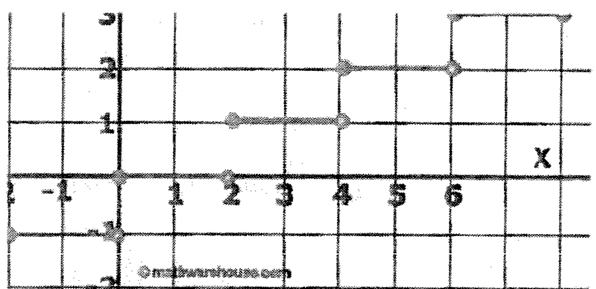
Solution Key

Greatest Integer Function Practice — Math 466

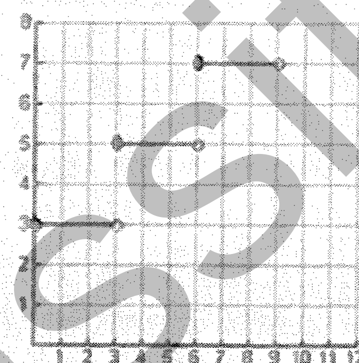
Find the rule for each graph.

$(h,K)=(0,0)$
$b=1/2 \quad a=1$
$b>0$, $ab>0$ } $a>0$



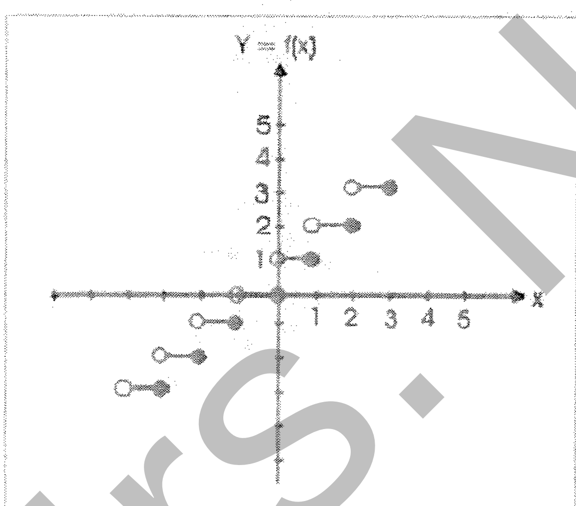
$f(x)=1\left[\frac{1}{2}(x)\right]$

$(h,K)=(0,3)$
$b=1/3$
$a=2$
$b>0$
$ab>0$
$a>0$



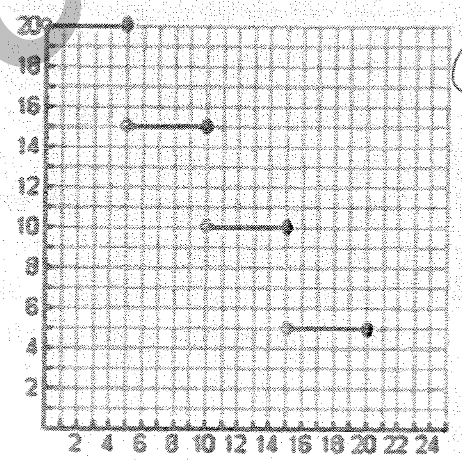
$f(x)=2\left[\frac{1}{3}(x)\right]+3$

$(h,K)=(1,1)$
$b=1 \quad a=1$
$b<0$, $ab>0$ } $a<0$



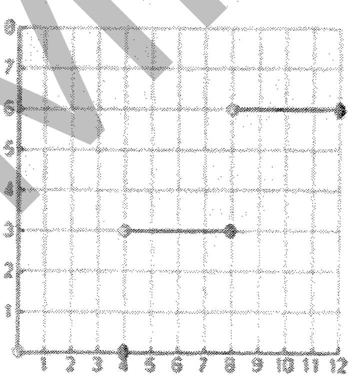
$f(x)=1[-1(x-1)]+1$

$(h,K)=(5,20)$
$b=1/5$
$a=5$
$b<0$
$ab<0$
$a>0$



$f(x)=5\left[-\frac{1}{5}(x-5)\right]+20$

$(h,K)=(4,0)$
$b=1/4 \quad a=3$
$b<0$, $ab>0$ } $a<0$



$f(x)=-3\left[-\frac{1}{4}(x-4)\right]$

Solution Key

GREATEST INTEGER FUNCTION PRACTICE

MATH 466

Graph the following functions.

1. $f(x) = \frac{1}{2}\left[\frac{1}{3}(x-2)\right] + 4$

L=3 H=1/2 ●—○ ↑

2. $f(x) = -2\left[\frac{1}{4}(x+1)\right] - 1$

L=4 H=2 ●—○ ↓

3. $f(x) = -[(x+1)] - 2$

L=1 H=1 ↓ ●—○

4. $f(x) = \frac{5}{4}\left[\frac{2}{3}(x)\right]$

L=1.5 H=1.25 ●—○ ↑

5. $f(x) = [x+1] - 5$

L=1 H=1 ↑ ●—○

6. $f(x) = -\frac{1}{4}\left[-\left(x - \frac{1}{4}\right)\right] + \frac{1}{4}$

L=1 H=1/4 ↑ ○—●

7. $f(x) = -10[10x + 100] - 20$

Difficult to draw

$f(x) = -10[10(x+10)] - 20$

L=1/10 H=10 ●—○ ↓

8. $f(x) = 2\left[-\frac{1}{5}x + 20\right] - 3$

Difficult to draw.

