

Chapter 5

Solids

CHALLENGE 5

- 5.1** Views of a solid
- 5.2** Perspectives of a solid
- 5.3** Prisms – Pyramids – Cylinders
- 5.4** Cones
- 5.5** Spheres

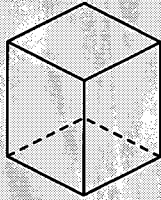
EVALUATION 5



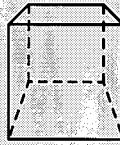
CHALLENGE 5

1. Here are three perspectives of a solid.

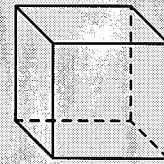
1.



2.



3.



Indicate which perspective is

- a) oblique 3 b) axonometric. 1 c) linear. 2

2. Explain how you can generate, by rotating around an axis,

- a) a cylinder with a radius of 3 cm and a height of 10 cm.

Rotate a rectangle with a width of 3 cm and a length of 10 cm around its length.

- b) a cone with a radius of 3 cm and a slant height of 5 cm.

Rotate a right triangle, with sides of 3 cm and 4 cm making the right angle, around the 4 cm side.

- c) a sphere with a 3 cm radius.

Rotate a half-disc with a radius of 3 cm around its diameter.

3. Explain how to construct a cone with a 6 cm radius and a height of 8 cm.

1. Construct a circular sector with a radius of 10 cm (the slant height).

The sector's angle is equal to 216° . $\left(\frac{\alpha}{360^\circ} = \frac{6}{10} \Rightarrow \alpha = 216^\circ\right)$

2. Construct a disc with a 6 cm radius.

4. Indicate the relation between

- a) the radius r , the height h and the slant height s of a cone.

$$s^2 = r^2 + h^2$$

- b) the side c of a square-based pyramid, its height h and its slant height s .

$$s^2 = \left(\frac{c}{2}\right)^2 + h^2$$

5.1 Views of a solid

ACTIVITY 1 Coded blueprint

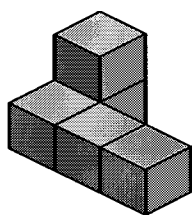
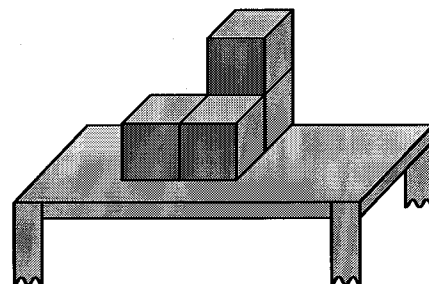
The solid placed on the table on the right is made up of four cubes.

The opposite faces of each cube are the same colour.

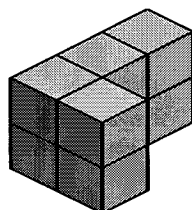
Each cube is placed on the table or stacked on top of another cube. The base of this solid is represented by the following blueprint.

The coded blueprint of the solid indicates in each square the number of cubes stacked up vertically.

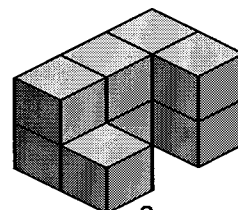
For each of the following solids, determine its coded blueprint.



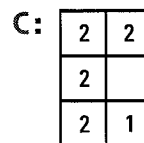
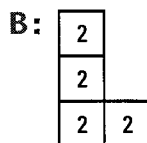
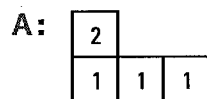
A



B



C

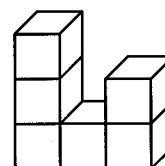
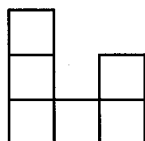


ACTIVITY 2 Different views of a solid

Using cubes, the object on the right is constructed.

Depending on from where you are looking, you can see different views of the object.

The front view of this object is:

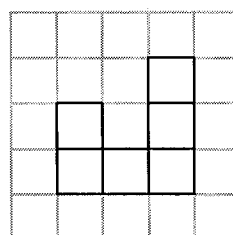
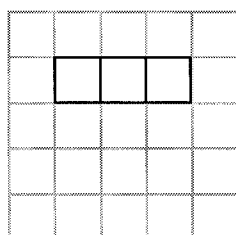
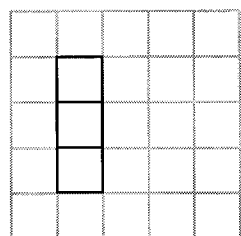


What would the view of this object be

a) from the right?

b) from the top?

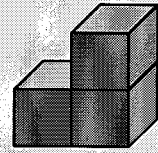
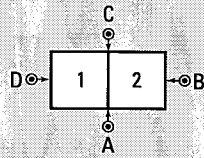
c) from the back?



CODED BLUEPRINT AND VIEWS OF A SOLID

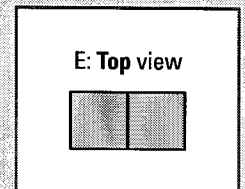
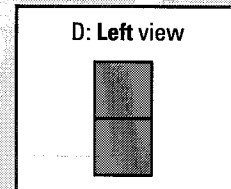
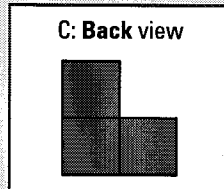
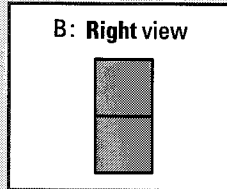
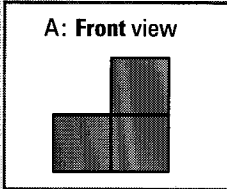
- The coded blueprint, associated with a solid made up of cubes, indicates in each square the number of cubes that are stacked up vertically.

Ex.: The solid on the right is described by the following coded blueprint:



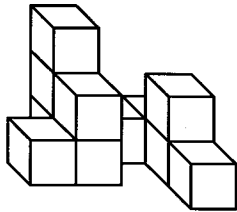
The opposite faces are the same colour.

- There are the following views:

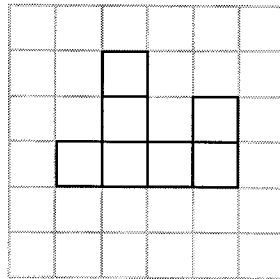


- Draw the requested views for each of the following solids.

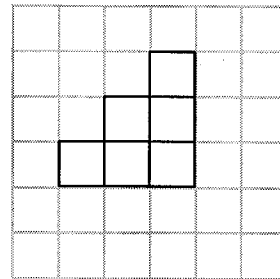
a)



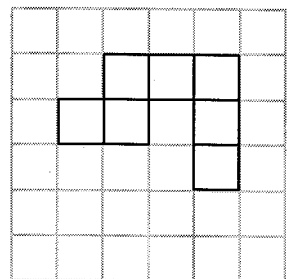
Front



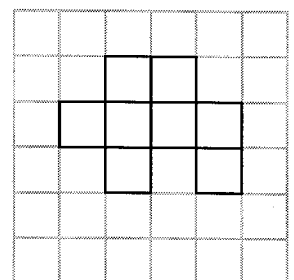
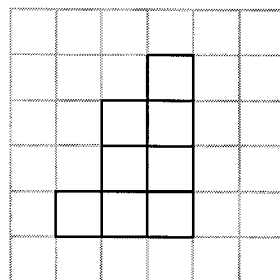
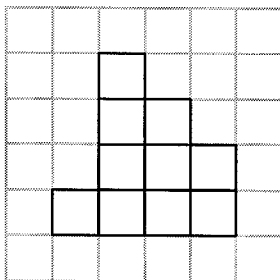
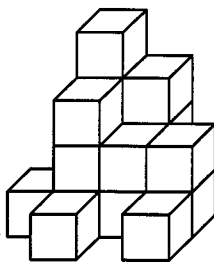
Right



