FACTORING

1 Example of an appropriate method

Polynomial representing the area of rectangle ABCD

Since APQD is a square, segment AD measures (x) cm.

Area of rectangle ABCD

$$\text{m } \overline{\text{AD}} \times \text{m } \overline{\text{DC}}$$

$$\text{m } \overline{\text{AD}} \times \left(\text{m } \overline{\text{DQ}} + \text{m } \overline{\text{QC}}\right)$$

$$x(x + x + 8)$$

$$x(2x + 8)$$

$$2x^2 + 8x$$

Value of x

Area of rectangle ABCD = 120 cm^2

$$2x^2 + 8x = 120$$

$$2x^2 + 8x - 120 = 0$$

$$2(x^2 + 4x - 60) = 0$$

$$x^2 + 4x - 60 = 0$$

$x^2 - 6x + 10x - 60 = 0$

$x(x - 6) + 10(x - 6) = 0$

$(x - 6)(x + 10) = 0$

$x = 6$ or $x = -10$ (impossible)

Area of rectangle PBCQ

$$\text{m } \overline{\text{PQ}} \times \text{m } \overline{\text{QC}}$$

$x(x + 8)$

$6(6 + 8)$

84 cm^2

Answer: The numerical area of rectangle PBCQ is **84** cm^2.

Note: Students who use an appropriate method in order to determine the polynomial representing the area of rectangle ABCD **and** then write the equation $2x^2 + 8x = 120$ have shown that they have a partial understanding of the problem.

2 Example of an appropriate method

Value of x

Since the square and the rectangle are equivalent, their areas are equal.

Area of the rectangle = Area of the square

$2x^2 - 7x - 30 = x^2$

$x^2 - 7x - 30 = 0$

$x^2 - 10x + 3x - 30 = 0$

$x(x - 10) + 3(x - 10) = 0$

$(x - 10)(x + 3) = 0$

$x = 10$ or $x = -3$ (impossible)

Algebraic expressions representing the dimensions of the rectangle

$2x^2 - 7x - 30$

$2x^2 - 12x + 5x - 30$

$2x(x - 6) + 5(x - 6)$

$(x - 6)(2x + 5)$

The rectangle measures $(x - 6)$ cm by $(2x + 5)$ cm.

Perimeter of the rectangle

Since $x = 10$, the rectangle measures 4 cm by 25 cm.

Perimeter: $2(4 + 25) = 58$ cm

Answer: The perimeter of the rectangle is **58** cm.

Note: Students who use an appropriate method in order to determine the value of x **or** the algebraic expressions representing the dimensions of the rectangle have shown that they have a partial understanding of the problem.

3 Example of an appropriate method

Lengths of the sides of rectangle ABCD

Area: $12x^2 + 28x - 5$

$12x^2 + 30x - 2x - 5$

$6x(2x + 5) - 1(2x + 5)$

$(2x + 5)(6x - 1)$

Rectangle ABCD measures $(2x + 5)$ units by $(6x - 1)$ units.

Lengths of the sides of rectangle FBGH

$2x + 5 - 6 = 2x - 1$

$6x - 1 - 6 = 6x - 7$

Rectangle FBGH measures $(2x - 1)$ units by $(6x - 7)$ units.

Area of rectangle FBGH

Area: $(2x - 1)(6x - 7)$

$12x^2 - 14x - 6x + 7$

$12x^2 - 20x + 7$

Answer: The polynomial $12x^2 - 20x + 7$ represents the area of rectangle FBGH.

Note: Students who use an appropriate method in order to determine the binomials representing the lengths of the sides of rectangle ABCD have shown that they have a partial understanding of the problem.

4 C

5 Example of an appropriate method

Factoring the numerator

$c^2 - 4 = (c - 2)(c + 2)$

Factoring the denominator

$c^2 + c - 6 = (c + 3)(c - 2)$

Simplifying the expression

$$\frac{c^2 - 4}{c^2 + c - 6} = \frac{(c - 2)(c + 2)}{(c + 3)(c - 2)} = \frac{c + 2}{c + 3}$$

Answer The simplified form of this expression is $\frac{c + 2}{c + 3}$.

6 The result is $3a$

7 Work : (example)

Height of the right prism with a rectangular base

Volume = Area of the base $\times$ height

Height = $\dfrac{2x^3 + x^2 - 22x + 24}{2x^2 + 5x - 2}$

Height = $x - 2$

Dimensions of the rectangular base of the right prism

Area = length $\times$ width

$2x^2 + 5x - 12 = (2x - 3)(x + 4)$

Result The dimensions of the base are $(2x - 3)$ m and $(x + 4)$ m and the height is $(x - 2)$ m.

8 Work : (example)

Area of the rectangular base of the right prism

Volume = area of the base × height

Area of the base = $\dfrac{6x^3 - 35x^2 + 19x + 30}{x - 5}$

Area of the base = $(6x^2 - 5x - 6)$ m^2

Dimensions of the rectangular base of the right prism

Area = length × width

$6x^2 - 5x - 6 = (3x + 2)(2x - 3)$

Result The dimensions are $(3x + 2)$ m and $(2x - 3)$ m.