$f(x) = 2[-1/5(x-100)] - 3$

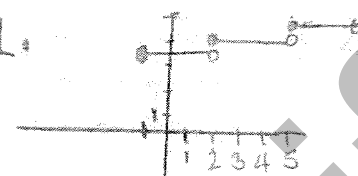
L=5 H=2 ○—● ↓

9. $f(x) = -1\left[-\frac{1}{3}x - 9\right]$

Difficult to draw.

$f(x) = -1[-1/3(x+27)]$

L=3 H=1 ○—● ↑

1.


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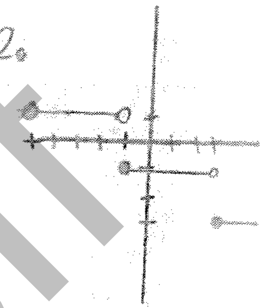

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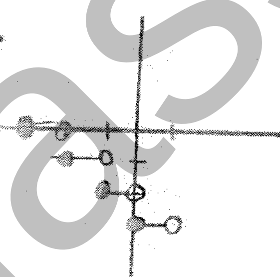

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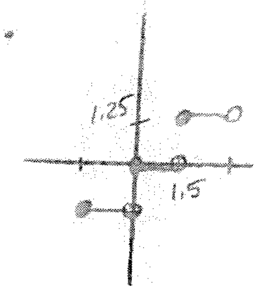

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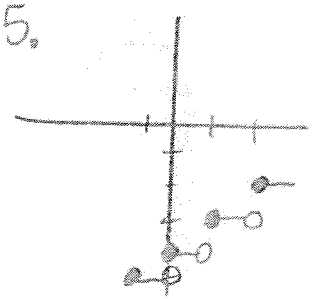

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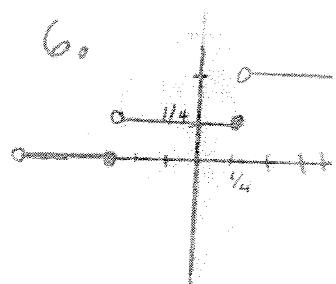

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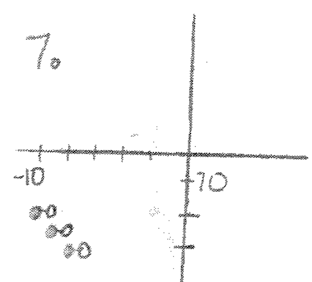

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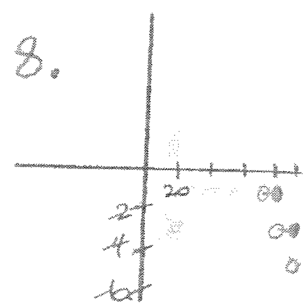

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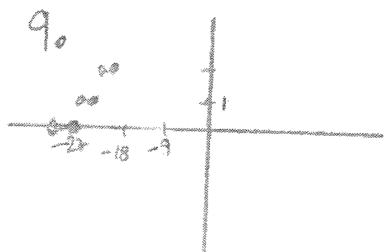


GREATEST INTEGER FUNCTION

REAL ESTATE AGENT

Phillip is a real estate agent. He receives a commission every time he sells a house.

Function f described below is used to determine his commission for the sale of a house.

$$f(x) = 1000\left[\frac{x}{25\,000}\right] + 1000$$

where x: sale price of the house, in dollars

$f(x)$: commission received, in dollars

Phillip sold a house. He received a commission of \$6 000 for its sale.

If the sale price of the house had been \$500 more, Phillip's commission would have been \$7 000.

What are the possible sale prices of this house?

$$6000 = 1000\left[\frac{x}{25000}\right] + 1000$$

$$5 = \left[\frac{x}{25000}\right]$$

$$5 \leq \frac{x}{25000} < 6$$

$$125000 \leq x < 150000$$

$$[125000, 150000[$$

$$7000 = 1000\left[\frac{x}{25000}\right] + 1000$$

$$6 = \left[\frac{x}{25000}\right]$$

$$[150000, 175000[$$

In order for Phillip to land in the second interval, the sale price of the house must have been in the interval [149500, 150000[. By adding 500\$ more to any value, you would

THE HEDGES

A gardener is hired to trim some cedar hedges. To calculate how much it will cost to trim a hedge, he uses function f described below.

$f(x) = -15[-x] + 25$

where x : length of the hedge, in metres
$f(x)$: cost of trimming the hedge, in dollars

Philip and Sebastian are two of the gardener's customers. Philip's hedge is 2.6 m longer than Sebastian's hedge.

Philip stated the following:

The cost of trimming my hedge will be $45 more than the cost of trimming Sebastian's hedge because my hedge is 2.6 m longer than Sebastian's.

Is Philip's statement true or false?
Explain why.