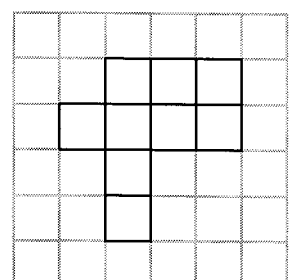
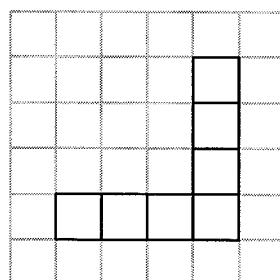
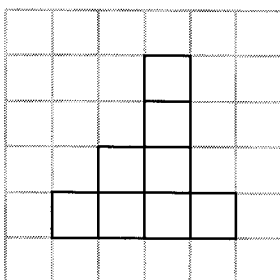
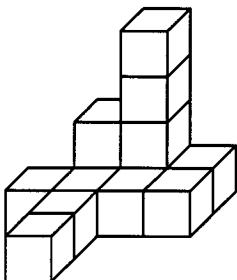
Top



b)



c)



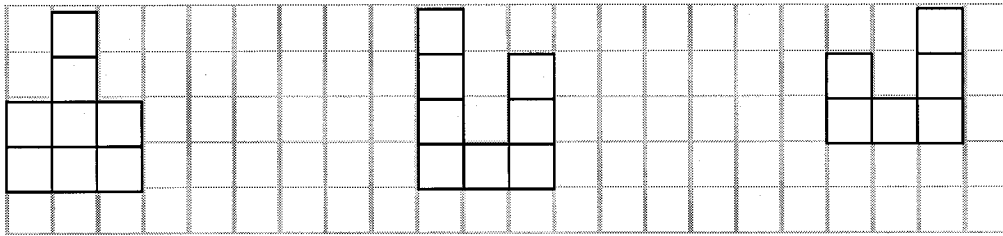
- 2.** Cubes are stacked to obtain an object represented by the coded blueprint on the right. Draw the requested views of this object.

		2
3		4
2	1	1

a) Left

b) Back

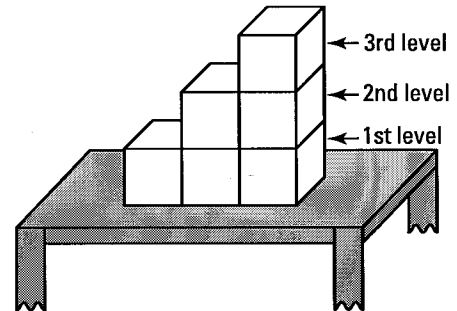
c) Top



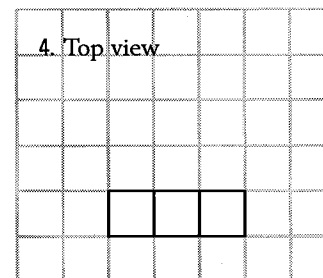
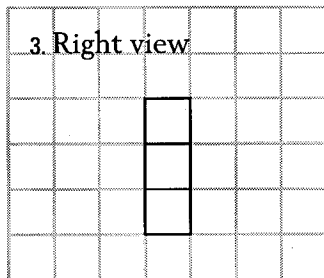
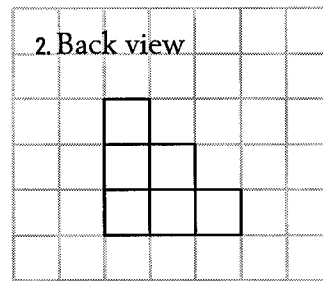
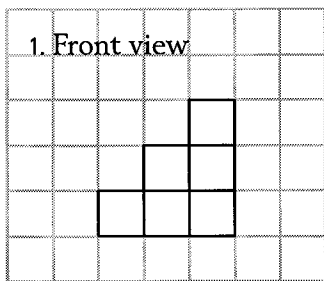
- 3.** The solid placed on the table on the right is made up of cubes. Each cube is on the table or stacked on top of another cube.

- a) How many total cubes are there in this solid? 6
 b) Determine this solid's coded blueprint.

1	2	3
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- c) Draw the solid from different views.

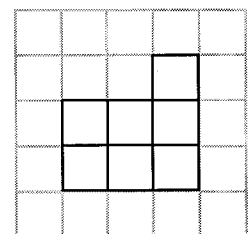


- 4.** The top view of a solid made up of identical cubes is represented by the following coded blueprint:

1	3		
2	1	1	
2			2

Right

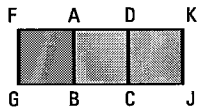
Front



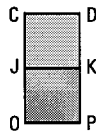
Draw the view of this solid from the right.

8. Complete the figures by placing the vertices on the following views.

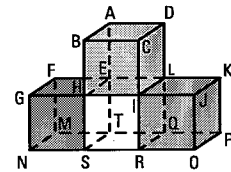
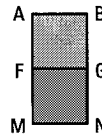
a) Top view



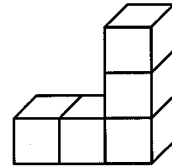
b) Right view

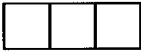



c) Left view




9. The following solid is observed from different views. In each case, indicate whether it is a front view, top view, right view, etc.



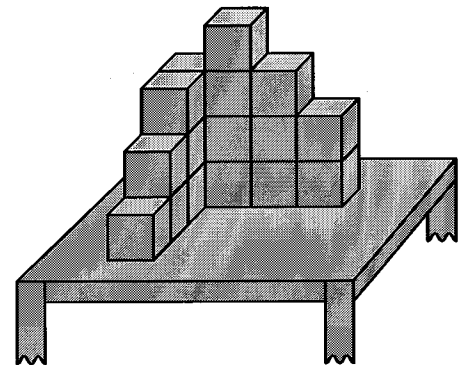
a) 
Top or bottom view

b) 
Right or left view

c) 
Front view

10. The solid on the right is made up of cubes. Each cube is placed on the table or stacked on top of another cube.

- a) How many cubes cannot be seen? 2
- b) How many cubes are on each of the following horizontal levels?
1st: 7 2nd: 6 3rd: 4 4th: 1
- c) How many total cubes are there in this solid? 18

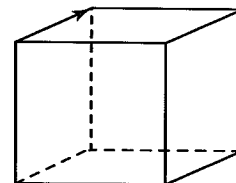


5.2 Perspectives of a solid

ACTIVITY 1 Oblique perspective

The face of a cube is represented on the right. Perform the translation of this face according to the translation vector and complete the cube's representation by drawing dotted lines for the invisible edges.

You have just represented the cube in oblique perspective.

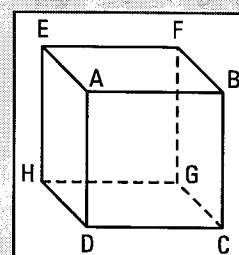


OBLIQUE PERSPECTIVE

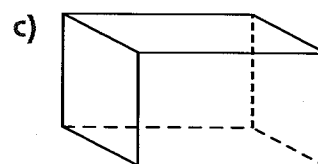
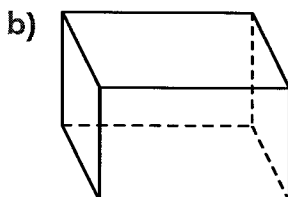
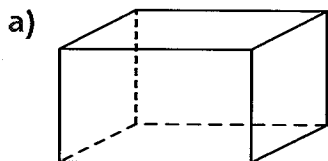
In the figure on the right, a cube has been represented in oblique perspective.

- The face ABCD is parallel to the sheet's plane.
- The edges AE, BF, DH and CG are called the receding lines. The receding lines are oblique and reduced in size compared to their actual length.

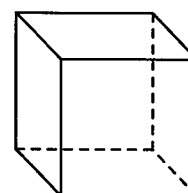
The cube is then usually completed with dotted lines to represent the unseen edges.



