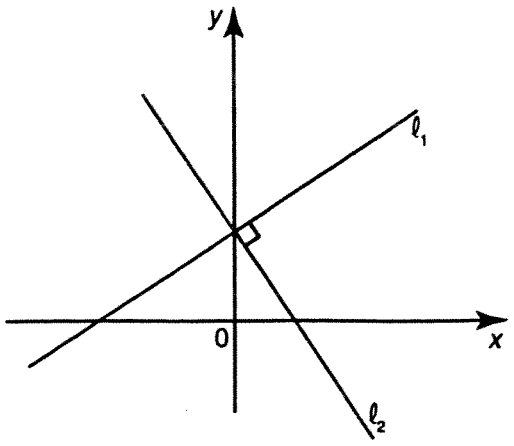
Assignment 466

EXAM 1

SECTION A

1. The two lines ℓ_1 and ℓ_2 are drawn in the following Cartesian plane. Line ℓ_1 is perpendicular to line ℓ_2. The point of intersection of the two lines ℓ_1 and ℓ_2 is on the *y*-axis.



The equation of line ℓ_1 is: $\frac{x}{a}+\frac{y}{b}=1$.

What is the equation of line ℓ_2?

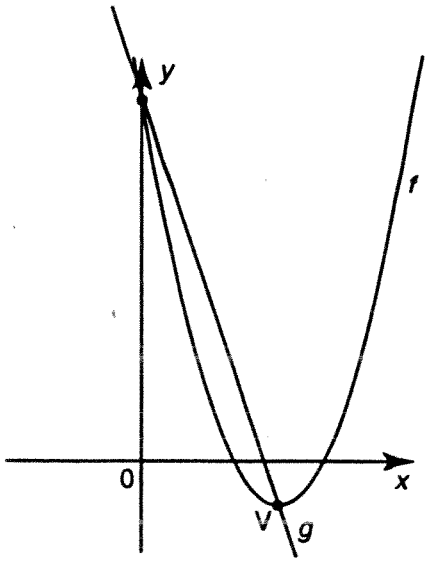
A) $y=-ax$

B) $y=-\frac{a}{b}x+b$

C) $y=\frac{a}{b}x+b$

D) $y=\frac{a}{b}x+a$

2. The quadratic function *f*, and the linear function *g*, are represented in the following Cartesian plane. Functions *f* and *g* have the same initial value. Function *g* passes through the vertex V of the parabola.



If the rule of function *f* is given by $f(x)=x^2-6x+8$, what is the rule of function *g*?

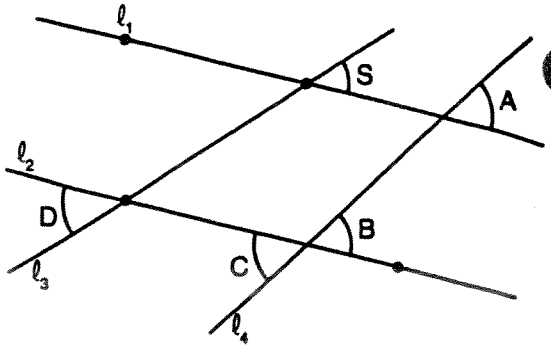
A) $y=3x+8$

B) $y=-3x$

C) $y=-3x+7$

D) $y=-3x+8$

4. In the figure on the right, the lines ℓ_1 and ℓ_2 are parallel.



Which angle is congruent to angle S?

A) Angle A

B) Angle B

C) Angle C

D) Angle D

SECTION B

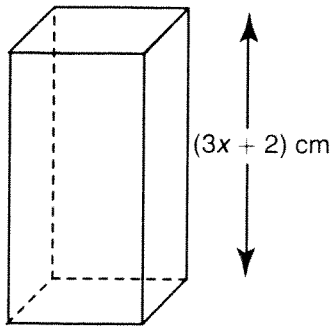
7. Determine the two ordered-pair solutions to the following system:

$$\begin{cases} y = -x^2 + 5x - 6 \\ 2x - y - 4 = 0 \end{cases}$$

The two ordered-pair solutions to the system are: _____

8. The following right rectangular prism has a volume of $(12x^3 + 8x^2 - 3x - 2)$ cm^3 and a height of $(3x + 2)$ cm.

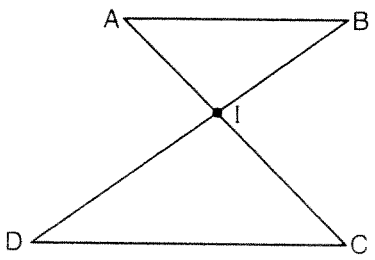
Determine the numerical value of the volume of this prism if the area of the base is equal to 15 cm^2.



The numerical value of the volume of this prism is: _____

9. Segments AB and CD on the right are parallel. The segments AC and BD intersect at point I.

Justify the statements which enable you to prove that triangles AIB and CID are similar.



