per metre?

2100 m²

3.1 Equations

ACTIVITY 1 Properties of equality

a) Given three real numbers a, b and c. Complete:

1. If $a = b$ then $a + c =$ ***b + c***
2. If $a = b$ then $a - c =$ ***b − c***
3. If $a = b$ then $a \times c =$ ***b × c***
4. If $a = b$ and $c \neq 0$ then $\frac{a}{c} =$ $\frac{b}{c}$

b) Referring to the answers of the last question, answer the following questions.

Do you get another equality when

1. You add the same number to each side of the equality? ***Yes***
2. You subtract the same number from each side of the equality? ***Yes***
3. You multiply each side of the equality by the same number? ***Yes***
4. You divide each side of the equality by the same non-zero number? ***Yes***

c) Complete:

1. If $x - 6 = 0$ then $x =$ ***6***
2. If $x + 10 = 0$ then $x =$ ***-10***
3. If $\frac{x}{2} = 10$ then $x =$ ***20***
4. If $5x = 10$ then $x =$ ***2***

PROPERTIES OF EQUALITY

- If you add the same number to each side of the equality, you get a new equality.

If	$a = b$
then	$a + c = b + c$

- If you subtract the same number from each side of the equality, you get a new equality.

If	$a = b$
then	$a - c = b - c$

- If you multiply each side of the equality by the same number, you get a new equality.

If	$a = b$
then	$a \times c = b \times c$

- If you divide each side of the equality by the same non-zero number, you get a new equality.

If	$a = b$
then	$a \div c = b \div c$

ACTIVITY 2 Equivalent equations

a) Two equations are equivalent when they have the same solution.

1. Are the equations $2x + 1 = 7$ and $2x = 6$ equivalent? Justify your answer.

Yes, they both have the same solution: 3

2. Explain why $3x - 2 = 10$ and $3x = 8$ are not equivalent.
They do not have the same solution.

b) Given an equation, the properties of equality enable you to obtain an equivalent equation with the same solution.

1. Justify, using the properties of equality, the following steps for isolating the unknown on one side of the equality.

① $3x - 6 = 9$
↓ ↓ ***You add 6 to each side***
② $3x = 15$
↓ ↓ ***You divide each side by 3***
③ $x = 5$

(The unknown x is isolated.)

2. Verify that "5" is the solution to each of the preceding equations.

SOLVING AN EQUATION

- To solve an equation, use the properties of equality in order to isolate the unknown on one side of the equality.

Ex.:

$3x - 2 = 10$	
$3x - 2 + 2 = 10 + 2$	Add 2 to each side
$3x = 12$	Reduce
$x = 4$	Divide each side by 3

Verify: $3 \times 4 - 2 = 10$
The solution to the equation is 4. Write S = {4}.

Ex.:

$5x + 4 = 10 + 3x$	
$5x + 4 - 3x = 10 + 3x - 3x$	Subtract $3x$ from each side
$2x + 4 = 10$	Reduce
$2x + 4 - 4 = 10 - 4$	Subtract 4 from each side
$2x = 6$	Reduce
$x = 3$	Divide each side by 2

Verify: $5 \times 3 + 4 = 19$
$10 + 3 \times 3 = 19$
The solution to the equation is 3. Thus S = {3}.

1. Solve the following equations.

a) $5x - 3 = 12$ ***S = {3}***
b) $-2x + 3 = -5$ ***S = {4}***
c) $12 = 4 - 2x$ ***S = {-4}***
d) $-15 = 3x + 3$ ***S = {-6}***
e) $3x + 1 = 5$ ***S = {$\frac{4}{3}$}***
f) $\frac{3}{2}x - 2 = 4$ ***S = {4}***
g) $\frac{3}{2}x - 1 = \frac{5}{4}$ ***S = {$\frac{3}{2}$}***
h) $1.5x - 0.2 = 0.8$ ***S = {$\frac{2}{3}$}***

2. Solve the following equations.

a) $2x + 3 = 3x - 4$ ***S = {7}***
b) $6x - 2 = 3x + 7$ ***S = {3}***
c) $(x - 1) + 2(x - 3) = x + 5$ ***S = {6}***
d) $3(4x + 1) = 2(3x - 1)$ ***S = {$-\frac{5}{6}$}***

e) $(2x-1)-(3x+2)=4x-3$ **$S=\{0\}$**

f) $-2(3x-5)+(x-3)=3(2x-1)$ **$S=\{\frac{10}{11}\}$**

g) $5(2x+1)-3(2x-1)=2(3x-2)$ **$S=\{6\}$**

h) $3-6(x-1)+4(3x-2)=2x+3$ **$S=\{\frac{1}{2}\}$**

3. Solve the following equations.

a) $\frac{3}{2}x-\frac{1}{3}=\frac{3}{4}x+\frac{1}{2}$ **$S=\{\frac{10}{9}\}$**

b) $-\frac{2}{3}x+\frac{1}{4}=\frac{3}{4}x+\frac{1}{2}$ **$S=\{-\frac{3}{17}\}$**

c) $\frac{x+1}{2}+\frac{x-1}{3}=-4$ **$S=\{-5\}$**

d) $\frac{2x+3}{3}=\frac{3x-1}{2}$ **$S=\{\frac{9}{5}\}$**

4. Solve the following equations.

a) $\frac{x-1}{2}+\frac{x+1}{3}=\frac{7}{6}$ **$S=\{\frac{8}{5}\}$**

b) $\frac{x-2}{3}-\frac{x+1}{5}=1$ **$S=\{14\}$**

c) $\frac{x+1}{3}+\frac{3}{2}=\frac{x-1}{2}-\frac{1}{6}$ **$S=\{15\}$**

d) $\frac{2x+1}{3}-\frac{3x-1}{2}=\frac{x-1}{4}-\frac{x+1}{3}$ **$S=\{\frac{17}{9}\}$**