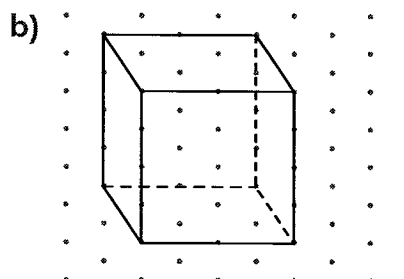
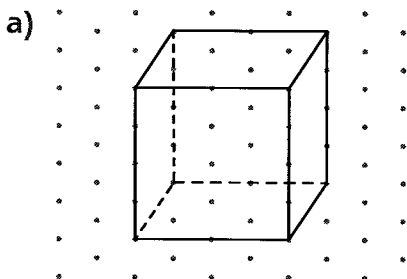
1. In the three figures below, three of a rectangular base prism's edges are drawn. Complete these figures to obtain the prism's representation in oblique perspective.



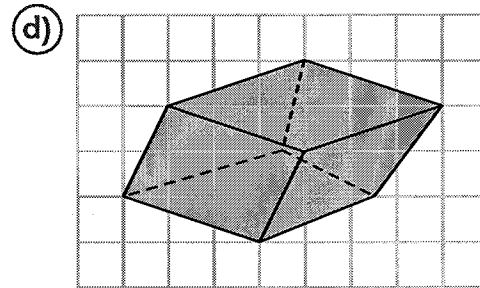
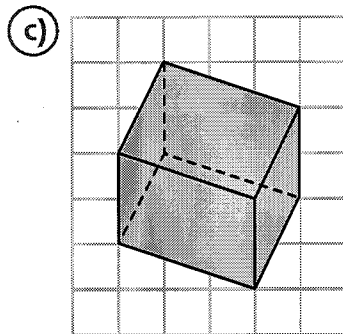
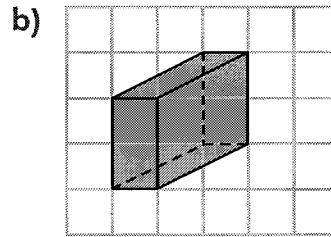
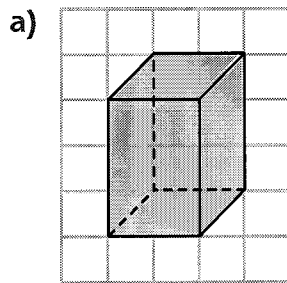
2. Consider the square on the right. Give the representation of a cube in oblique perspective using this square as one of its faces.



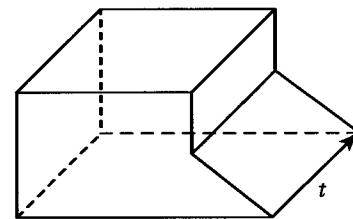
3. Using the triangular grid paper, complete the following figures to obtain an oblique perspective of the cube.



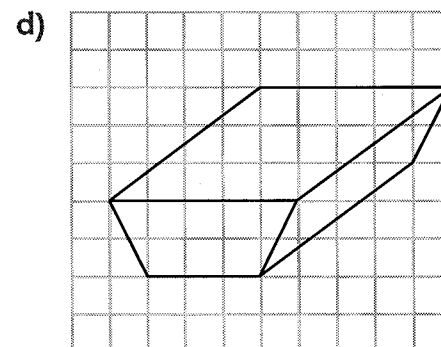
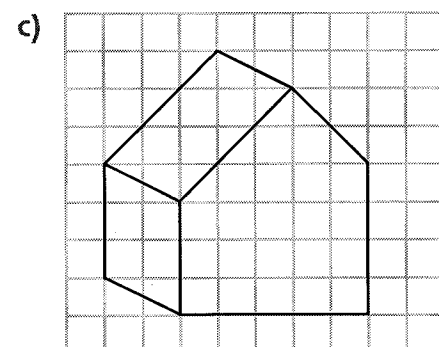
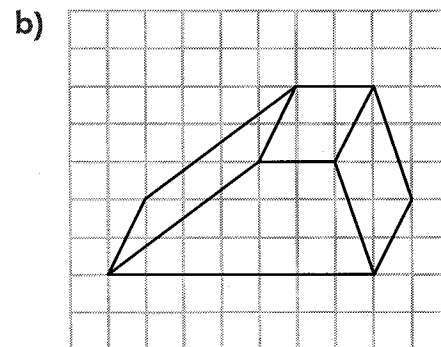
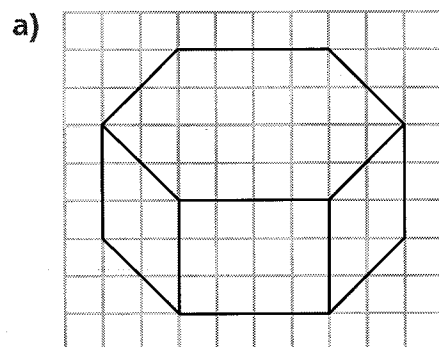
4. Among the following figures, which ones do not represent a prism drawn in oblique perspective?



5. Using the translation vector, complete the following figure to obtain an oblique perspective of the trapezoidal base prism.



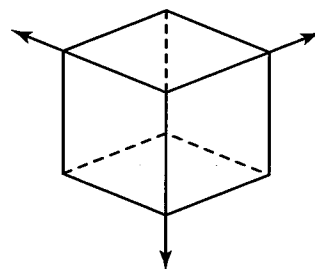
6. For each of the figures below, the base of a right prism is represented. Complete these figures to obtain an oblique perspective of the solid.



ACTIVITY 2 Axonometric perspective

Consider the three axes represented on the right and the edges of a cube. Complete the drawing of the cube by using the given axes and by drawing parallel lines to the axes.

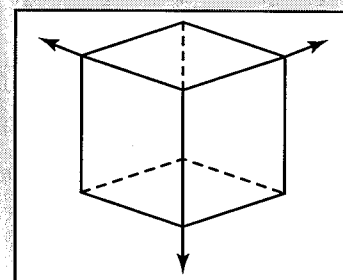
You have just represented the cube in axonometric perspective.



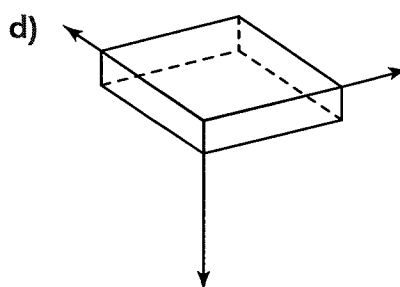
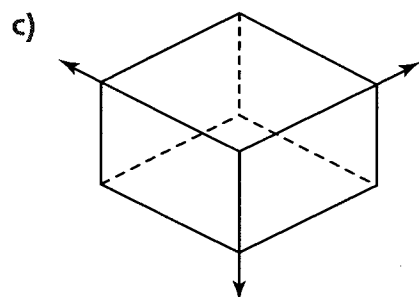
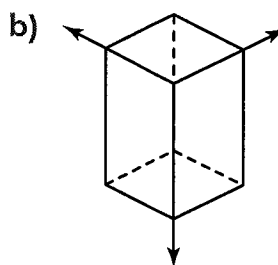
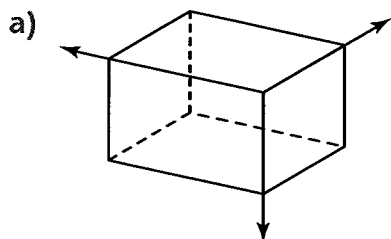
AXONOMETRIC PERSPECTIVE

In the figure on the right, a cube is represented in axonometric perspective.

