# GREATEST INTEGER
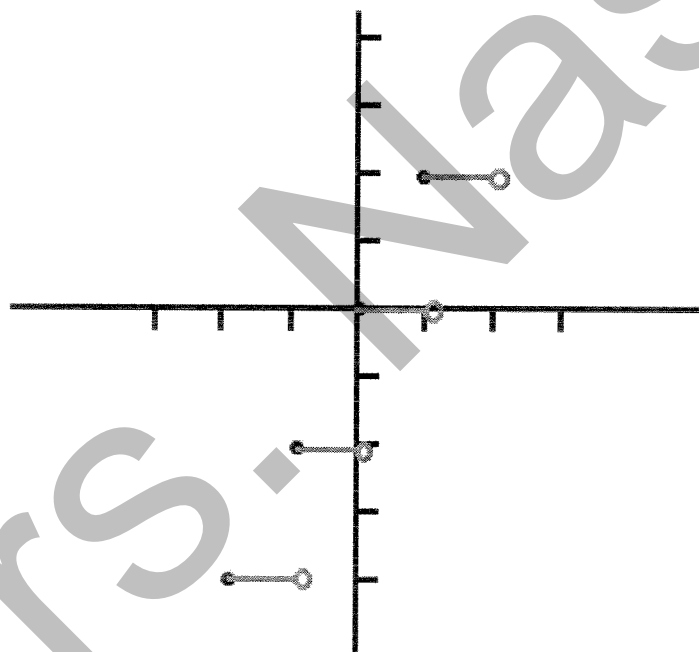
# FUNCTION

# PACKET

# MATH 466

# MISS NASSIF

Solve for x

ANSWERS:

1.) $[x - 2] = 3$

2.) $[2x - 5] = 9$

3.) $[x + 8] = -34$

4.) $-2[x + 1] = 8$

5.) $5[2x + 3] = 10$

6.) $-5\left[\frac{2}{3}x - 9\right] = -20$

7.) $\frac{3}{2}\left[\frac{x}{4} - \frac{3}{5}\right] = -1$

8.) $1.25\left[\frac{x}{2} + 1\right] = -2.5$

9.) $\frac{3}{4}\left[x - \frac{2}{3}\right] = 9$

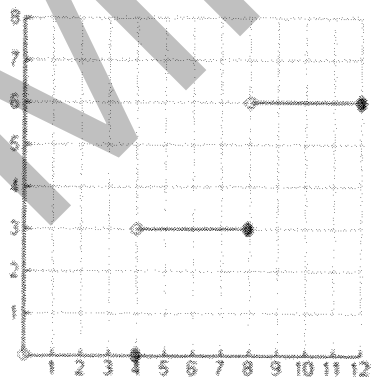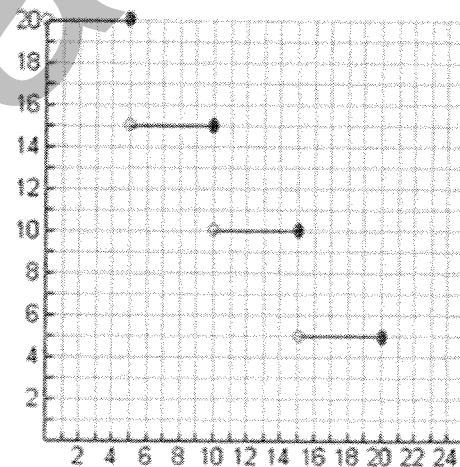10.) $\left[\frac{x}{2} + \frac{3}{4}\right] = \frac{2}{7}$
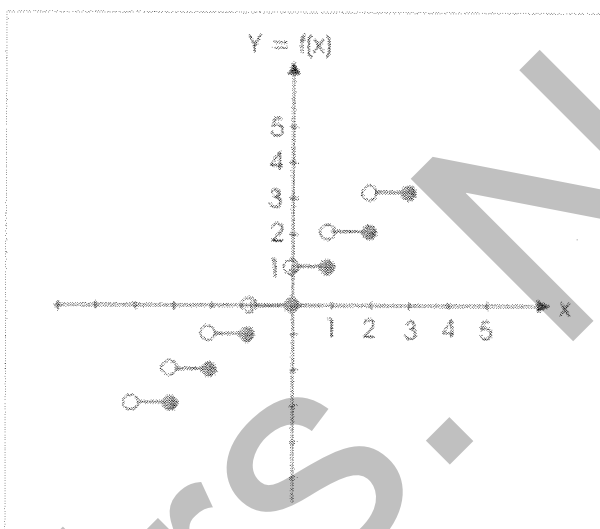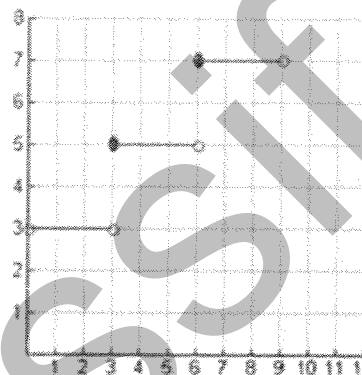
1.) $[5, 6[$