5. Solve the following equations.

a) $-5(2x+1)+3(x-2)=2(4x-1)-3(2x-3)$ **$S=\{-2\}$**

b) $\frac{2}{3}(x+1)-\frac{3}{2}(4x-2)=\frac{5}{6}(12x+3)-\frac{3}{4}$ **$S=\{\frac{1}{8}\}$**

c) $\frac{2x-3}{4}=3x$ **$S=\{-\frac{3}{10}\}$**

d) $\frac{x+1}{3}+\frac{x-2}{5}=1$ **$S=\{2\}$**

e) $\frac{2x-1}{3}-\frac{3x+1}{2}=\frac{5}{6}$ **$S=\{-2\}$**

6. The following equations either have a unique solution, infinite solutions or no solution. Solve them.

a) $3(2x+1)-(x-3)=6$ **$S=\{0\}$**

b) $2(3x+1)=3(2x-1)$ **$S=\varnothing$**

c) $2(3x-1)-3(2x+1)=-5$ **$S=\mathbb{R}$**

d) $\frac{3x+1}{2}=\frac{6x-3}{4}$ **$S=\varnothing$**

7. Nancy is 2 years older than her brother Eric. In 5 years, the sum of their ages will be equal to 40 years. What is the present age of each?

x: Eric's age; (x + 7) + (x + 5) = 40; x = 14. Eric is 14 years old and Nancy is 16 years old.

8. Today, Frank is 4 years older than Maria. Six years ago, Frank was twice as old as Maria. What is Frank's present age?

x: Maria's age; (x −2) = 2(x −6); x = 10. Frank is 14 years old.

9. Steve is 4 years older than Rose. In 5 years from now, the sum of their ages will be equal to twice the sum of their ages from 5 years ago. What is Steve's present age?

x: Rose's age; (x + 9) + (x + 5) = 2 [(x − 1) + (x − 5)]; x = 13. Steve is 17 years old.

10. In a class of 30 students, there are 6 more girls than boys. How many girls are there in the class?

x: number of boys; x + (x + 6) = 30; x = 12. There are 18 girls in the class.

11. Joanne has \$24 in her "piggy" bank in quarters, \$1 and \$2 coins. She has 2 fewer \$2 coins than \$1 coins and twice as many quarters as \$1 coins. How many coins of each type does she have?

x: number of \$1 coins; $2(x - 2) + 1(x) + 0.25(2x) = 24$; $x = 8$.

8 \$1 coins; 6 \$2 coins and 16 quarters.

12. Johnny buys 8 oranges and gets 20 cents back in change from a \$5 bill. What is the price of an orange?

x: price of an orange; $8x + 0.20 = 5$; $x = 0.60$. An orange costs 60 cents.

13. The width of a rectangle measures 4 cm less than the length. What is the area of the rectangle if its perimeter is equal to 100 cm?

x: length; $4x - 8 = 100$; $x = 27$; length = 27 cm; width = 23 cm; Area = 621 cm^2.

14. The area of a triangle is 42 cm^2. If one of its bases is 12 cm, what is the height relative to that base?

x: height; $12 \times x = 84$; $x = 7$. The height measures 7 cm.

15. A prism has a rectangular base with dimensions 6 cm by 4 cm. Determine the height of the prism if its total area is equal to 108 cm^2.

x: height; $48 + 20x = 108$; $x = 3$. The height measures 3 cm.

16. A salesman has a base salary of \$175 per week and a commission of \$25 per item sold. How many items did he sell in a week if his total salary was \$500?

x: number of items sold; $175 + 25x = 500$; $x = 13$. He sold 13 items.

17. Denise, Evelyn and Fran work at a convenience store. They earn an hourly rate of \$8. In one week, Denise worked 4 hours more than Evelyn whereas Fran worked 9 hours less than Denise. Together, they earned a total salary of \$856. What was Fran's salary that week?

x: number of hours worked by Evelyn; $8(3x - 1) = 856$; $x = 36$. Fran's salary was \$248.

18. Eric pays \$5 for 4 apples and 3 oranges. Knowing that an orange costs 15 cents more than an apple, how much will Nathalie pay for 3 apples and 5 oranges?

x: price of an apple; $4x + 3(x + 0.15) = 5$; $x = 0.65$. Nathalie will pay \$5.95.

19. Sylvia buys 6 pencils. If each pencil had cost 10 cents less, she could have bought 4 more pencils for the same amount. What is the price of one pencil?

x: price of a pencil; $6x = 10(x - 10)$; $x = 25$. Each pencil costs 25 cents.

20. Over the course of one term, Julia's average in math is 74. She can only remember 4 of the 5 marks she received that term: 70, 64, 80, 84. What is the mark she can't remember?

x: missing mark; $\frac{x + 298}{5} = 74$; $x = 72$.

21. Corinne must sell all the chocolate bars in a box to raise money. If she sells each chocolate bar for \$3.50, she exceeds her goal by \$4, whereas if she sells them for 50 cents less, she will be \$8 short of her goal. How many chocolate bars are in the box?

x: number of chocolate bars in a box; $3.5x - 4 = 3x + 8$; $x = 24$.