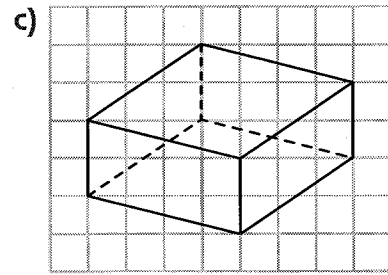
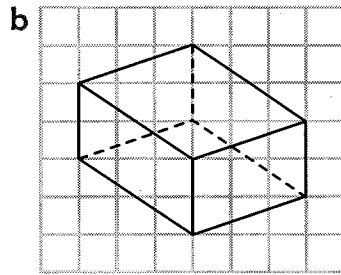
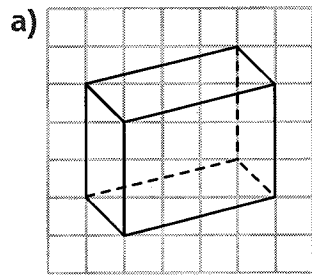
- The edges of the cube are parallel to the axes.
- The edges in the plane are not all congruent.



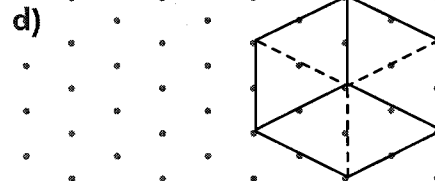
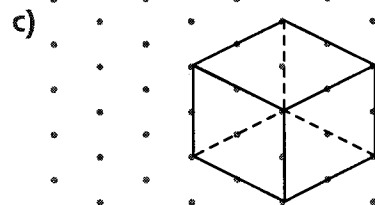
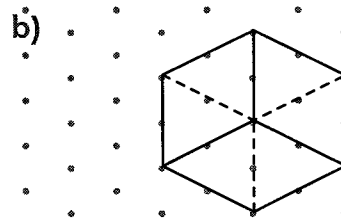
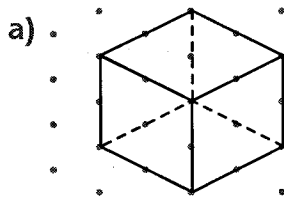
7. In the four figures below, three edges of rectangular base prisms are drawn. Complete these figures to obtain an axonometric perspective of each prism.



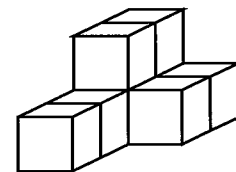
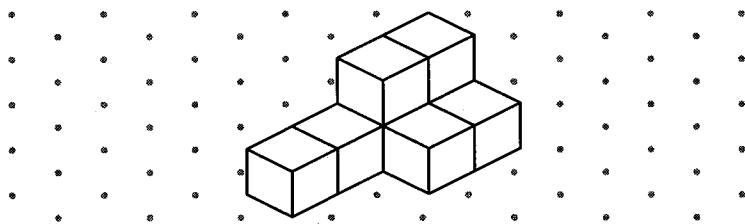
8. Complete the following figures to obtain an axonometric perspective of the rectangular base prism with the given edges.



9. Using the triangular grid paper below, complete the axonometric perspective of the cube with the given edges.



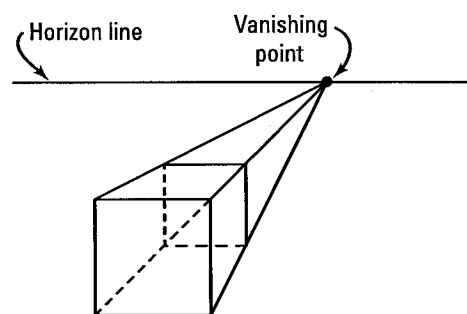
10. A solid composed of cubes is given in oblique perspective. Use the triangular grid paper below to draw this solid in axonometric perspective.



ACTIVITY 3 Linear perspective

Consider the face of a cube, a line and a point on this line as illustrated in the figure on the right.

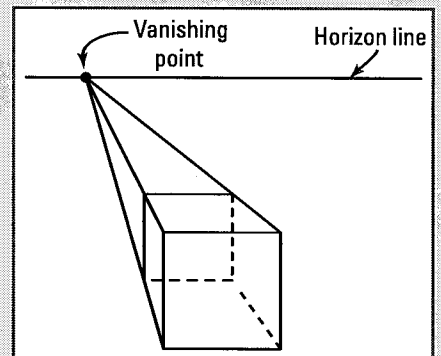
Join each of the vertices of the cube's face to the point called the **vanishing point**. Within these lines and from a distance which you judge to be appropriate from a vertex, draw the back face of the cube using parallel lines to each of the edges of the cube's given face. You have just drawn the cube in **linear perspective**.



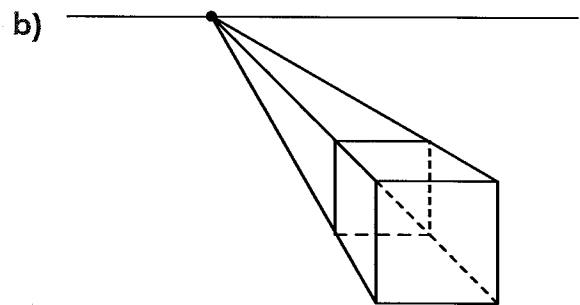
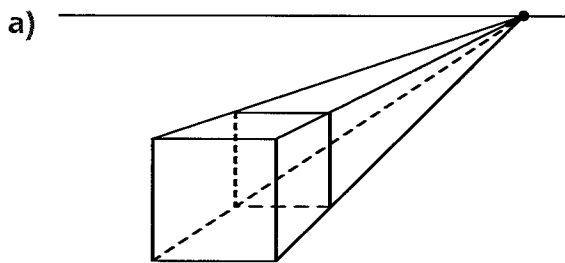
LINEAR PERSPECTIVE

In the figure on the right, a cube is represented in linear perspective where a vanishing point is given.

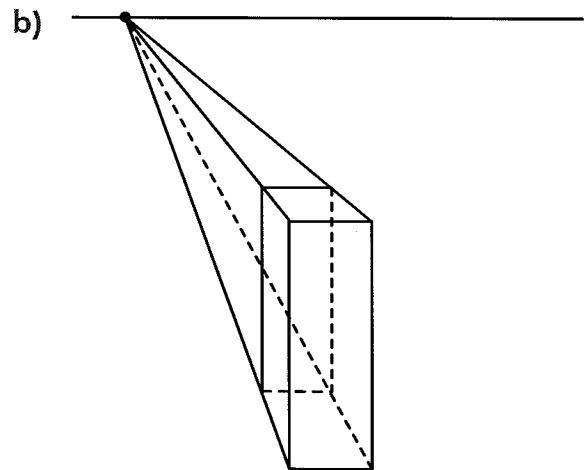
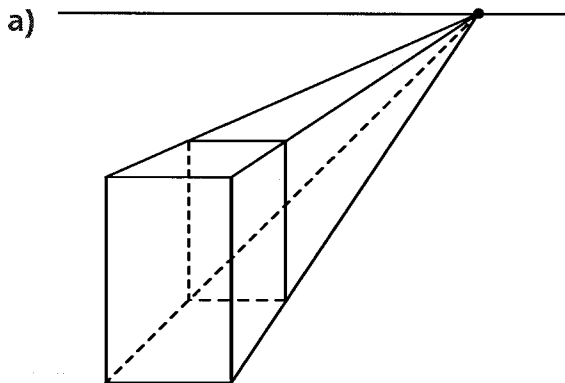
- One of the faces is parallel to the sheet's plane.
- The receding lines are reduced in size.