2.) $[7, \quad 7.5[$

3.) $[-42, -41[$

4.) $[-5, -4[$

5.) $[-\frac{1}{2}, 0[$

6.) $[19.5, 21[$

7.) NO SOLN

8.) $[-4, -6[$

9.) $[\frac{38}{3}, \frac{41}{3}[$

10.) NO SOLN

Find the rule for each graph.
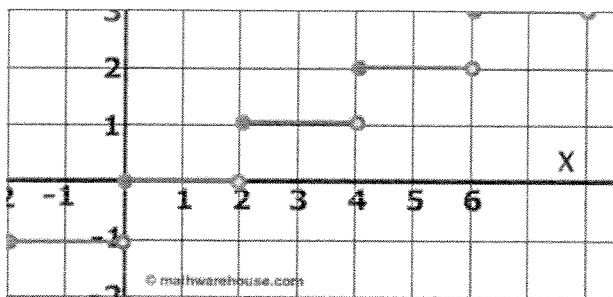
Graph the following functions.

1. $f(x) = \frac{1}{2}\left[\frac{1}{3}(x-2)\right] + 4$

2. $f(x) = -2\left[\frac{1}{4}(x+1)\right] - 1$

3. $f(x) = -[(x+1)] - 2$

4. $f(x) = \frac{5}{4}\left[\frac{2}{3}(x)\right]$

5. $f(x) = [x+1] - 5$

6. $f(x) = -\frac{1}{4}\left[-\left(x - \frac{1}{4}\right)\right] + \frac{1}{4}$

# GREATEST INTEGER FUNCTION

## REAL ESTATE AGENT

Phillip is a real estate agent. He receives a commission every time he sells a house.

Function $f$ described below is used to determine his commission for the sale of a house.

$$f(x) = 1000 \left[ \frac{x}{25\,000} \right] + 1000$$

where  $x$ :  sale price of the house, in dollars
$f(x)$ :  commission received, in dollars

Phillip sold a house. He received a commission of $6 000 for its sale.

If the sale price of the house had been $500 more, Phillip's commission would have been $7 000.

What are the possible sale prices of this house?

## THE HEDGES

A gardener is hired to trim some cedar hedges. To calculate how much it will cost to trim a hedge, he uses function $f$ described below.

$$f(x) = -15 \left[ -x \right] + 25$$

where  $x$ : length of the hedge, in metres
$f(x)$ : cost of trimming the hedge, in dollars

Philip and Sebastian are two of the gardener's customers. Philip's hedge is 2.6 m longer than Sebastian's hedge.

Philip stated the following:

> The cost of trimming my hedge will be $45 more than the cost of trimming Sebastian's hedge because my hedge is 2.6 m longer than Sebastian's.

Is Philip's statement true or false?
Explain why.

## TWO NOVELS FOR LISA

A bookstore offers its customers a loyalty card. With each purchase, the bookstore stamps the card with one or more stars. The value of the purchase, before taxes, determines the number of stars stamped on the card. When the card is full, the customer receives a gift certificate.