→ Come up with a counterexample for which ~~its~~ proves he's wrong.
→ Using an example to prove he's right, is insufficient to prove he is right. you would have to use all the possibilities
→ Do not erase any of your guesses! Marks are given.

Length Sebast	Length Philip	Cost Seb	Cost philip	Difference in Cost
20.1 m	22.7 m	340$	370$	30$.

This example contradicts Philip's statement. FALSE

Two novels for Lisa

A bookstore offers its customers a loyalty card. With each purchase, the bookstore stamps the card with one or more stars. The value of the purchase, before taxes, determines the number of stars stamped on the card. When the card is full, the customer receives a gift certificate.

Function f described below is used to determine the number of stars stamped on the card according to the value of the purchase, before taxes.

$f(x) = -[-0.04x]$

where x: value of the purchase, before taxes, in dollars

$f(x)$: number of stars stamped on the card

$\text{dom } f =]0, +\infty[$

Lisa stated the following:

If I buy two novels at different prices, my card will be stamped with the same number of stars, regardless of whether I buy them separately or together.

Is Lisa's statement true or false?

Explain why.

Price of 1st Novel	Price of 2nd Novel	Total Stars for Books Separately Bought	Total Stars for Books Together
17$	28$	1+2=3 Stars	f(45)=2 Stars

This example contradicts Lisa's Statement. Buying the books together gives her 2 stars while buying them Separately would give her 3! It's not the Same!

REWARDS

John is Sophia's and Mark's father. To reward them for doing their chores, John gives them tokens that they can trade in for money at the end of the month.

To determine the value of the reward that Sophia and Mark earn when they trade in their tokens, John uses function f described below:

$$f(x) = 4\left[\frac{1}{10}(x-1)\right] + k$$

where $\quad x$: the number of tokens traded in

$f(x)$: the value of the reward, in dollars

Last month:

- Sophia had $\boxed{\text{53 tokens}}$ that she traded in for a reward of \$24 → used to find K.
- Mark traded in his tokens for a reward of \$32
- if Sophia and Mark had combined their tokens, they would have traded them in for a reward of \$52

What are the possible numbers of tokens that Mark earned last month?

step 1

(53, 24)

$24 = 4\left[\frac{1}{10}(\overset{53}{x}-1)\right] + K$

$24 = 4\left[\frac{1}{10}(52)\right] + K$

$24 = 4[5.2] + K$

$24 = 20 + K$

$K = 4$

step 2 Mark: Find # of tokens

$32 = 4\left[\frac{1}{10}(x-1)\right] + 4$

$32 - 4 = 4\left[\frac{1}{10}(x-1)\right]$

$\frac{28}{4} = \left[\frac{1}{10}(x-1)\right]$

$7 \leq \frac{1}{10}(x-1) < 8$

$70 \leq x-1 < 80$

$71 \leq x < 81$

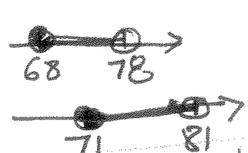
$[71, 81[$

Step 3 How many tokens necessary for 52\$ Reward?

$52 = 4\left[\frac{1}{10}(x-1)\right] + 4$

$x \in [121, 131[$

Subtract Sofia's tokens (53) → $[68, 78[$



common interval $[71, 78[$

A MODIFIED PROGRAM

A store offers its customers a loyalty reward program. For each purchase they make, customers earn points that they can then exchange for different products.

Points are earned based on the value of the purchase. Function f described below can be used to determine the number of points earned.

$$f(x) = a\left[\frac{1}{20}x\right]$$

where x: value of the purchase, in dollars

$f(x)$: number of points earned

$f(275)=26$

In February, Richard made a purchase of $275. He earned 26 points. Use to find Rule

In March, he made a purchase of $131. How many points?

The store decided to modify its reward program as follows:

"Earn 3 points for every $15 spent."

This change came into effect starting in April.

In May, Richard made a purchase under the new program and earned the same number of points that he earned in March under the old program.

In dollars, what are the possible values of the purchase Richard made in May?

step1

$$26 = a\left[\frac{1}{20}(275)\right]$$

$$26 = [13.75]a$$

$$26 = 13a$$

$$a = 2$$

$$f(x) = 2\left[\frac{1}{20}x\right]$$

step2

$$f(131) = 2\left[\frac{1}{20}(131)\right]$$

$f(131) = 12$ points

step3

$$g(x) = 3\left[\frac{x}{15}\right]$$

$$12 = 3\left[\frac{x}{15}\right]$$

$$4 \le \frac{x}{15} < 5$$

$$60 \le x < 75$$

$[60, 75[$

The possible values of his purchase are $[60, 75[$.

BAGGAGE FEES

Clara, David and Mia are taking a plane together to go on vacation. They have to pay baggage fees.

To determine these fees, the airline uses the function f described below:

$f(x) = a\left[0.5(x-1)\right] + k$

where x: mass of the checked baggage, in kilograms

$f(x)$: baggage fees, in dollars

Information regarding each person's baggage is presented below:

use to find Rule

- The mass of Clara's checked baggage is 2.8 kg, and she pays \$5 in baggage fees. $\rightarrow f(2.8)=5$
- The mass of David's checked baggage is 14.3 kg, and he pays \$19.40 in baggage fees. $\Rightarrow f(14.3)=19.40$
- Mia pays \$14.60 in baggage fees.