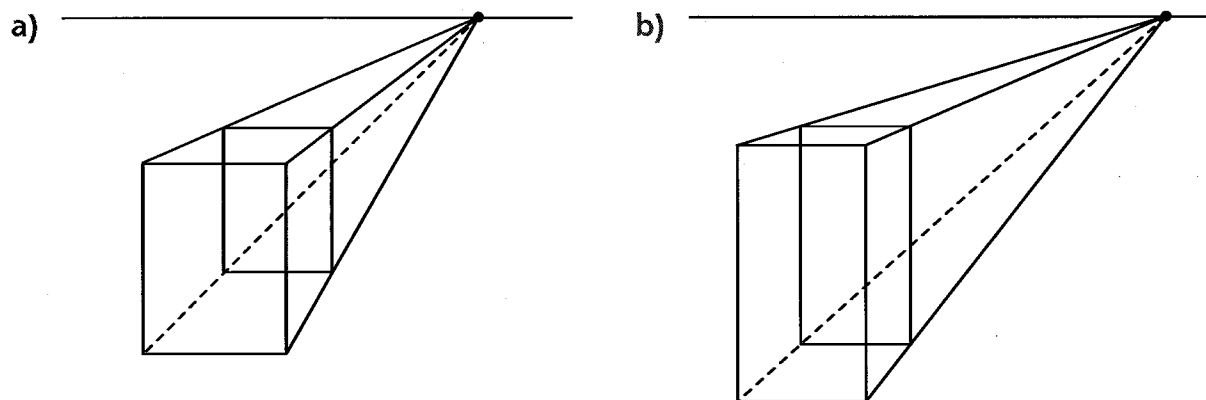
11. Given the face of a cube and the vanishing point, complete the following figures to obtain a linear perspective of the cube.



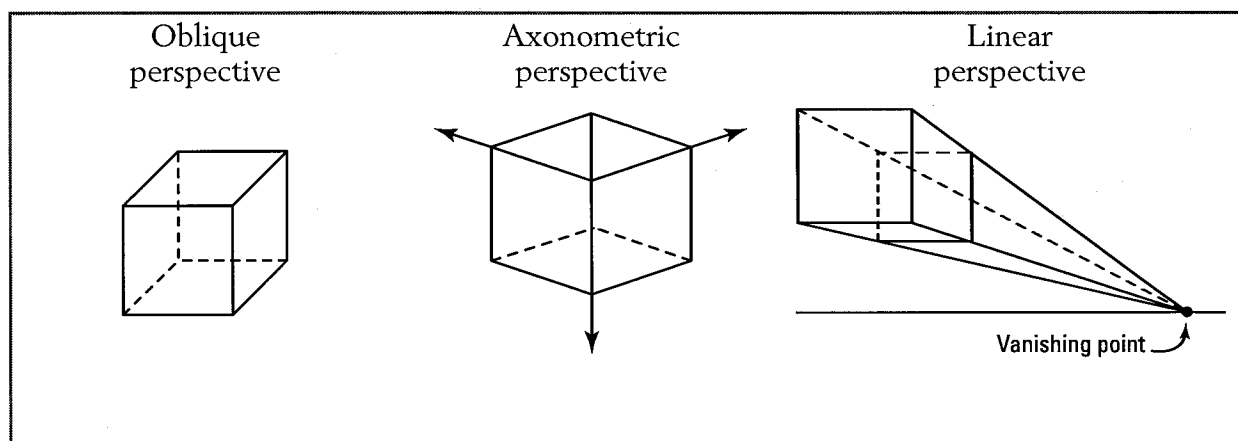
12. Draw a right square base prism in linear perspective, given the front face and the vanishing point.



13. Draw a right square base prism in linear perspective, given the back face and the vanishing point.



14. Draw a cube with 1.5 cm edges in three different perspectives.



5.3 Prisms – Pyramids – Cylinders

ACTIVITY 1 Describing a solid

a) Consider the given right prism.

1. Indicate the nature of this prism's bases.

The bases are triangles.

2. Are the bases parallel and congruent? Yes

3. Indicate the nature of this prism's lateral faces.

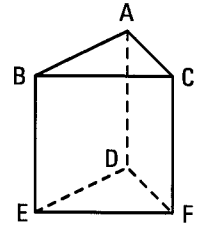
The lateral faces are rectangles.

4. Are the lateral faces congruent and perpendicular to the bases?

Yes

5. Name one edge whose length is the height of this prism.

The edge AD, BE or CF.



b) Consider the given right regular pyramid.

1. Indicate the nature of the base. A square

2. Indicate the nature of the lateral faces.

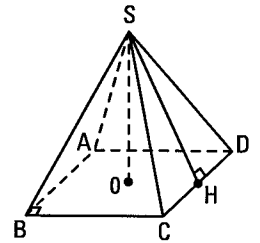
The lateral faces are isosceles triangles.

3. Are the lateral faces congruent?

Yes

4. What does the height SH of the triangle SCD represent for the pyramid? The slant height

5. The point O is located at the centre of the base. What does the segment SO represent for the pyramid? The height of the pyramid



c) Consider the given right circular cylinder.

1. Indicate the nature of the bases.

The bases are discs.

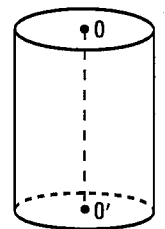
2. Are the bases parallel and congruent? Yes

3. Is the lateral surface plane or rounded?

Rounded

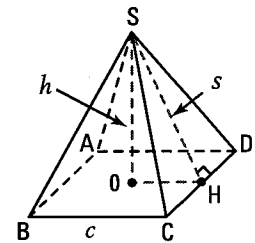
4. What does the segment OO' joining the two centres of the bases represent for the cylinder?

The height of the cylinder



ACTIVITY 2 A remarkable triangle within the pyramid

Consider the triangle SOH in the given right square base pyramid. The point O is located at the centre of the base.



- a) What is the nature of the triangle SOH? A right triangle
- b) What relation can be deduced between the height h of the pyramid, the side length c of the base and the slant height s ? Justify your answer.

Since triangle SOH is a right triangle at O, the Pythagorean Theorem states that

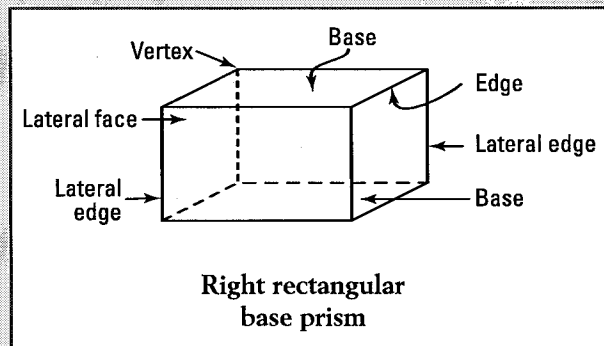
$$h^2 + \left(\frac{c}{2}\right)^2 = s^2 \text{ or } h^2 + \frac{c^2}{4} = s^2$$

- c) Find the slant height of the pyramid if $c = 6$ cm and $h = 4$ cm.

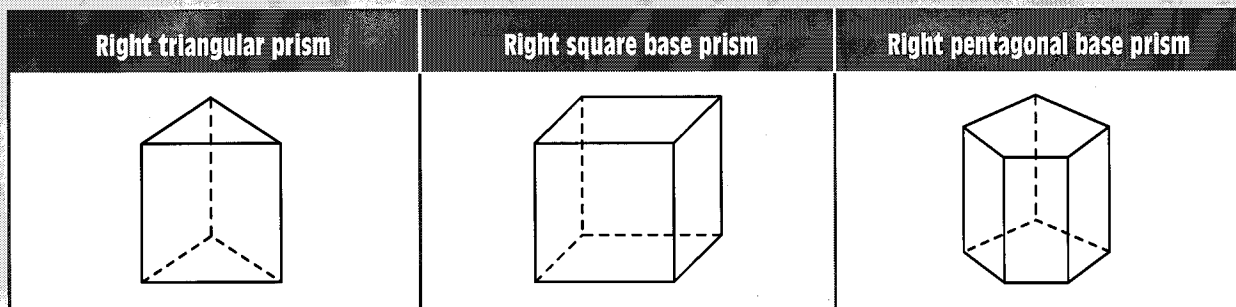
$$s = 5 \text{ cm}$$

PRISMS

- A right prism is a solid limited by
 - two parallel and congruent polygons called the prism's bases;
 - rectangular lateral faces that are perpendicular to the bases.
- Properties of right prisms:
 - The bases are parallel and congruent.
 - The lateral faces are perpendicular to the bases.
 - The lateral edges are parallel, congruent and perpendicular to the bases.
- The height of the prism is the length of one of the lateral edges.