Function $f$ described below is used to determine the number of stars stamped on the card according to the value of the purchase, before taxes.

$$f(x) = -\lfloor -0.04\, x \rfloor$$

where
$x$: value of the purchase, before taxes, in dollars
$f(x)$: number of stars stamped on the card
$\text{dom } f = \,]0, +\infty[$

Lisa stated the following:

*If I buy two novels at different prices, my card will be stamped with the same number of stars, regardless of whether I buy them separately or together.*

Is Lisa's statement true or false?

Explain why.

## REWARDS

John is Sophia's and Mark's father. To reward them for doing their chores, John gives them tokens that they can trade in for money at the end of the month.

To determine the value of the reward that Sophia and Mark earn when they trade in their tokens, John uses function $f$ described below:

$$f(x) = 4 \left\lfloor \frac{1}{10}(x-1) \right\rfloor + k$$

where
$x$: the number of tokens traded in
$f(x)$: the value of the reward, in dollars

Last month:

- Sophia had 53 tokens that she traded in for a reward of $24
- Mark traded in his tokens for a reward of $32
- if Sophia and Mark had combined their tokens, they would have traded them in for a reward of $52

What are the possible numbers of tokens that Mark earned last month?

## A MODIFIED PROGRAM

A store offers its customers a loyalty reward program. For each purchase they make, customers earn points that they can then exchange for different products.

Points are earned based on the value of the purchase. Function $f$ described below can be used to determine the number of points earned.

$$f(x) = a\left[\frac{1}{20}x\right]$$

where    $x$:  value of the purchase, in dollars
         $f(x)$:  number of points earned

In February, Richard made a purchase of $275. He earned 26 points.

In March, he made a purchase of $131.

The store decided to modify its reward program as follows:

    "Earn 3 points for every $15 spent."

This change came into effect starting in April.

In May, Richard made a purchase under the new program and earned the same number of points that he earned in March under the old program.

In dollars, what are the possible values of the purchase Richard made in May?

## BAGGAGE FEES

Clara, David and Mia are taking a plane together to go on vacation. They have to pay baggage fees.

To determine these fees, the airline uses the function $f$ described below:

$$f(x) = a\left[0.5(x-1)\right] + k$$

where    $x$:  mass of the checked baggage, in kilograms
         $f(x)$:  baggage fees, in dollars

Information regarding each person's baggage is presented below:

* The mass of Clara's checked baggage is 2.8 kg, and she pays $5 in baggage fees.

* The mass of David's checked baggage is 14.3 kg, and he pays $19.40 in baggage fees.

* Mia pays $14.60 in baggage fees.

If only one person had checked in all of Clara's, David's and Mia's baggage at the same time, what would have been the minimum baggage fees paid?
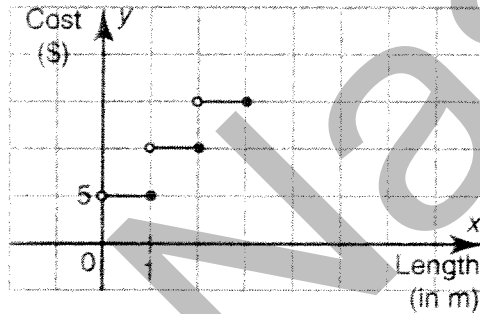
## TWO LOTS TO BE FENCED IN

A farmer wants to fence in two lots, one rectangular and the other in the shape of a right trapezoid.

The two lots are represented below and have the same area (in m²).



$(a + 10)$

$(5a + 10)$

$(a + 10)$

$(2a + 4)$

$(6a + 20)$
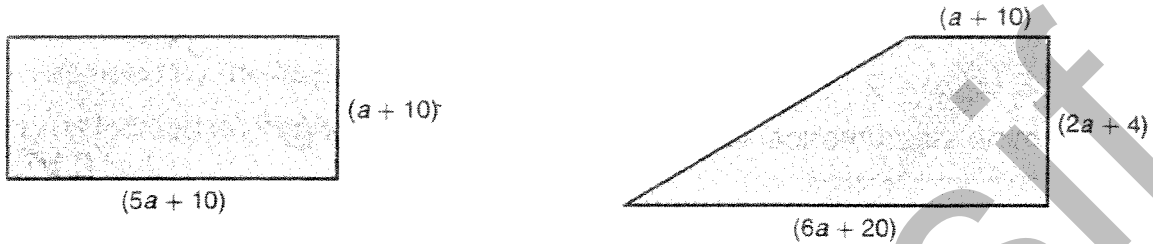
The greatest integer function represented below gives the cost (in $) of the fence as a function of the length of the fence (in m).