There are 24 chocolate bars in a box.

22. Val and Nancy have a total of \$300. They would have the same amount if Val gave \$30 to Nancy. How much do they each have?

x: Val's amount; $x - 30 = 300 - (x - 30)$; $x = 180$. Val has \$180 and Nancy has \$120.

23. Anthony leaves Montreal at 8 a.m. to go to Toronto and travels at an average speed of 60 km/h. One hour later, Ben also leaves Montreal for Toronto and travels at an average speed of 100 km/h. At what time, and at what distance from their starting point, does Ben pass Anthony?

At 10:30 a.m., at 150 km from Montreal.

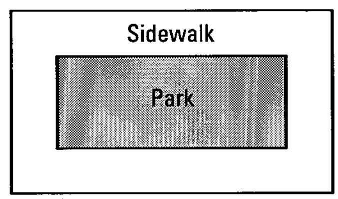
24. By mixing yellow paint costing \$1.20 per litre with blue paint costing \$1.60 per litre, we get a 30 litre mixture of green paint that costs \$1.36 per litre. How many litres of yellow paint and blue paint did we use?

x: number of litres of yellow paint; $1.20x + 1.60(30 - x) = 30 \times 1.36$; $x = 18$.

18 litres of yellow paint and 12 litres of blue paint.

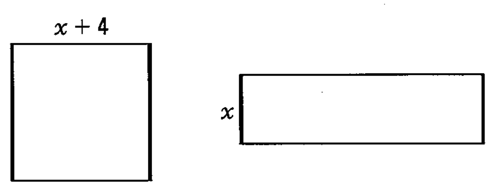
25. The length of a rectangular park is 5m more than triple its width. The park is surrounded by a 3m wide sidewalk. With the sidewalk, the area of the park increases by 1986 m^2.

What is the area of the park without the sidewalk? ***19 600 m^2***



26. The square and rectangle on the right have the same area. The length of the rectangle is 12 m more than its width. What is the perimeter of each figure?

square: 32 m, rectangle: 40 m

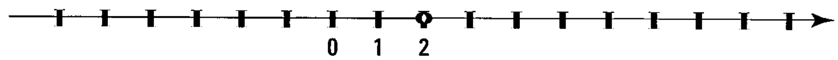


3.2 Inequalities

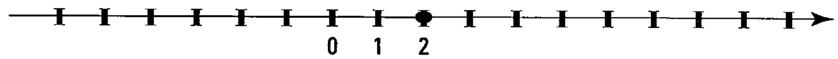
ACTIVITY 1 Inequality relation $<, \leqslant, >, \geqslant$

a) Represent, on the real number line, the set of all numbers

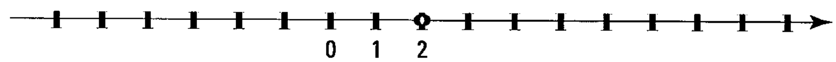
1. less than 2.



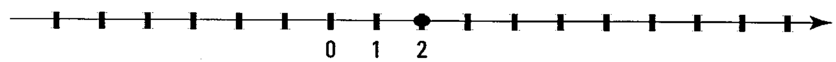
2. less than or equal to 2.



3. greater than 2.



4. greater than or equal to 2.



b) Each of the sets above is an interval. Describe these intervals in set-builder notation and then in interval notation with brackets.

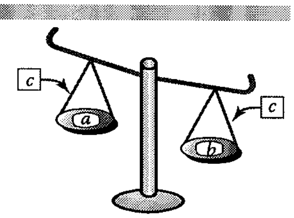
1. $\{x \in \mathbb{R} \mid x < 2\} =]-\infty, 2[$
2. $\{x \in \mathbb{R} \mid x \leqslant 2\} =]-\infty, 2]$
3. $\{x \in \mathbb{R} \mid x > 2\} =]2, +\infty[$
4. $\{x \in \mathbb{R} \mid x \geqslant 2\} = [2, +\infty[$

ACTIVITY 2 Order and addition

a) Given any three real numbers a, b and c.

If $a \leqslant b$, compare

1. the numbers $a + c$ and $b + c$. $a + c \leqslant b + c$
2. the numbers $a - c$ and $b - c$. $a - c \leqslant b - c$



b) When you add or subtract the same number from each side of the inequality $a \leqslant b$, is the newly obtained inequality in the same or opposite direction as the original inequality?

Same direction.

ACTIVITY 3 Order and multiplication

Consider the inequality $2 \leqslant 4$.

a) 1. Multiply each side of the inequality by the positive number 5. $10 \leqslant 20$

2. Is the newly obtained inequality in the same direction or opposite direction as the original inequality? ***Same direction.***

b) 1. Multiply each side of the inequality by the negative number –5. $-10 \geqslant -20$

2. Is the newly obtained inequality in the same direction or opposite direction as the original inequality? ***Opposite direction.***

c) Given any three real numbers a, b and c. Complete the following with the appropriate inequality sign.

1. If $a \leqslant b$ and $c > 0$ then ac $\underline{\quad \leqslant \quad}$ bc
2. If $a \leqslant b$ and $c < 0$ then ac $\underline{\quad \geqslant \quad}$ bc
3. If $a \leqslant b$ and $c > 0$ then $\frac{a}{c}$ $\underline{\quad \leqslant \quad}$ $\frac{b}{c}$
4. If $a \leqslant b$ and $c < 0$ then $\frac{a}{c}$ $\underline{\quad \geqslant \quad}$ $\frac{b}{c}$

PROPERTIES OF INEQUALITY

- **Addition and subtraction**
  Given any three real numbers a, b and c.