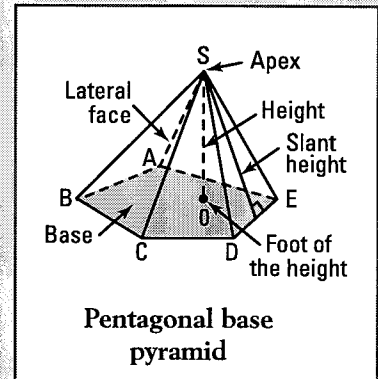


Ex.:



PYRAMIDS

- A pyramid is a solid limited by
 - a polygon called the base of the pyramid;
 - lateral faces in the shape of triangles.
- The height of each lateral face is called slant height.
- A pyramid is **right and regular** when its base is a regular polygon and when the foot of the height is the centre of the base. The lateral faces are then **congruent isosceles triangles**.

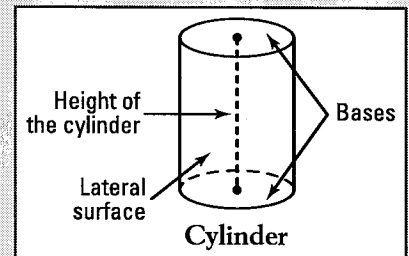


Ex. :

Right square base pyramid	Right regular hexagonal base pyramid	Right regular triangular base pyramid

CYLINDERS

- A right circular cylinder is limited by
 - two discs with the same radius and parallel called the cylinder's bases;
 - a curved surface called the lateral surface or cylindrical surface.
- The radius of the base is called the cylinder's radius.
- The segment joining the centres of the bases is the **height** of the cylinder.



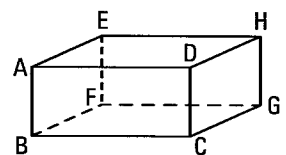
In order to simplify classification, it is generally agreed upon to use the words **prism**, **pyramid** and **cylinder** to represent any right prism, right regular pyramid or right circular cylinder.

1. a) Name the solid on the right which is laying on its base.

It is a rectangular base prism.

- b) Complete the description.

1. BCGF is a base. 2. A is a vertex.
3. \overline{AB} is an edge. 4. ABCD is a face.

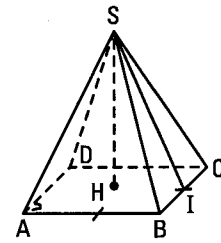


2. a) Name the given right solid.

It is a square base pyramid.

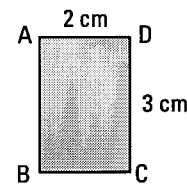
b) Complete the description.

1. ABCD is the base. 2. S is the apex.
 3. \overline{SH} is the pyramid's height. 4. \overline{SI} is the slant height.



3. a) Explain how to generate a cylinder with a height of 3 cm using the rectangle on the right. What will the cylinder's radius be?

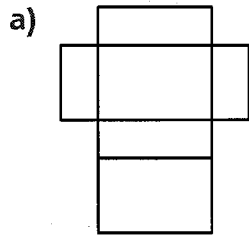
We rotate the rectangle one complete revolution around its length. The radius is 2 cm.



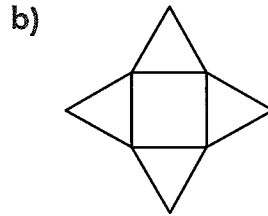
b) Explain how to generate a cylinder with a height of 2 cm using the same rectangle. What will the cylinder's radius be?

We rotate the rectangle one complete revolution around its width. The radius is 3 cm.

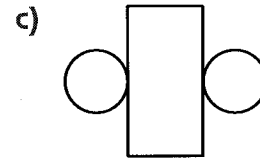
4. The nets of various solids are represented below. Name each solid based on its net.



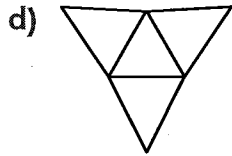
Rectangular base prism



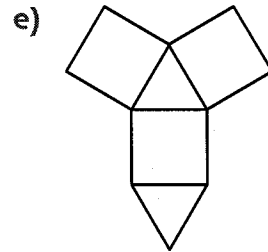
Square base pyramid



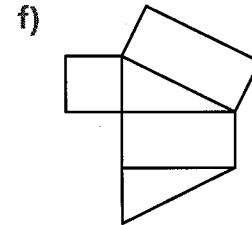
Cylinder



Triangular base pyramid



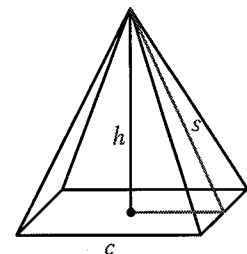
Triangular base prism



Triangular base prism

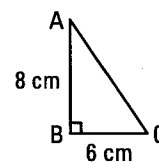
5. The pyramid on the right with height h and slant height s has a base with side length c . Complete the following table.

h	c	s
8	12	10
12	10	13
4	6	5



1. a) Explain how to generate a cone with a height of 8 cm using the triangle on the right. What will its radius be?

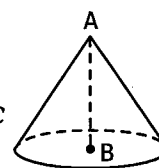
A complete revolution of this triangle around the side AB. $r = 6$ cm



- b) Which point represents
1. the cone's apex? A 2. the centre of its base? B

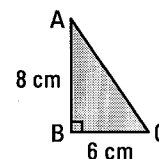
- c) Which side of the triangle generates
1. the base of the cone? Side BC 2. the conical surface? The hypotenuse AC

- d) Draw a view of this generated cone.



2. a) Explain how to generate a cone with a height of 6 cm using the same triangle as the preceding question. What will its radius be?

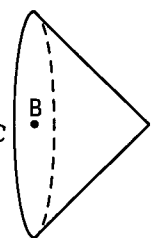
A complete revolution of this triangle around the side BC. $r = 8$ cm



- b) Which point represents
1. the cone's apex? C 2. the centre of its base? B

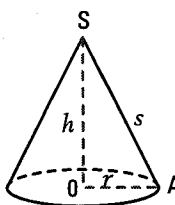
- c) Which side of the triangle generates
1. the base of the cone? Side AB 2. the conical surface? The hypotenuse AC

- d) Draw a view of this generated cone in perspective.



ACTIVITY 3 A remarkable triangle within the cone

The cone on the right has height h and radius r . The segment SO represents the height of the cone, segment OA represents a radius and segment SA represents a slant height of length s .



- a) What is the nature of triangle SOA? Right in O

- b) What is the relation between the slant height s , the height h and the radius r of the cone? Justify your answer.

$s^2 = h^2 + r^2$ (Pythagorean Theorem applied to triangle SOA)

- c) Determine

1. s if $h = 4$ cm and $r = 3$ cm. $s = 5$ cm

2. h if $s = 6$ cm and $r = 3,6$ cm. $h = 4,8$ cm

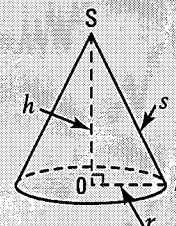
3. r if $s = 7$ cm and $h = 5,6$ cm. $r = 4,2$ cm

PROPERTIES OF CONES

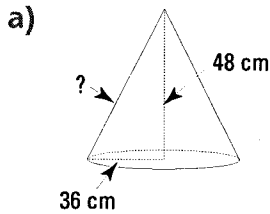
The cone on the right has radius r , height h and slant height s .