If only one person had checked in all of Clara's, David's and Mia's baggage at the same time, what would have been the minimum baggage fees paid?

Step 1

$f(2.8) = a[0.5(\overset{2.8}{x}-1)] + k = 5$

$f(2.8) = a[0.5(1.8)] + k = 5$

$a(0) + k = 5$

$k = 5$

Step 2

$f(14.3) = a[0.5(x-1)] + 5 = 19.40$

$a[0.5(\overset{14.3}{x}-1)] = 14.40$

$a[6.65] = 14.40$

$6a = 14.40$

$a = 2.4$

Step 3 Rule

$f(x) = 2.4[0.5(x-1)] + 5$

Step 4 weight of mia's Baggages

$14.60 = 2.4[0.5(x-1)] + 5$

$x \in [9, 11[$ kg minimum 9 kg

Step 5

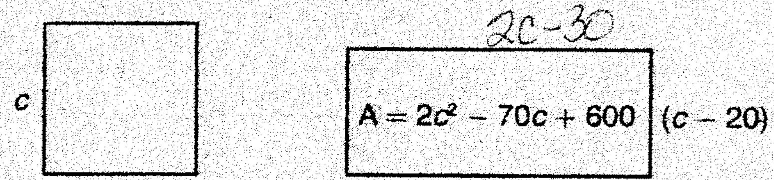
2.8 kg
+14.3 kg
9 kg
26.1 kg

$f(26.1) = 33.80\$$

cost of all Baggages is 33.80\$

A LANDSCAPING ARRANGEMENT

A landscaping architect wants to create two equivalent flowerbeds, one rectangular and one square, as represented in the following figure.

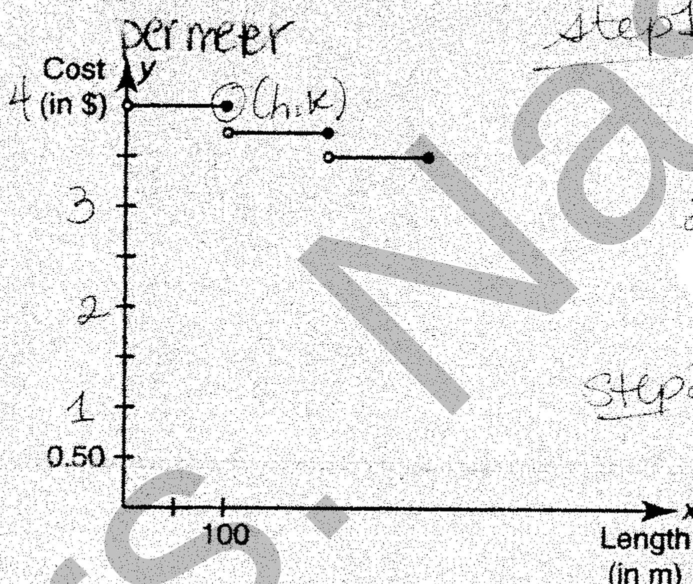


The square flowerbed, to be planted with shrubs, has a side length of *c* metres.

The rectangular flowerbed, to be planted with flowers, has an area represented by the polynomial $2c^2 - 70c + 600$ and a width represented by the binomial $(c - 20)$.

The architect wants to surround both flowerbeds with a fence.

The graph below gives the cost of the fence according to the length, in metres, of the fence.



step1 Find side of Rectangle

$2c^2 - 70c + 600$

$2(c^2 - 35c + 300)$

$(c-20)\underbrace{(2c-30)}_{\text{width}}$

Step2 Find 'c'

$c^2 = 2c^2 - 70c + 600$

$0 = c^2 - 70c + 600$

$0 = (c-10)(c-60)$

$c = 10 \quad c = 60$

What is the total cost of the fence surrounding the flowerbed of flowers?