If $a \leqslant b$ then $a + c \leqslant b + c$	Ex.: $3 \leqslant 5 \Rightarrow 3 + 2 \leqslant 5 + 2$.
If $a \leqslant b$ then $a - c \leqslant b - c$	Ex.: $3 \leqslant 5 \Rightarrow 3 - 2 \leqslant 5 - 2$.

When you add (or subtract) the same number on each side of an inequality, you get an inequality in the **same direction**.

- **Multiplication and division**
  Given any three real numbers a, b and c.

If $a \leqslant b$ and $c > 0$ then $ac \leqslant bc$	Ex.: $3 \leqslant 5$ and $2 > 0 \Rightarrow 6 \leqslant 10$.
If $a \leqslant b$ and $c < 0$ then $ac \geqslant bc$	Ex.: $3 \leqslant 5$ and $-2 < 0 \Rightarrow -6 \geqslant -10$.
If $a \leqslant b$ and $c > 0$ then $\frac{a}{c} \leqslant \frac{b}{c}$	Ex.: $6 \leqslant 10$ and $2 > 0 \Rightarrow 3 \leqslant 5$.
If $a \leqslant b$ and $c < 0$ then $\frac{a}{c} \geqslant \frac{b}{c}$	Ex.: $6 \leqslant 10$ and $-2 < 0 \Rightarrow -3 \geqslant -5$.

If you multiply (or divide) both sides of an inequality by the same **positive** number, you get an inequality in the **same direction**, whereas if you multiply (or divide) both sides of an inequality by the same **negative** number, you get an inequality in the **opposite direction**.

Note: The rules above are stated with the inequality $a \leqslant b$. Similar rules exist for $a < b$, $a \geqslant b$ and $a > b$.

1. Complete with the appropriate symbol.

a) If $a \leqslant b$ then $a + 10$ $\underline{\ \leqslant\ }$ $b + 10$

b) If $a \leqslant b$ then $a - 5$ $\underline{\ \leqslant\ }$ $b - 5$

c) If $a > b$ then $a + 1$ $\underline{\ >\ }$ $b + 1$

d) If $a \geqslant b$ then $a - 2$ $\underline{\ \geqslant\ }$ $b - 2$

e) If $a \leqslant b$ then $5a$ $\underline{\ \leqslant\ }$ $5b$

f) If $a < b$ then $-10a$ $\underline{\ >\ }$ $-10b$

g) If $a > b$ then $\frac{a}{2}$ $\underline{\ >\ }$ $\frac{b}{2}$

h) If $a \geqslant b$ then $\frac{a}{-3}$ $\underline{\ \leqslant\ }$ $\frac{b}{-3}$

2. In each of the following cases, compare the difference $a - b$ with the real number 0:

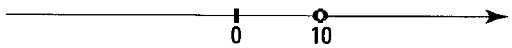
1. $a \geqslant b$. $\underline{\ a - b \geqslant 0\ }$
2. $a > b$. $\underline{\ a - b > 0\ }$
3. $a \leqslant b$. $\underline{\ a - b \leqslant 0\ }$
4. $a < b$. $\underline{\ a - b < 0\ }$

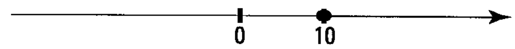
ACTIVITY 4 Building an inequality

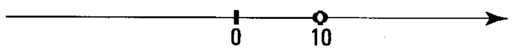
a) Let x represent the temperature at noon.
Translate the following statements using one of the inequality symbols ($<, \leqslant, >, \geqslant$).
At noon,

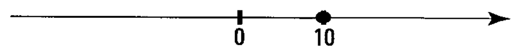
1. the temperature is more than 10°. ***x > 10***
2. the temperature is at least 10°. ***x ⩾ 10***
3. the temperature is less than 10°. ***x < 10***
4. the temperature is at most 10°. ***x ⩽ 10***

b) Represent each of the preceding situations on the real number line.

1. 

2. 

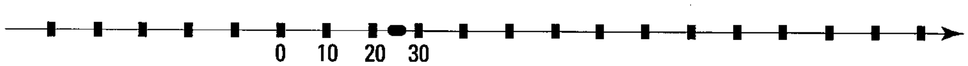
3. 

4. 

ACTIVITY 5 Building an inequality

The length of a rectangular field measures 10 m more than its width. The perimeter of the field is at least 120 m. Let x represent the width of the field.

1. Establish an inequality translating this situation. ***4x + 20 ⩾ 120***
2. Two possible values for the length of the field are suggested: 20 m or 30 m. Which one of the two suggested values is possible? Justify your answer.
   20 m is rejected as a solution since 4 × 20 + 20 = 100 and 100 < 120.
   30 m is accepted as a solution since 4 × 30 + 20 = 140 and 140 ⩾ 120.
3. There exists an infinite number of solutions for the length x of the field. What is the minimal solution? ***25 m***
4. Represent the set of all possible solutions for the width x of the field on the real number line.



INEQUALITIES WITH ONE UNKNOWN

- An inequality with one unknown is a statement with one variable that is constructed using one of the inequality symbols ($<, \leqslant, >, \geqslant$).
  Ex.: $2x - 6 \geqslant 10$ is an inequality.
  x is the unknown.
  $2x - 6$ is the left side (or 1st side) of the inequality.
  10 is the right side (or 2nd side) of the inequality.