Thus,

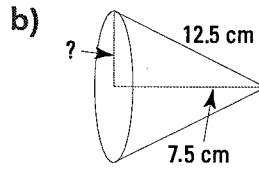
$s^2 = h^2 + r^2$



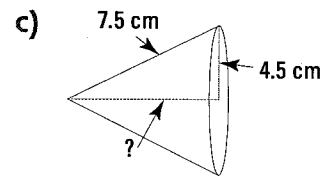
3. Find the missing dimension.



60 cm



10 cm



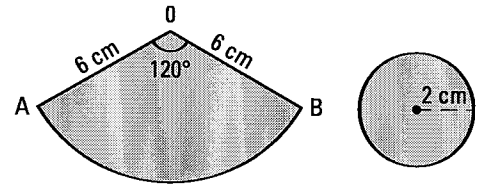
6 cm

ACTIVITY 4 Net of a cone (Constructing cones)

The net of a cone is composed of a circular sector and a disc.

a) Draw on construction paper

1. a circular sector with a radius of 6 cm and a central angle measuring 120° .
2. a disc with a 2 cm radius.

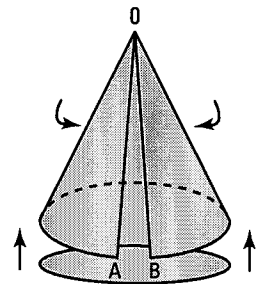


- b) 1. Calculate the length of the arc AB. 4π cm
2. Calculate the circumference of the circle with a 2 cm radius. 4π cm
3. Verify that the length of the arc AB is equal to the circumference of the circle.

- c) 1. Cut out the circular sector and fold it as illustrated on the right.
2. Cut out the disc and glue it to the arc of the circular sector to complete the construction of the cone.

d) What do the following become for the cone?

1. The surface of the circular sector AOB. The lateral surface of the cone
2. The radius of the circular sector AOB. The slant height of the cone



ACTIVITY 5 Net of a cone (Finding the angle of the circular sector)

The net of a cone with slant height $s = 4$ cm and radius $r = 0.8$ cm is represented on the right.

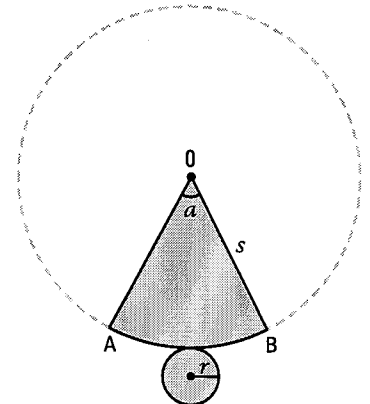
- a) 1. What can be said about the measure of the arc AB and the circumference of the cone's base? They are equal.
2. What is the measure of the arc AB? 1.6π cm
- b) Determine the circumference C of the large disc with radius s (represented as dotted). 8π cm
- c) The central angle a , the measure of the arc AB and the circumference C of the large disc form the proportion:

$$\frac{a}{360^\circ} = \frac{m\widehat{AB}}{C}$$

Determine the measure of the angle a using this proportion.

$$\frac{a}{360^\circ} = \frac{1.6\pi}{8\pi} \Rightarrow a = 72^\circ$$

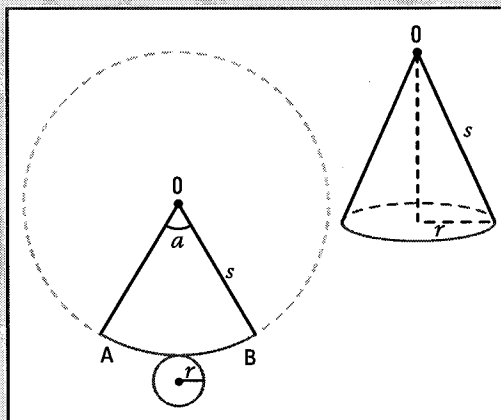
- d) Verify the proportion $\frac{a}{360^\circ} = \frac{r}{s}$ $\frac{72^\circ}{360^\circ} = \frac{0.8}{4}$ True



NET OF A CONE

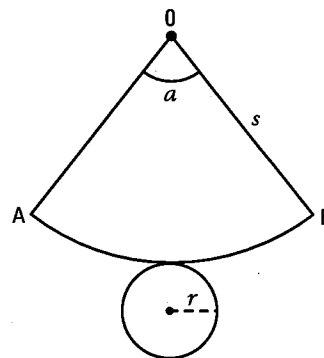
- The net of a cone with radius r and slant height s includes:
 - a circular sector of radius s (slant height of the cone) and central angle a , representing the conical surface.
 - a disc of radius r , representing the base of the cone.
- The following proportions are obtained:

$$\frac{a}{360^\circ} = \frac{\widehat{mAB}}{2\pi s} \quad \text{and} \quad \frac{a}{360^\circ} = \frac{r}{s}$$



4. Complete the following table using the given net of a cone and the proportion $\frac{r}{s} = \frac{a}{360^\circ}$.

a ($^\circ$)	r (cm)	s (cm)	\widehat{mAB} (cm)
60°	2	12	4π
90°	3	12	6π
45°	0.5	4	π
30°	2	24	4π
45°	1	8	2π



5. Explain the procedure for constructing a cone with a 3 cm radius and height of 4 cm.

1. Calculate the slant height of the cone: $s^2 = h^2 + r^2$; $s^2 = 25$; $s = 5$ cm.

2. Calculate the central angle of the circular sector.

$$\frac{a}{360^\circ} = \frac{r}{s} \Rightarrow \frac{a}{360^\circ} = \frac{3}{5} \Rightarrow a = 216^\circ$$

3. Draw this circular sector with a 5 cm radius (slant height of the cone).

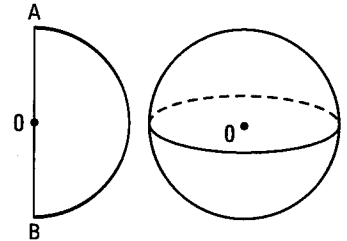
4. Draw a disc with a 3 cm radius.

5.5 Sphere

ACTIVITY 1 Sphere generated by a rotation

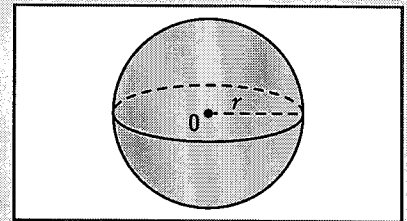
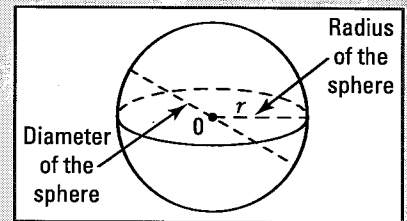
A complete revolution of the semi-disc on the right around the diameter AB will generate a solid.

- a) What is this solid called? A sphere Draw it.
- b) Is the surface generated by the semi-circle AB straight or curved? Curved
 What is this surface called? Spherical surface



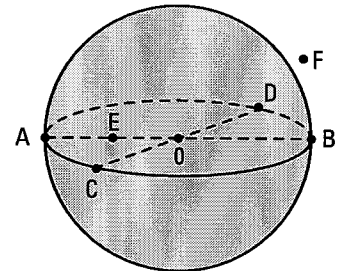
SPHERES

- The spherical surface with centre O and radius r is the set of all points in space located at a distance from the centre equal to r .
 - Any segment joining the centre of the sphere to a point on the spherical surface is a radius of the sphere.
 - Any segment joining two points on the spherical surface and passing through the centre of the sphere is a diameter of the sphere.
- A sphere with centre O and radius r is the set of all points in space whose distance to the centre is less than or equal to r .



1. Consider the sphere with centre O and a radius of 3 cm.

- a) Complete.
- \overline{OA} is a radius.
 - \overline{AB} is a diameter.
 - E is a point inside the sphere.
 - C is a point on the sphere.
 - F is a point outside the sphere.