What is the total cost of fencing in both lots?

## A LANDSCAPING ARRANGEMENT

A landscaping architect wants to create two equivalent flowerbeds, one rectangular and one square, as represented in the following figure.

$$c \quad \boxed{\phantom{xxx}} \qquad \boxed{A = 2c^2 - 70c + 600}\,(c - 20)$$
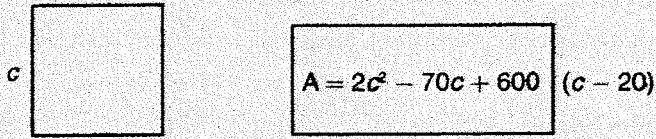
The square flowerbed, to be planted with shrubs, has a side length of $c$ metres.

The rectangular flowerbed, to be planted with flowers, has an area represented by the polynomial $2c^2 - 70c + 600$ and a width represented by the binomial $(c - 20)$.

The architect wants to surround both flowerbeds with a fence.

The graph below gives the cost of the fence according to the length, in metres, of the fence.
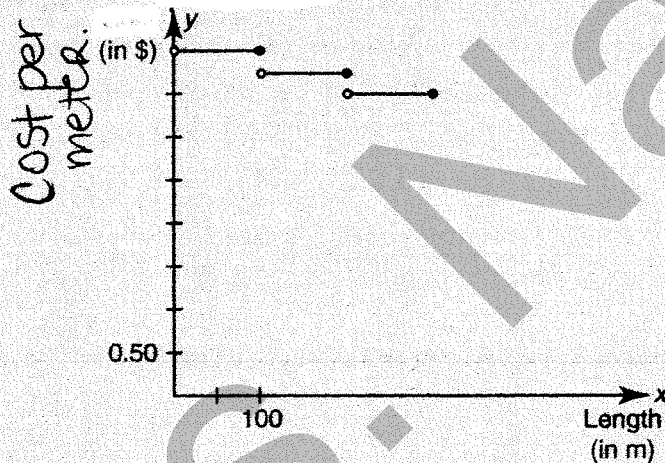


What is the total cost of the fence surrounding the flowerbed of flowers?

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**GREATEST INTEGER FUNCTION**

**QUIZ**

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**1**

The function $f$ is defined by the following rule:

$$f(x) = 3\left[-\frac{(x-1)}{2}\right] + 6$$

What are the zeros of this function?
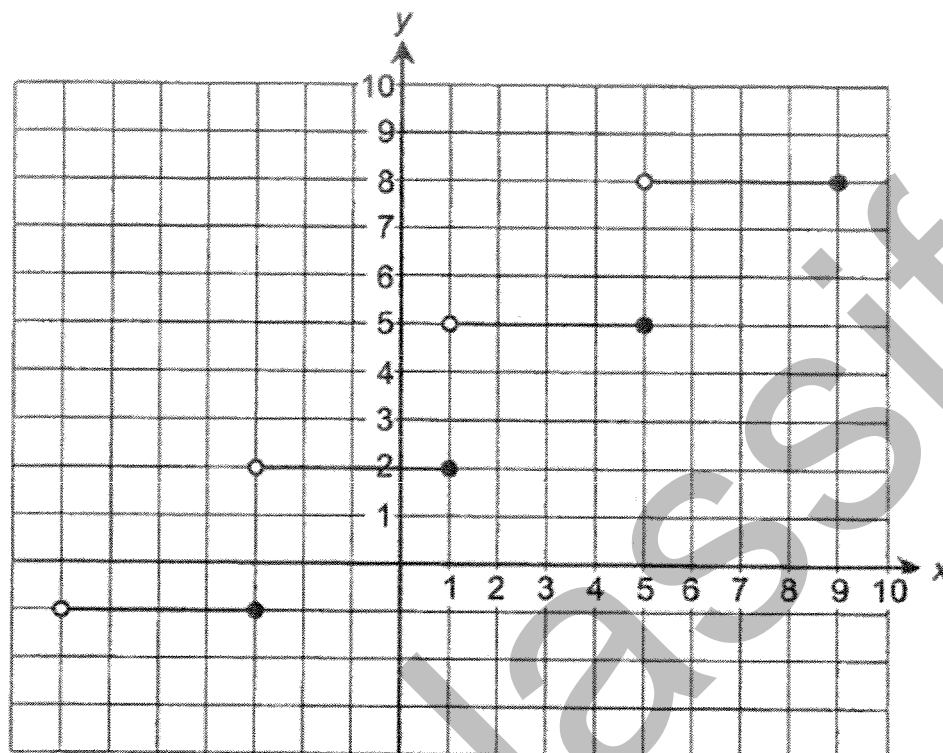
**2**  **The graph of a step function is shown below.**
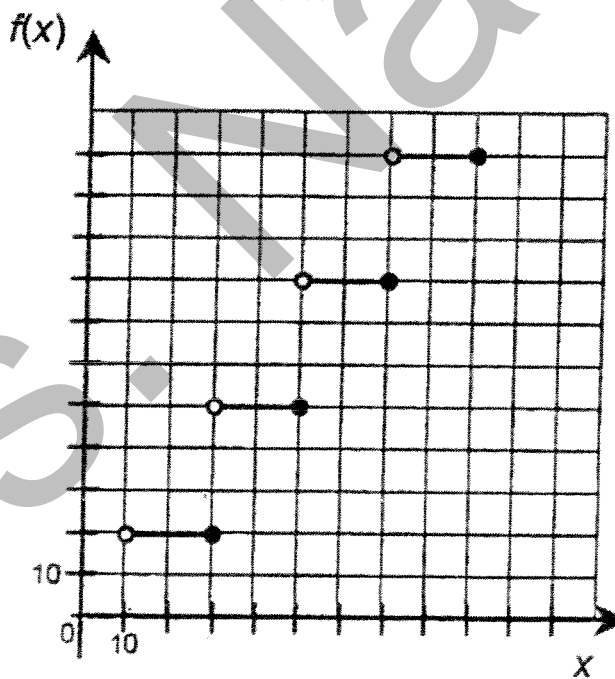


For what values of x is     $f(x) \geq 0$?

**3**

**What is the rule of correspondence of the function graphed below?**



A) $f(x) = -3\left[\dfrac{-1(x-1)}{4}\right] + 2$

C) $f(x) = 3[4(x-1)] + 2$

B) $f(x) = 3\left[\dfrac{-1(x-1)}{4}\right] + 2$

D) $f(x) = -3\left[\dfrac{-1(x-1)}{4}\right] + 5$

**4** A greatest integer function is defined by:

$$f(x) = 2[3x - 5] + 1$$

What is the range of this function?
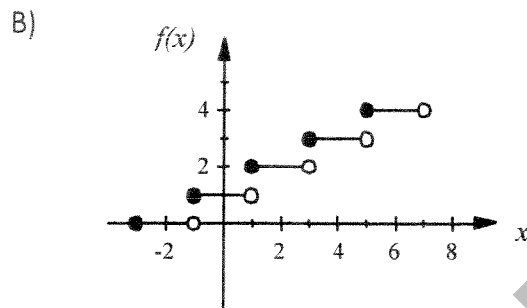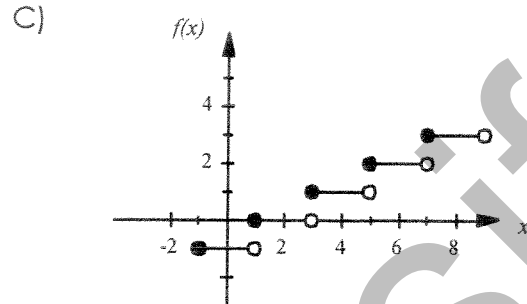
A) $\{y \in \Re \mid y = 2n, n \in Z\}$