step3. Perimeter Square $= 60 \times 4 = 240\text{m}$

Perimeter Rect $= 2(90 + 40) = 260\text{ m}$

Total: 500m

step4 $f(x) = a[b(x-h)] + k$

$(h,k) = (100, 4)$

o—• $b < 0$

$a = |0.25| \to a = 0.25$

$b = |100| \to b = -100$

$ab < 0$

$f(x) = 0.25[-\frac{1}{100}(x-100)] + 4$

$f(500) = 0.25[-\frac{1}{100}(500-100)] + 4$

$= 0.25(-4) + 4$

$= -1 + 4$

$= 3\$$

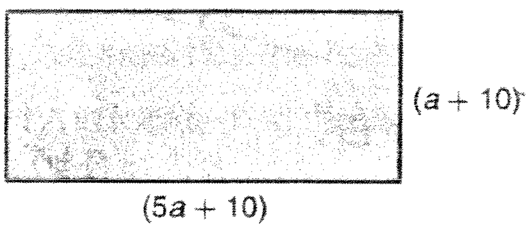
step5

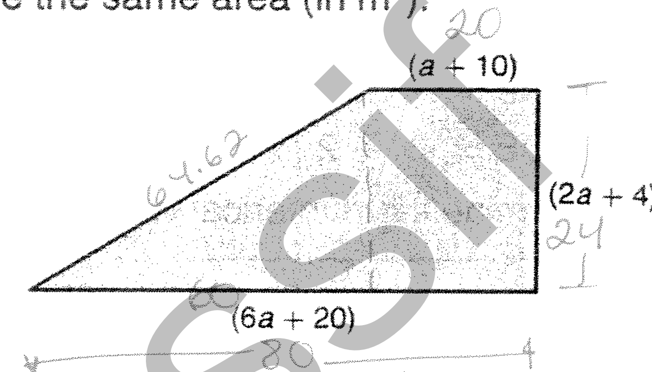
$3\$ \times 500\text{m} = 1500\$$ total

Two lots to be fenced in

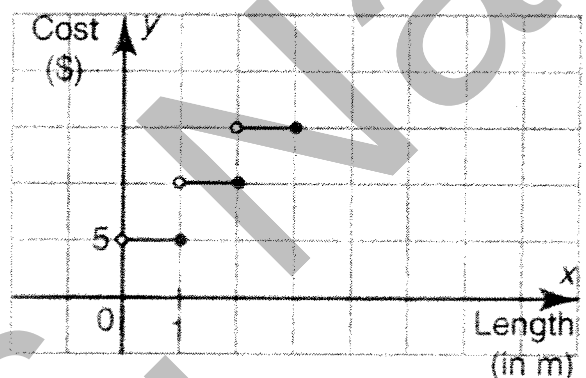
A farmer wants to fence in two lots, one rectangular and the other in the shape of a right trapezoid.

The two lots are represented below and have the same area (in m²).





The greatest integer function represented below gives the cost (in $) of the fence as a function of the length of the fence (in m).



step1

What is the total cost of fencing in both lots?

$A = (5a+10)(a+10)$
$A = 5a^2+60a+100$
Rect

$A_{trap} = \frac{(B+b)h}{2}$

$= \frac{[(6a+20)+(a+10)](2a+4)}{2}$

$= \frac{(7a+30)(2a+4)}{2}$

$= 7a^2+44a+60$

step2 Solve for a

$5a^2+60a+100 = 7a^2+44a+60$

$0 = 2a^2-16a-40$

$0 = a^2-8a-20$

$(a-10)(a+2)=0$

$a=10 \quad a \neq -2$

step3
find hyp.

$60^2+24^2=c^2$
$c=64.62$ m

step4 Total P.
$P_{rect}=160$
$P_{trap}=188.62$
Total $= 348.62$ m

step 4
$f(x) = -5[-(x-1)]+5$
$f(348.62) = -5[-(348.62-1)]+5$
$= 1745\$$

15. REAL ESTATE AGENT

A. EXAMPLE OF APPROPRIATE REASONING

SALE PRICES OF HOUSES FOR WHICH PHILLIP RECEIVES A COMMISSION OF $6 000

$$1000\left[\frac{x}{25\,000}\right]+1000=6\,000$$

$$1000\left[\frac{x}{25\,000}\right]=5\,000$$

$$\left[\frac{x}{25\,000}\right]=5$$

$$\frac{x}{25\,000}\geq 5 \quad \text{and} \quad \frac{x}{25\,000}<6$$

$$x\geq 125\,000 \quad \text{and} \quad x<150\,000$$

Phillip receives a commission of $6 000 if the sale price of a house is at least $125 000, but less than $150 000.

SALE PRICES OF HOUSES FOR WHICH PHILLIP RECEIVES A COMMISSION OF $7 000

In the rule of function f, parameter a is equal to 1000 and parameter b is equal to $\frac{1}{25\,000}$.

The commission increases by $1 000 for each $25 000 increase in the sale price of a house.

Phillip receives a commission of $7 000 if the sale price of a house is at least $150 000, but less than $175 000.

POSSIBLE SALE PRICES OF THE HOUSE

p: sale price of the house, in dollars

$p\geq 125\,000$ and $p<150\,000$ { Phillip receives a commission of $6 000

$p+500\geq 150\,000$ and $p+500<175\,000$
$p\geq 149\,500$ and $p<174\,500$ { Phillip would have received a commission of $7 000 if the sale price had been $500 more.

The sale price of the house is at least $149 500 because $p\geq 125\,000$ and $p\geq 149\,500$.
The sale price of the house is less than $150 000 because $p<150\,000$ and $p<174\,500$.

CONCLUSION

In dollars, the possible sale prices of this house are the values included in the interval $[149\,500,\ 150\,000[$.

or

The possible sale prices of this house are prices greater than or equal to $149 500, but less than $150 000.