3. In a class, there are 3 more girls than boys. Let x represent the number of boys in the class. Translate each of the following situations.

a) There are at least 30 students in the class. ***2x + 3 ⩾ 30***

b) There are 25 students or less in the class. ***2x + 3 ⩽ 25***

c) There are 28 students or more in the class. ***2x + 3 ⩾ 28***

d) There are at most 26 students in the class. ***$2x + 3 \leq 26$***

e) There are less than 30 students in the class. ***$2x + 3 < 30$***

f) There are more than 32 students in the class. ***$2x + 3 > 32$***

g) There are at least 20 students and at most 25 students in the class. ***$20 \leq 2x + 3 \leq 25$***

h) There are more than 23 students and at most 28 students in the class. ***$23 < 2x + 3 \leq 28$***

4. A "piggy" bank contains 20 coins worth \$1 or \$2. Let x represent the number of \$1 coins.

a) Establish an inequality to indicate that the bank contains a maximum of \$30. ***$40 - x \leq 30$***

b) Give two possible values for x. ***Varied answers***

c) Give two impossible values for x. ***Varied answers***

d) What is the minimal value for x? ***10***

5. John is an appliance salesman. He earns a weekly base salary of \$175 and a \$25 bonus per appliance sold. He earns a weekly total salary of less than \$250.

a) Translate this situation using an inequality, after correctly identifying the variable.
x: number of appliances sold that week, $25x + 175 < 250$.

b) How many appliances could John have sold? ***0, 1 or 2.***

ACTIVITY 6 Solving an inequality

a) Consider the inequality $3x + 4 \geqslant 10$.

1. Using the properties of inequality, justify the following steps in isolating the unknown x on one side of the inequality.

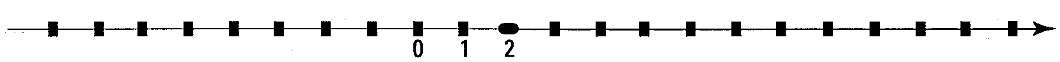
$3x + 4 \geqslant 10$ → ***Subtract 4 from each side; the direction of the inequality does not change.***

$3x \geqslant 6$ → ***Divide each side by the positive number 3; the direction of the inequality does not change.***

$x \geqslant 2$

(The unknown x is isolated.)

2. The inequality where the unknown is isolated indicates the possible solutions for x. Represent the set of possible solutions on the real number line.



b) Consider the inequality $-2x - 3 \geqslant 1$.

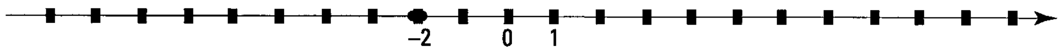
1. Using the properties of inequality, justify the following steps in isolating the unknown x on one side of the inequality.

$-2x - 3 \geqslant 1$ → ***Add 3 on each side; the direction of the inequality does not change.***

$-2x \geqslant 4$ → ***Divide each side by the negative number -2; the direction of the inequality becomes opposite.***

$x \leqslant -2$

2. The inequality where the unknown is isolated indicates the possible solutions for x. Represent the set of possible solutions on the real number line.



c) Consider the inequality $x + 1 < 4x + 7$.

1. Using the properties of inequality, justify the following steps to isolating the unknown x on one side of the inequality.

The unknown x is on both sides.

$$x + 1 < 4x + 7$$

Subtract 4x from each side; the direction of the inequality does not change.

$$\downarrow \qquad \downarrow$$

$$x + 1 - 4x < 4x + 7 - 4x$$

Reduce each side

$$\downarrow \qquad \downarrow$$

$$-3x + 1 < 7$$

Subtract 1 from each side; the direction of the inequality does not change.

$$\downarrow \qquad \downarrow$$

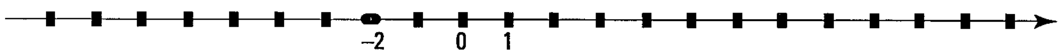
$$-3x < 6$$

Divide each side by the negative number -3; the direction of the inequality becomes opposite.

$$\downarrow \qquad \downarrow$$

$$x > -2$$

2. The inequality where the unknown is isolated indicates the possible solutions for x. Represent the set of possible solutions on the real number line.



SOLVING AN INEQUALITY

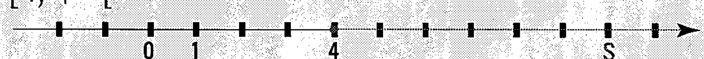
- The **solution** to an inequality, with x as the unknown, is any value of x which verifies the inequality.
  Ex.: 3 is a solution to the inequality $2x + 1 \geqslant 4$, since the inequality $2 \times 3 + 1 \geqslant 4$ is true.
- **Solving** an inequality is to find the set of all possible solutions.
- To solve an inequality, use the properties of inequality in order to **isolate** the unknown on one side.

Ex.:

$2x - 3 \geqslant 5$	
$2x - 3 + 3 \geqslant 5 + 3$	Add 3 to each side. The direction of the inequality does not change.
$2x \geqslant 8$	Reduce
$x \geqslant 4$	Divide each side by the positive number 2. The direction of the inequality does not change.

The set S of all possible solutions is expressed in three different ways:

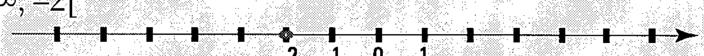
1. in **set-builder** notation: $S = \{x \in \mathbb{R} \mid x \geqslant 4\}$
2. in **interval notation**: $S = [4, +\infty[$
3. on the **real number line**:



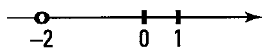
Ex.:

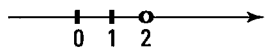
$-3x + 6 > 12$	
$-3x + 6 - 6 > 12 - 6$	Subtract 6 from each side. The direction of the inequality does not change.
$-3x > 6$	Reduce
$x < -2$	Divide each side by the negative number -3. The direction of the inequality becomes opposite.