C) $\{y \in \Re \mid y = n + 1, n \in Z\}$

B) $\{y \in \Re \mid y = 2n + 1, n \in Z\}$

D) $\{y \in \Re \mid y = 3n - 5, n \in Z\}$

**5** Which of the graphs below represents the function

$$f(x) = \left\lceil \frac{x-1}{2} \right\rceil + 2 \,?$$

A)



C)



B)



D)
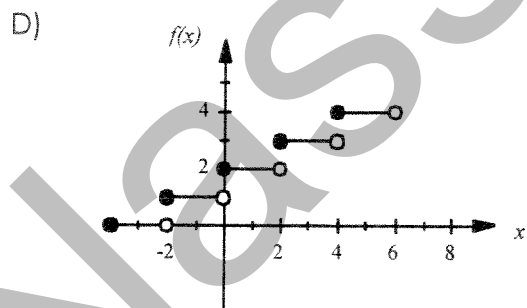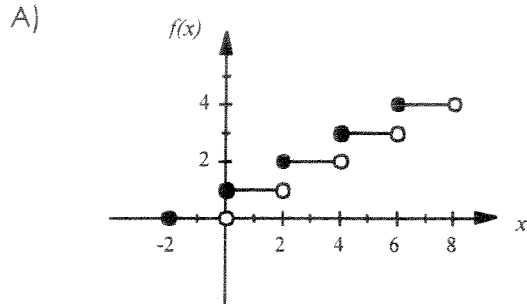


**6** The weekly salary s(n) of a car salesperson is established by the equation

$$s(n) = 200 \left\lceil \frac{1}{2}(n+3) \right\rceil + 200$$

where n is the number of cars sold in a week. For what amount of cars will the salesperson make 1200$ in salary?
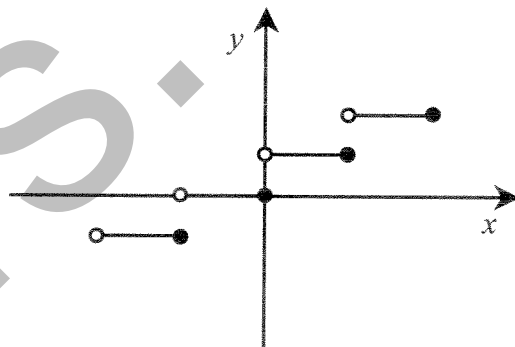
**7** The cost C, in dollars, to send a parcel is given by the function C(x) = [2.75x] + 1.25 where x is the mass in kg.

How much will it cost Danielle to send a parcel that weighs 4.4 kg?

**8**    Which of graphs below represents the equation $f(x) = 2[-2x] + 2$?

A)



C)



B)