6. The hedges

Example of appropriate reasoning

Comparison of the cost of trimming Philip's hedge and Sebastian's hedge according to the length of each hedge

	Length of Sebastian's hedge (m)	Length of Philip's hedge (m)	Cost of trimming Sebastian's hedge ($)	Cost of trimming Philip's hedge ($)	Difference between the cost of trimming Philip's hedge and the cost of trimming Sebastian's hedge
1st possibility	10	$10+2.6=$ 12.6	$f(10)=$ $-15[-10]+25=$ $-15(-10)+25=$ $150+25=$ 175	$f(12.6)=$ $-15[-12.6]+25=$ $-15(-13)+25=$ $195+25=$ 220	\$220 − \$175 = \$45
2nd possibility	10.1	$10.1+2.6=$ 12.7	$f(10.1)=$ $-15[-10.1]+25=$ $-15(-11)+25=$ $165+25=$ 190	$f(12.7)=$ $-15[-12.7]+25=$ $-15(-13)+25=$ $195+25=$ 220	\$220 − \$190 = \$30

In the second possibility, the difference between the cost of trimming Philip's hedge and the cost of trimming Sebastian's hedge is \$30 and not \$45.

Therefore, this is a counterexample.

Conclusion

☐ Philip's statement is true.

☑ Philip's statement is false.

Explanation

There is at least one example that contradicts Philip's statement. (See the second possibility.)

Note: Students can conclude that Philip's statement is false after giving only one example if it is a counterexample.

5. Two novels for Lisa

Example of appropriate reasoning

Comparing the number of stars stamped on the card according to whether you buy the novels separately or together

	Price of the first novel	Price of the second novel	Number of stars stamped on the card when you buy two novels separately	Number of stars stamped on the card when you buy two novels together	Observations
1st possibility	\$18	\$25	$f(18)+f(25)=$ $-[-0.72]+-[-1]=$ $-(-1)+-(-1)=$ $1+1=$ 2	$f(18+25)=$ $f(43)=$ $-[-1.72]=$ $-(-2)=$ 2	The number of stars stamped on the card is the same regardless of whether you buy the novels separately or together.
2nd possibility	\$18	\$26	$f(18)+f(26)=$ $-[-0.72]+-[-1.04]=$ $-(-1)+-(-2)=$ $1+2=$ 3	$f(18+26)=$ $f(44)=$ $-[-1.76]=$ $-(-2)=$ 2	More stars are stamped on the card if you buy the novels separately. This is a counterexample that contradicts Lisa's statement.

Conclusion

☐ Lisa's statement is true.

☑ Lisa's statement is false.

Explanation

There is at least one example that contradicts Lisa's statement. (See 2nd possibility.)

Note: Students can conclude that Lisa's statement is false after giving only one example if it is a counterexample.

. **REWARDS**

EXAMPLE OF APPROPRIATE REASONING

RULE OF FUNCTION f

$f(x) = 4\left[\frac{1}{10}(x-1)\right] + k$

$24 = 4\left[\frac{1}{10}(53-1)\right] + k$ {According to the information on Sophia's reward, $f(53) = 24$.

$24 = 4[5.2] + k$

$24 = 4(5) + k$

$4 = k$

Rule of function f: $f(x) = 4\left[\frac{1}{10}(x-1)\right] + 4$

- **NUMBER OF TOKENS FOR WHICH THE REWARD IS \$32**

Find the values of x for which $f(x) = 32$.

$32 = 4\left[\frac{1}{10}(x-1)\right] + 4$

$28 = 4\left[\frac{1}{10}(x-1)\right]$

$7 = \left[\frac{1}{10}(x-1)\right]$

$7 \le \frac{1}{10}(x-1)$ and $\frac{1}{10}(x-1) < 8$

$70 \le x-1$ and $x-1 < 80$

$71 \le x$ and $x < 81$

Number of tokens for which the reward is \$32: at least 71 but less than 81

➢ **NUMBER OF TOKENS FOR WHICH THE REWARD IS \$52**

Find the values of x for which $f(x) = 52$.

$52 = 4\left[\frac{1}{10}(x-1)\right] + 4$

$48 = 4\left[\frac{1}{10}(x-1)\right]$

$12 = \left[\frac{1}{10}(x-1)\right]$

$12 \le \frac{1}{10}(x-1)$ and $\frac{1}{10}(x-1) < 13$

$120 \le x-1$ and $x-1 < 130$

$121 \le x$ and $x < 131$

Number of tokens for which the reward is \$52: at least 121 but less than 131

➢ **POSSIBLE NUMBERS OF TOKENS THAT MARK EARNED LAST MONTH**

Where m: number of tokens that Mark earned last month

$71 \le m$ and $m < 81$

in addition, $121 \le m + 53$ and $m + 53 < 131$

$68 \le m$ and $m < 78$

Minimum number: $71 \le m$ and $68 \le m$
We can deduce that $71 \le m$.

Maximum number: $m < 81$ and $m < 78$
We can deduce that $m < 78$.