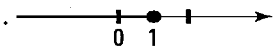
Thus, the solution set
– defined in set-builder notation is: $S = \{x \in \mathbb{R} \mid x < -2\}$
– defined in interval notation is: $S =]-\infty, -2[$
– on the real number line:

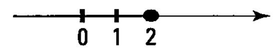


6. Solve the following inequalities and represent the solution set in three ways.
1. in set-builder notation 2. as an interval 3. on the real number line

a) $5x > -10$
1. **$S = \{x \in \mathbb{R} \mid x > -2\}$**
2. **$S =]-2, +\infty[$**
3. 

b) $-5x > -10$
1. **$S = \{x \in \mathbb{R} \mid x < 2\}$**
2. **$S =]-\infty, 2[$**
3. 

c) $x - 2 \leqslant -1$
1. **$S = \{x \in \mathbb{R} \mid x \leqslant 1\}$**
2. **$S =]-\infty, 1]$**
3. 

d) $x + 1 \leqslant 3$
1. **$S = \{x \in \mathbb{R} \mid x \leqslant 2\}$**
2. **$S =]-\infty, 2]$**
3. 

7. Solve the following inequalities. Give the solution set in interval notation.

a) $3x - 1 < 5$ **$S =]-\infty, 2[$**

b) $\frac{x}{4} + 1 > 5$ **$S =]16, +\infty[$**

c) $-4x + 1 \geqslant -11$ **$S =]-\infty, 3]$**

d) $\frac{3x+1}{2} \leqslant 5$ **$S =]-\infty, 3]$**

e) $2x \geqslant x - 1$ **$S = [-1, +\infty[$**

f) $-2(x + 1) \leqslant -4$ **$S = [1, +\infty[$**

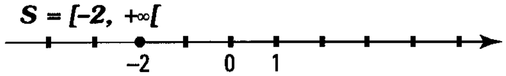
g) $\frac{-x}{4} + 1 \geqslant 3$ **$S =]-\infty, -8]$**

h) $\frac{3}{4}x - 1 \leqslant 2$ **$S =]-\infty, 4]$**

8. Represent on the real number line the values of x such that

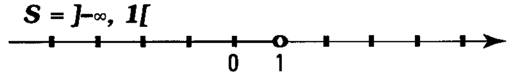
a) $3x + 10 \geqslant x + 6$

$S = [-2, +\infty[$



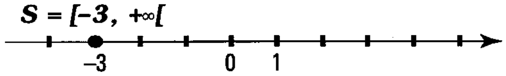
b) $5x - 1 < 3x + 1$

$S =]-\infty, 1[$



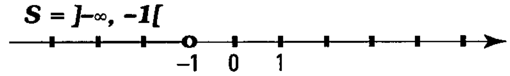
c) $3x - 2 \leqslant 4x + 1$

$S = [-3, +\infty[$



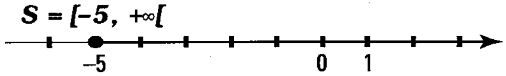
d) $-2x + 1 > 3x + 6$

$S =]-\infty, -1[$



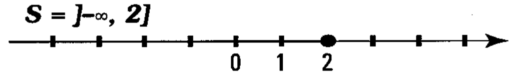
e) $2(x - 1) - 3(x + 1) \leqslant 0$

$S = [-5, +\infty[$



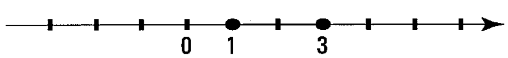
f) $-2(x - 3) \geqslant 3(x - 1) - 1$

$S =]-\infty, 2]$

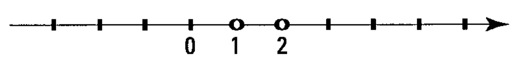


9. Represent on the real number line the values of x such that

a) $3 \leqslant 2x + 1 \leqslant 7$



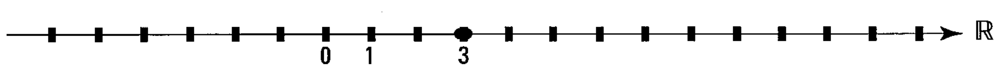
b) $-4 < -3x + 2 < -1$

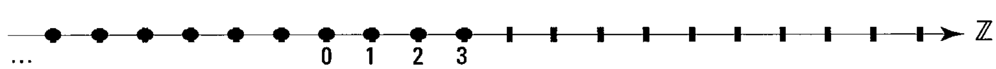


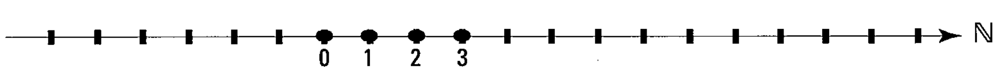
ACTIVITY 7 Solving an inequality in a given domain

Given an inequality with one variable, the domain is the set of all possible values for the unknown.

a) Consider the inequality $x + 1 \leqslant 4$. Represent, on a number line, the solution set for this inequality when the domain is:

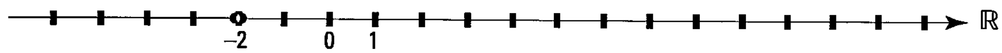
1. $\mathbb{R}$ 

2. $\mathbb{Z}$ 

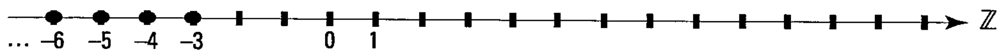
3. $\mathbb{N}$ 

b) Consider the inequality $-3x > 6$. Represent, on a number line, the solution set for this inequality when the domain is:

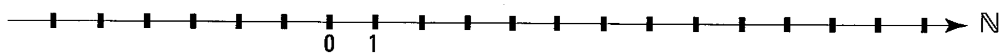
1. $\mathbb{R}$



2. $\mathbb{Z}$



3. $\mathbb{N}$



S = Ø

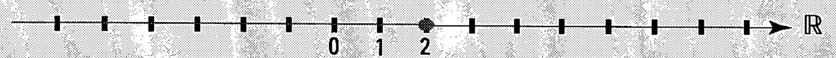
SOLVING AN INEQUALITY IN A GIVEN DOMAIN

- Solving an inequality in a given domain consists of finding the solutions that are in that domain.

Ex.: The solution set of the inequality $5x \leqslant 10$ is

(Only an interval uses brackets.)

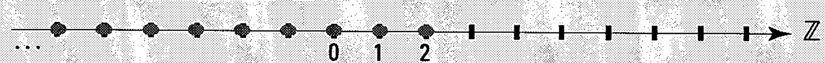
1. when the domain is $\mathbb{R}$:



$S =]-\infty, 2]$

2. when the domain is $\mathbb{Z}$:

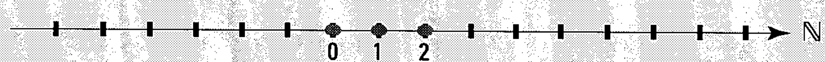
(The solution set is infinite, but not an interval.)



$S = \{..., 0, 1, 2\}$

3. when the domain is $\mathbb{N}$:

(The solution set is finite.)



$S = \{0, 1, 2\}$

- When the domain is not specified, we assume the domain to be all real numbers.

10. Solve, in $\mathbb{Z}$, the following inequalities.

a) $2x - 5 \leqslant 0$ ***S = {..., –1, 0, 1, 2}***

b) $-2x + 3 > -1$ ***S = {..., – 1, 0, 1}***

c) $3x - 1 > 8$ ***S = {4, 5, 6, ...}***

d) $-3x + 1 \leqslant 5$ ***S = {–1, 0, 1 ...}***

11. Solve, in $\mathbb{N}$, the following inequalities and indicate if the solution set is empty, finite or infinite.

a) $2x + 1 > 11$ ***S = {6, 7, 8, ...}; infinite***

b) $2x - 1 < 5$ ***S = {0, 1, 2}; finite***

c) $-2x + 3 \geqslant 7$ ***S = Ø; empty***

d) $-2x + 3 < 5$ ***S = ℕ; infinite***

12. Jacky earns a weekly base salary of \$175. He also earns a commission of \$50 for each computer he sells. How many computers could he have sold in a week if he received a total salary of less than \$825?

x: number of computers sold; 175 + 50x < 825; x < 13.

He sold 12 computers or less.

13. The length of a rectangular field measures 15 m less than twice the width. If the perimeter measures at least 120 m, determine the minimum width of the field.

x: width; $6x - 30 \geqslant 120$; $x \geqslant 25$. The minimum width is 25 m.

14. Helen and Caroline work in a library. They earn an hourly wage of $12. In one week, Caroline worked 5 hours more than Helen. Together, they receive a total salary of more than $780. How many hours could Caroline have worked?

x: number of Helen's hours; $12x + 12(x + 5) > 780$; $x > 30$.

Caroline worked more than 35 hours.

15. In a class of at least 30 students, there are 4 more girls than boys. What is the minimum number of girls in this class?

x: number of boys; $x + (x + 4) \geqslant 30$; $x \geqslant 13$. There is a minimum of 17 girls in the class.

16. The length of a rectangular field measures 10 m more than its width. The perimeter of the field is more than 80 m but less than 100 m. In what interval will the width of the field be?

x: width; $80 < 4x + 20 < 100$; $15 < x < 20$; $x \in]15, 20[$

17. A prism has a rectangular base with dimensions 8 cm by 12 cm. In what interval is the height of the prism if the total area of the prism is more than 352 cm^2, but less than 392 cm^2?

x: height of the prism; $352 < 192 + 40x < 392$; $4 < x < 5$; $x \in]4, 5[$

18. A taxi driver charges an initial fee of $1.25 and then $0.75 per km traveled. In what interval is the distance traveled if the cost of the trip is more than $11 but less than $14?

x: distance traveled; $11 < 0.75x + 1.25 < 14$; $13 < x < 17$; $x \in]13, 17[$

19. In one term, Julian receives marks of 70 and 78 on his first two French tests. What must his mark be on the third test for the average of the three tests to be at least 75?

x: mark on the third test; $\frac{148 + x}{3} \geqslant 75$; $x \geqslant 77$

He must obtain a mark of least 77 on the third test.