D)
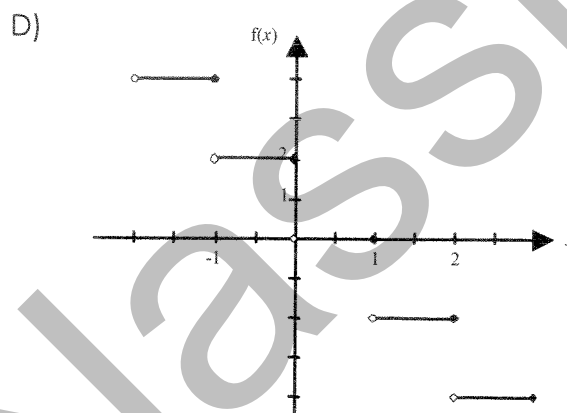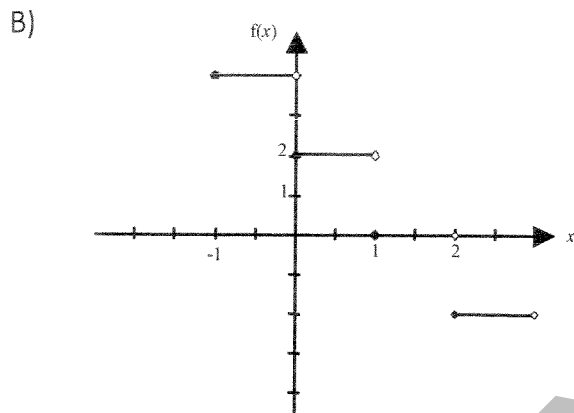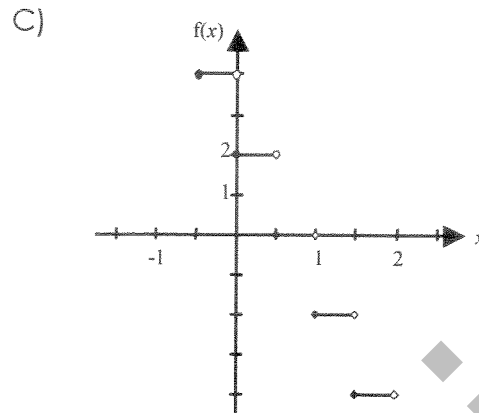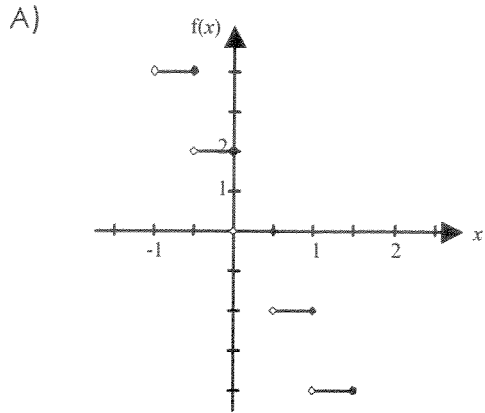


**9**    The rule of function $f$ represented in the Cartesian plane below is $f(x) = r[sx]$.



Which of the following statements is true?
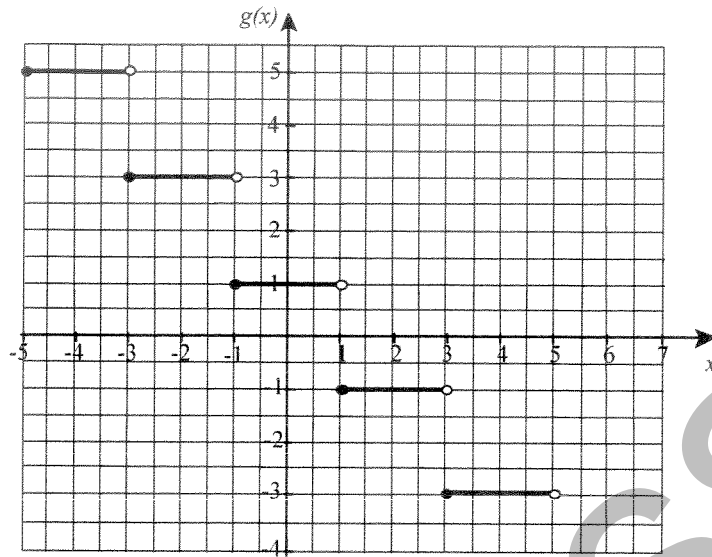
A)    $r > 0$ and $s > 0$

C)    $r < 0$ and $s > 0$

B)    $r > 0$ and $s < 0$

D)    $r < 0$ and $s < 0$

**10** The graph of the function $g(x) = a[bx - 1] + k$ is represented below.



Which of the following are possible values for the parameters a and b of the function $g(x)$?

A)  $a < 0$

   $0 < b < 1$

B)  $a < 0$

   $b > 1$

C)  $a > 0$

   $0 < b < 1$

D)  $a > 0$

   $b > 1$

**11** The basic greatest integer function, $f(x) = [x]$, has been transformed into $g(x) = a[b(x - h)] + k$.

Which of the following statements concerning the role of the parameters is false?

A)  Parameter $a$ affects the distance between the steps.

B)  Parameter $b$ affects the length of the steps.

C)  Parameter $h$ and $k$ represent a horizontal and a vertical translation respectively.

D)  If parameter $a < 0$, and parameter $b < 0$, then function $g(x)$ decreases over $\Re$.