9 Example of an appropriate method

Area of the rectangular base of the right prism

Volume = area of the base × height

Area of the base = $\dfrac{x^3 + 4x^2 + x - 6}{x - 1}$

Area of the base = $x^2 + 5x + 6$

Dimensions of the rectangular base of the right prism

Area = length × width

$x^2 + 5x + 6 = (x + 3)(x + 2)$

Result The dimensions are $(x + 3)$ m and $(x + 2)$ m.

10 B

Name : _____

Group : _____

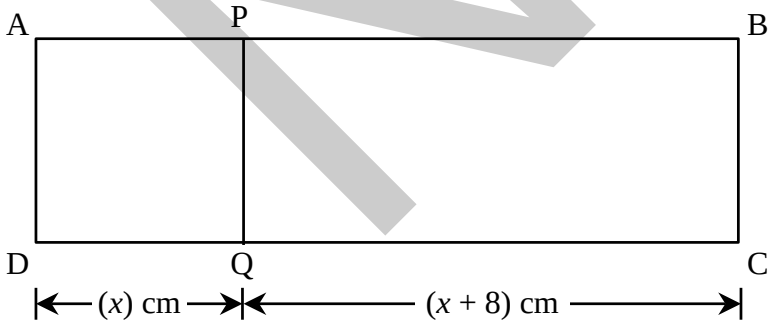
Date : _____

568436 - Mathematics

Question Booklet

1 In the figure below, line segment PQ divides rectangle ABCD into the following two quadrilaterals: square APQD and rectangle PBCQ.

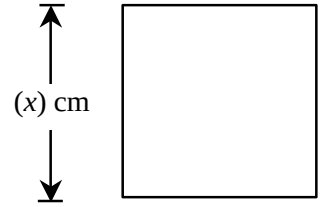
The area of rectangle ABCD is 120 cm^2. In addition, $m\overline{DQ}$ = (x) cm and $m\overline{QC}$ = (x + 8) cm.

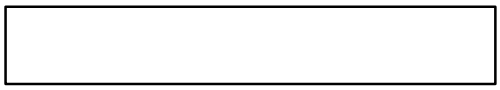


What is the numerical area of rectangle PBCQ?

Show all your work.

2 The square and the rectangle shown below are equivalent figures. Each side of the square measures (x) cm. The area of the rectangle is $(2x^2 - 7x - 30)$ cm^2.





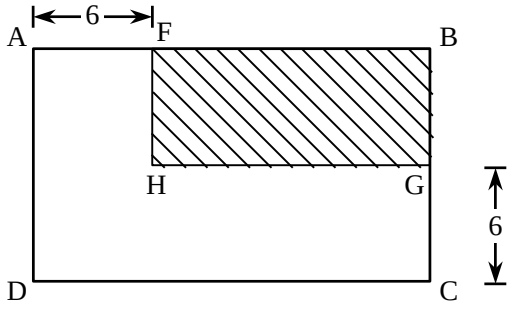
Area: $(2x^2 - 7x - 30)$ cm^2

What is the perimeter of the rectangle?

Your final answer must be a number.

Show all your work.

3 In the following diagram, quadrilaterals ABCD and FBGH are rectangles. In addition, $m\overline{AF}$ = 6 units and $m\overline{GC}$ = 6 units.



The polynomial $12x^2 + 28x - 5$ represents the area of rectangle ABCD.

What polynomial represents the area of rectangle FBGH?

Show all your work.

4 In the following algebraic expression, the denominators are not equal to zero.

$$\frac{x}{x+9}+\frac{3x+27}{x^2+18x+81}$$

Which of the following expressions is equivalent to this algebraic expression?

A) $\frac{1}{3}$

B) $x + 3$

C) $\frac{x+3}{x+9}$

D) $\frac{x+3}{(x+9)^2}$

5 In the following algebraic expression, the denominator is not equal to zero.

$$\frac{c^2-4}{c^2+c-6}$$

Simplify the expression.

Show all your work.

6 Simplify the following algebraic expression.

$$\frac{a^2 - 1}{a^2 + a - 2} \div \frac{2a + 2}{6a^2 + 12a}$$

7 The volume of a right prism with a rectangular base is $(2x^3 + x^2 - 22x + 24)$ m^3. The area of its base is $(2x^2 + 5x - 12)$ m^2.

What are the three dimensions of this prism?

(Express each dimension as a binomial.)

Show your work.

8 The volume of a right prism with a rectangular base is $(6x^3 - 35x^2 + 19x + 30)$ m^3. Its height is $(x - 5)$ m.

What are the dimensions of the base of this prism?

(Express each dimension as a binomial.)

Show your work.

9 The volume of a right prism with a rectangular base is $(x^3 + 4x^2 + x - 6)$ m^3. Its height is $(x - 1)$ m.

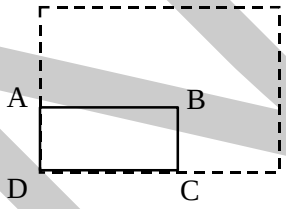
What are the dimensions of the base of this prism?

(Express each dimension as a binomial.)

Show your work.

10 In the figure below, the area of rectangle ABCD in square units is expressed by the trinomial $2x^2 - 11x + 12$, the measure of its sides being binomials.

Sides DA and DC are each extended 4 units to form a new rectangle.



In square units, what algebraic expression represents the area of the new rectangle?

A) $2x^2 + 1$

B) $2x^2 + x$

C) $2x^2 - 11x + 28$

D) $2x^2 + 23x + 56$