Possible numbers of tokens that Mark earned last month: at least 71 but less than 78

➢ **CONCLUSION**

The possible numbers of tokens that Mark earned last month are the numbers of tokens grea
than or equal to 71 but less than 78.

or

The possible numbers of tokens that Mark earned last month are 71, 72, 73, 74, 75, 76 and

A MODIFIED PROGRAM

EXAMPLE OF APPROPRIATE REASONING

RULE OF FUNCTION f

$f(x) = a\left[\frac{1}{20}x\right]$

$26 = a\left[\frac{1}{20}(275)\right]$ $\begin{cases}\text{According to the information on the purchase made in February,}\\ f(275) = 26.\end{cases}$

$26 = a[13.75]$

$26 = a(13)$

$2 = a$

Rule of f: $f(x) = 2\left[\frac{1}{20}x\right]$

NUMBER OF POINTS RICHARD EARNED IN MARCH

Number of points Richard earned in March: $f(131) = 2\left[\frac{1}{20}(131)\right] = 2[6.55] = 2(6) = 12$

RULE OF THE FUNCTION FOR DETERMINING THE NUMBER OF POINTS EARNED UNDER THE MODIFIED PROGRAM

where x: value of the purchase, in dollars

$g(x)$: number of points earned under the modified program

Form of the rule: $g(x) = a[bx]$

$|b| = \frac{1}{\text{Length of each step}} = \frac{1}{15}$

$|a|$ = Number of points earned for every \$15 spent = 3

The lower limit of the step is included in the interval since points are earned only for every \$15 spent.
Therefore, $b > 0$.
Function g is increasing.
Therefore, a and b have the same sign.

Rule of g: $g(x) = 3\left[\frac{1}{15}x\right]$

POSSIBLE VALUES OF THE PURCHASE RICHARD MADE IN MAY

Find the values of x for which $g(x) = 12$. $\begin{cases}\text{In May, Richard earned the same}\\ \text{number of points as he did in March.}\end{cases}$

$12 = 3\left[\frac{1}{15}x\right]$

$4 = \left[\frac{1}{15}x\right]$

$4 \le \frac{1}{15}x$ and $\frac{1}{15}x < 5$

$60 \le x$ and $x < 75$

or

x	$g(x)$
$[0, 15[$	0
$[15, 30[$	3
$[30, 45[$	6
$[45, 60[$	9
$[60, 75[$	12

Possible values of the purchase Richard made in May: values greater than or equal to \$60, but less than \$75

CONCLUSION

In dollars, the possible values of the purchase Richard made in May are values greater than or equal to \$60, but less than \$75.

Baggage fees

Example of appropriate reasoning

Rule of function f

$f(x) = a[0.5(x-1)] + k$

$5 = a[0.5(2.8-1)] + k$ {According to the information on Clara's baggage, $f(2.8) = 5$.

$5 = a[0.9] + k$

$5 = a(0) + k$

$5 = k$

$f(x) = a[0.5(x-1)] + 5$

$19.4 = a[0.5(14.3-1)] + 5$ {According to the information on David's baggage, $f(14.3) = 19.4$.

$14.4 = a[6.65]$

$14.4 = a(6)$

$2.4 = a$

Rule of f: $f(x) = 2.4[0.5(x-1)] + 5$

Possible masses of Mia's baggage

Find the values of x for which $f(x) = 14.6$.

$14.6 = 2.4[0.5(x-1)] + 5$

$9.6 = 2.4[0.5(x-1)]$

$4 = [0.5(x-1)]$

$4 \le 0.5(x-1)$ and $0.5(x-1) < 5$

$8 \le x-1$ and $x-1 < 10$

$9 \le x$ and $x < 11$

Possible masses of Mia's baggage: at least 9 kg but less than 11 kg

Minimum baggage fees if only one person had checked in all of Clara's, David's and Mia's baggage at the same time

Since the values of parameters a and b are greater than 0, function f is increasing.
Consequently, the minimum baggage fees are related to the minimum mass.
Minimum mass of Mia's baggage: 9 kg
Total minimum mass of all the baggage: $2.8\text{ kg} + 14.3\text{ kg} + 9\text{ kg} = 26.1\text{ kg}$

$f(26.1) = 2.4[0.5(26.1-1)] + 5 = 2.4[12.55] + 5 = 2.4(12) + 5 = 33.80$

Minimum baggage fees if only one person had checked in all of Clara's, David's and Mia's baggage at the same time: \$33.80

Conclusion

If only one person had checked in all of Clara's, David's and Mia's baggage at the same time, the minimum baggage fees paid would have been \$33.80.

TWO LOTS TO BE FENCED IN

Value of a

Area of rectangle = Area of trapezoid

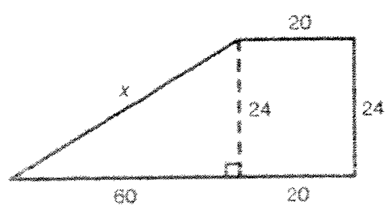
$$(5a + 10)(a + 10) = \frac{(7a + 30) \cdot (2a + 4)}{2}$$
$$5a^2 + 60a + 100 = 7a^2 + 44a + 60$$
$$2a^2 - 16a - 40 = 0$$
$$a^2 - 8a - 20 = 0$$
$$(a - 10)(a + 2) = 0$$
$$a = 10 \text{ or } a = -2$$

Since the length of the rectangle and the height of the trapezoid cannot be zero, we have: $a = 10$.