20. Jimmy and Sidney have a total of $300 combined. If Jimmy has at least $50 more than Sidney, determine in what interval is the amount that Jimmy has.

x: Jimmy's amount; $x - (300 - x) \geqslant 50$; $x \geqslant 175$

$x \in [175, 300]$

EVALUATION 3

1. Solve the following equations.

a) $3x + 2 = 11$ ***x = 3***

b) $\frac{3}{5}x - \frac{1}{2} = 1$ ***x = $\frac{5}{2}$***

c) $3(x - 2) + 2(x + 4) = 9$ ***x = $\frac{7}{5}$***

d) $5(2 - 3x) - 2(x - 4) = 3(x + 2)$ ***x = $\frac{3}{5}$***

e) $\frac{x + 5}{3} = \frac{x - 1}{9}$ ***x = −8***

f) $\frac{3x + 1}{5} - \frac{x - 3}{2} = \frac{9}{10}$ ***x = −8***

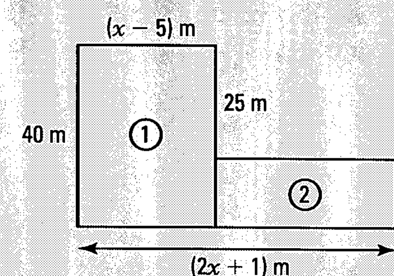
2. The admission price to an art exhibit is \$6 for children and \$12 for adults. If, in one day, there were 170 admissions and a total of \$1770 collected, how many tickets of each category were sold? ***45 tickets for children and 125 tickets for adults.***

3. Peter sells plants at \$12 each and flowers at \$2 each at a flea market. If, in one day, he collected the sum of \$870 and sold 3 flowers more than twice the number of plants, how many plants did he sell? ***54 plants.***

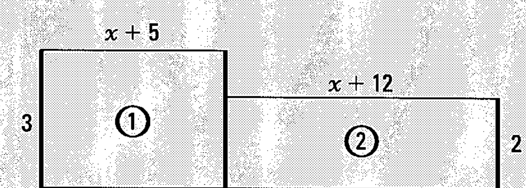
4. The length of a rectangular field measures 8 km more than twice its width. What is the area of the field if its perimeter is 91 km? ***412.5 km²***

5. Lucy is nine years less than three times the age of her daughter Chloe. In five years, she will be twice her daughter's age. How old is Chloe? ***14 years old.***

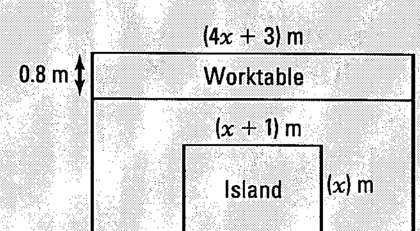
6. In the figure on the right, the area of rectangle ① is equal to twice the area of rectangle ②. What is the numerical value of the perimeter of rectangle ②? ***118 m***



7. Mrs. Connors has partitioned the land that she received as a gift into two rectangular lots of equal area as indicated in the figure on the right. What is the numerical value of the perimeter of her land? (The distances are in kilometers).
76 km



8. Mrs. Douglas has installed a worktable in her kitchen as indicated in the figure on the right. If the total area of the worktable is 8.48 m², what is the area of the island installed in the centre of the kitchen? (The answer is a numerical value).
5.51 m²



9. Find three consecutive natural numbers that have a sum of 327.

108, 109, 110

10. Solve the following inequalities and give the solution set in interval notation.

a) $2x - 3 > 5$ **$S =]4, +\infty[$**

b) $\frac{x}{2} - 9 \leqslant 1$ **$S =]-\infty, 20]$**

c) $3 - 4x \leqslant 19$ **$S = [-4, +\infty[$**

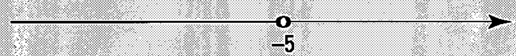
d) $\frac{3x - 2}{5} < 2$ **$S =]-\infty, 4[$**

e) $-2(x - 1) > -4$ **$S =]-\infty, 3[$**

f) $\frac{3}{4}x + 2 \geqslant 5$ **$S = [4, +\infty[$**

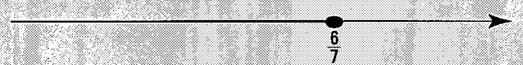
11. Solve the following inequalities and represent the solution set on the real number line.

a) $3x + 2 < x - 8$



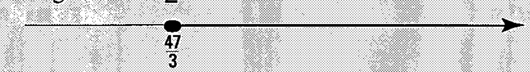
b) $-2x + 1 \geqslant 3x - 5$

c) $3(2x - 1) + (x - 3) \leqslant 0$



d) $2(2x - 5) \geqslant 3(x - 2)$

e) $\frac{x + 1}{5} \leqslant \frac{x - 9}{2}$



f) $2(x - 5) - 5(x + 2) \geqslant 1 - 4x$

12. Solve, in $\mathbb{Z}$, the following inequalities.

a) $3x + 2 < 11$

$S = \{..., -2, -1, 0, 1, 2\}$

b) $-3x + 4 \leqslant -2$

$S = \{2, 3, 4, 5, ...\}$

c) $x - 5 < 2x + 3$

$S = \{-7, -6, -5, ...\}$

d) $3(2x - 1) + (x + 2) \geqslant x - 3$

$S = \{0, 1, 2, 3, ...\}$

13. Two photocopier rental companies advertise their rates in the following manner. Company A: \$0.06 per copy and \$75 per month for renting the photocopier.

Company B: \$0.03 per copy and \$135 per month for renting the photocopier.

For how many copies is the amount charged by company A less than the amount charged by company B?

For less than 2000 copies.

14. In a school of at least 244 students, there are 4 girls more than twice the number of boys. What is the minimum number of girls in this school?

164 girls.

15. The length of a rectangular field measures 5 m less than triple its width. If the perimeter of the field is less than 326 m, what is the maximum length of this field?

121 m