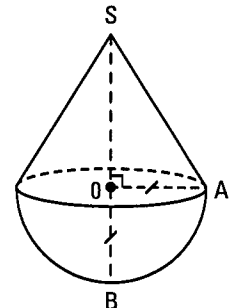
- b) Complete.
- $m\overline{OC}$ = 3 cm.
 - $m\overline{CD}$ = 6 cm.
 - $m\overline{OE}$ < 3 cm.
 - $m\overline{OF}$ > 3 cm.

2. a) Describe with precision the solid on the right.

It is a hemisphere topped by a right circular cone.

b) Complete the description.

- O is the foot of the cone's height and the centre of the hemisphere.
- \overline{SO} is the height of the cone.
- \overline{OA} is the radius of the cone and the hemisphere.
- \overline{SA} is the slant height of the cone.

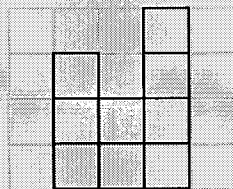


EVALUATION 5

1. Consider the solid represented on the right.
Draw the requested views of this solid.



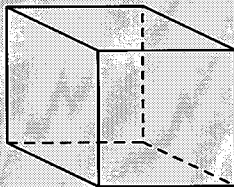
2. Consider the solid composed of identical cubes whose coded right. Represent the view of this solid from the left.



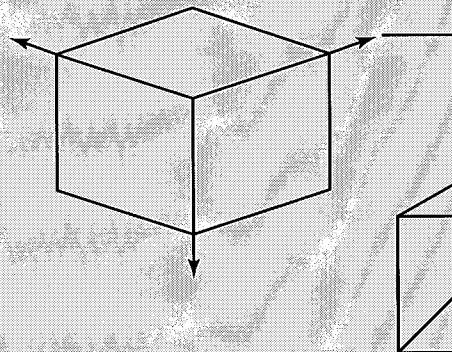
3. Complete the following figures to obtain the desired perspective

- a) a cube

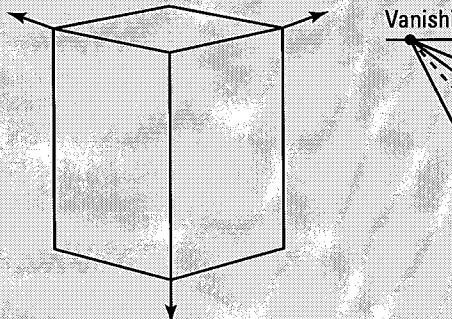
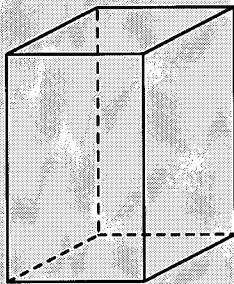
oblique perspective



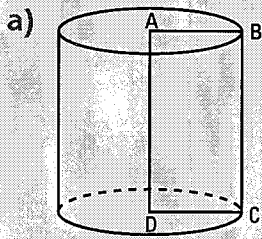
axonometric perspective



- b) a square base prism

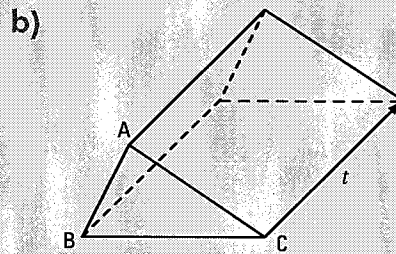


4. Give the name of the solid generated by the transformations indicated below and draw the solid.



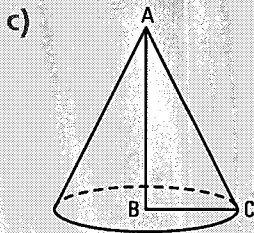
Rotation of the rectangle around the axis AD.

Cylinder



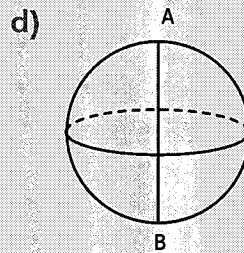
Translation of the triangle ABC with the translation vector t .

Triangular base prism



Rotation of the triangle ABC around the axis AB.

Cone

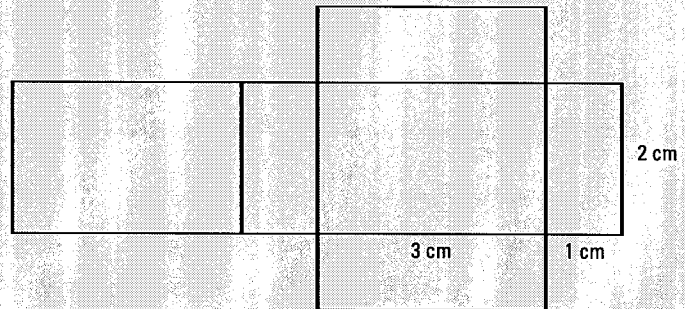
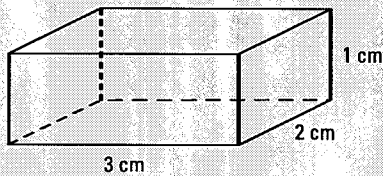


Rotation of the semi-circle around the axis AB.

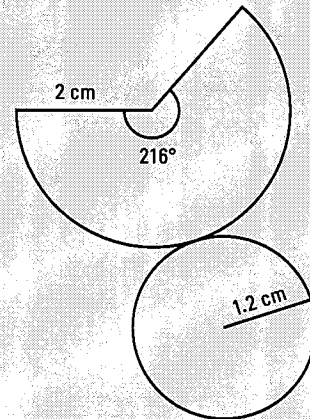
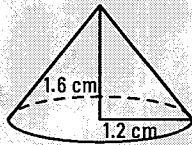
Sphere

5. Draw the net of each of the following solids using the given dimensions.

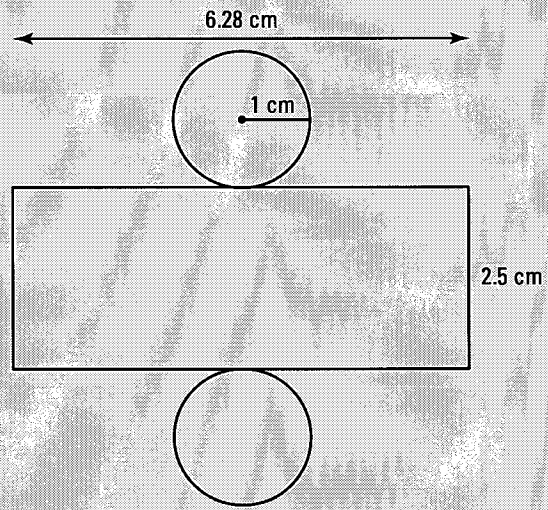
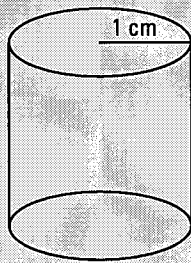
a) Rectangular base prism



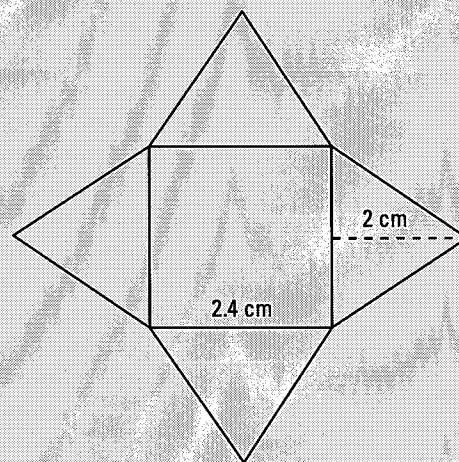
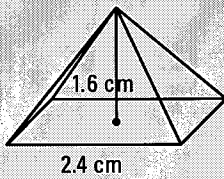
b) Cone



c) Cylinder



d) Square base pyramid



6. The circular sector of a cone's net has a radius of 13 cm. If the radius of the cone is 5 cm, what is the angle of this sector?

138.5°