Perimeter of rectangle

- Length of rectangle = $5(10) + 10 = 60$ m
- Width of rectangle = $10 + 10 = 20$ m
- $P = 2(60 + 20) = 160$ m

Perimeter of trapezoid



We have: $24^2 + 60^2 = x^2$ (Pythagorean theorem)

$x = 64.62$ m

$P = 64.62 + 20 + 24 + 80$
$= 188.62$ m

Total length of the fence

Total length of the fence = $160 + 188.62 = 348.62$ m

Rule of the greatest integer function

$y = a[b(x - h)] + k$ — x: length of fence (in m)

- $(h, k) = (1, 5)$ — y: cost (in \$)
- $ab > 0$, since the function is increasing.
- $b < 0$, since the steps are open on the left and closed on the right.
- $a < 0$ since $b < 0$ and $ab > 0$
- Step length $= \frac{1}{|b|} = 1 \Rightarrow b = -1$
- Step height $= |a| = 5 \Rightarrow a = -5$

The rule of the greatest integer function is: $y = -5\,[-(x - 1)] + 5$

- **Cost of the fence**

$y = -5[-(348.62 - 1)] + 5$ (Total length of the fence is equal to 348.62 m)

$y = -5[-347.62] + 5$

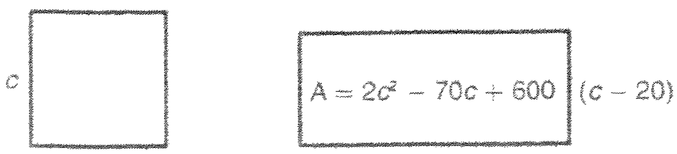
$y = (-5)(-348) + 5$

$y = \$1745$

- **CONCLUSION**

The total cost of fencing both lots is **$1745**.

A LANDSCAPING ARRANGEMENT



► Dimensions of the rectangular flowerbed

$2c^2 - 70c + 600$
$= 2c^2 - 40c - 30c + 600$
$= 2c(c - 20) - 30(c - 20)$
$= (c - 20)(2c - 30)$

The dimensions of the rectangle are $(c - 20)$ and $(2c - 30)$.

► Value of *c*

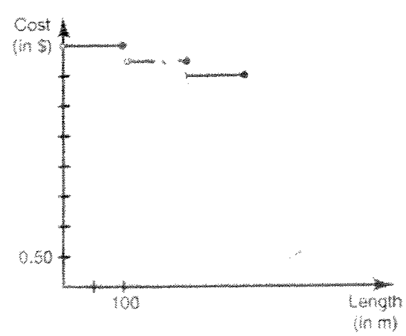
$2c^2 - 70c + 600 = c^2$ (The figures are equivalent and therefore have the same area)
$c^2 - 70c + 600 = 0$
$(c - 10)(c - 60) = 0$
$c - 10 = 0$ or $c - 60 = 0$
$c = 10$ or $c = 60$
$c = 10$ is a solution to be rejected since the dimension $(c - 20)$ cannot be negative.
We get: $c = 60$

► Perimeter of each flowerbed

- Perimeter of the rectangular flowerbed
  $P_1 = 2(2c - 30) + 2(c - 20) = 6c - 100$
  If $c = 60$, $P_1 = 6(60) - 100 = 260$ m
- Perimeter of the square flowerbed
  $P_2 = 4c = 4(60) = 240$ m
- The total length of fencing is 500 m.

► Rule of the greatest integer function

- $(h, k) = (100, 4)$
- $ab < 0$ (The function is decreasing)
- $b < 0$ (The steps are open on the left and closed on the right)
- $a > 0$ ($ab < 0$ and $b < 0$)
- Step length $= 100 = \frac{1}{|b|} \Rightarrow b = \frac{1}{100}$
- Step height $= 0.25 = |a| \Rightarrow a = 0.25$
- The rule of the function is: $y = 0.25\left[-\frac{1}{100}(x - 100)\right] + 4$



► Cost of the fence

If $x = 500$, $y = 0.25\left[-\frac{1}{100}(500 - 100)\right] + 4 = \$3$ per metre

Cost of the fence $= 500 \times 3 = \$1500$.

► CONCLUSION

The total cost of the fence is **$1500**.

Solution key Greatest Integer Function Quiz

1) $0 = 3\left[-\frac{1}{2}(x-1)\right] + 6$

$-\frac{6}{3} = \left[-\frac{1}{2}(x-1)\right]$

$-2 \leq -1/2(x-1) < -1$

$5 \geq x > 3$

$]3,5]$

2) $]10,\infty[$

3) A

4) B

5) $f(x) = \left[\frac{1}{2}(x-1)\right] + 2$

B

L=2 H=1 (1,2)

b>0 ab>0

6) $1200 = 200\left[\frac{1}{2}(n+3)\right] + 200$

$5 \leq \frac{1}{2}(n+3) < 6$

$7 \leq n < 9$

$[7,9[$

7) $C(x) = [2.75(4.4)] + 1.25$

$= 13.25\$$

wrong if 13.35\$

8) A

L=1/2 H=2 b<0 ab<0

9) s<0 since

rs>0 since ↑ so r<0

D

10) b>0 ↓ ab<0 } a<0

A

11) D