STATEMENT	JUSTIFICATION
1. $\angle AIB \cong \angle CID$	
2. $\angle IAB \cong \angle ICD$	
3. $\triangle AIB \sim \triangle CID$	Two triangles with two congruent angles are similar.

10. The rule of a quadratic function f is $f(x) = -2x^2 + x + 3$.

What is the range of function f?

The range of function f is: _____

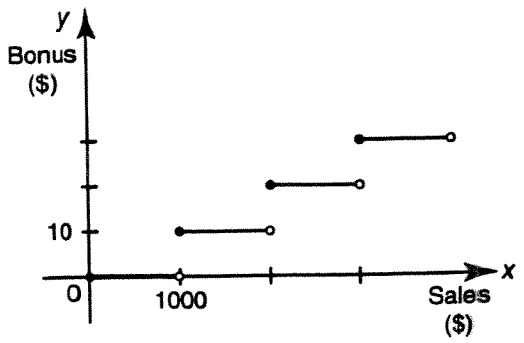
SECTION C

11. THE SALESWOMAN'S BONUS

The salespersons at a clothing store receive, at the end of the week, a bonus according to the amount of sales they made that week.

The graph on the right illustrates the bonus received according to the amount of sales.

Julie, a saleswoman in this store, sold $12 850 in merchandise this week. What will be the amount of Julie's bonus at the end of the week?

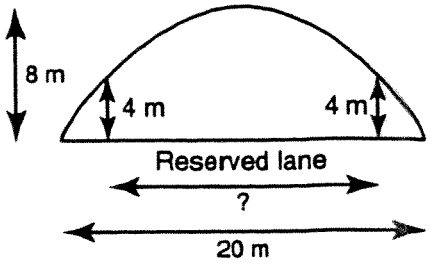


15. THE LANE UNDER A TUNNEL

The cross section of a parabolic shaped tunnel is represented on the right.

The tunnel has a width of 20 m and a maximum height of 8 m. The lane reserved for traffic is indicated at the tunnel's entrance.

What is, to the nearest unit, the width of this lane reserved for traffic?



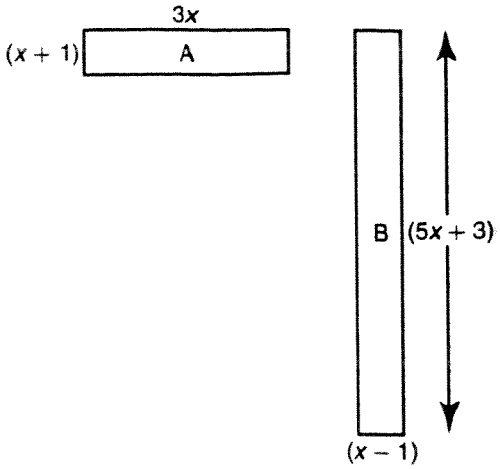
16.

THE RECTANGLE WITH THE SMALLEST PERIMETER

The rectangles A and B on the right are equivalent.

The dimensions, in cm, of rectangle A are: $3x$ and $(x + 1)$, and those of rectangle B are: $(5x + 3)$ and $(x - 1)$.

Of all rectangles equivalent to rectangle A, find the dimensions of the one with the smallest perimeter.



EXAM 2

SECTION A

1. Consider the line ℓ defined by the equation: $\frac{x}{m} + \frac{y}{n} = 1$, $m > 0$, $n > 0$, $m \neq n$.

Which one of the following equations is that of a line with the same slope as line ℓ?

A) $y = \frac{n}{m}x + b$

B) $nx - my + c = 0$

C) $mx + ny = 1$

D) $nx + my + k = 0$

2. Given that the divisor is non-zero, consider the following division:

$$(2x^3 + x^2 - 5x + 2) \div (x + 2)$$

What are the factors of the resulting quotient of this division?

A) $(2x + 1)$ and $(x - 1)$

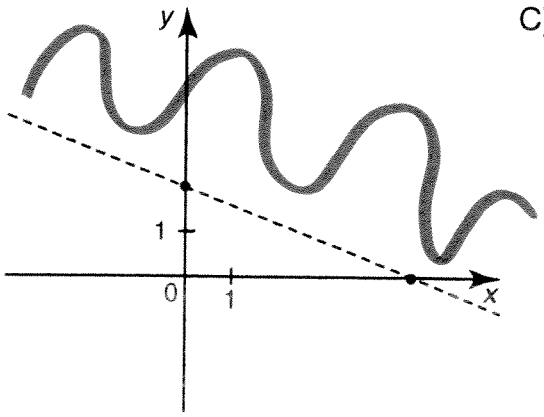
B) $(2x - 1)$ and $(x + 1)$

C) $(2x - 1)$ and $(x - 1)$

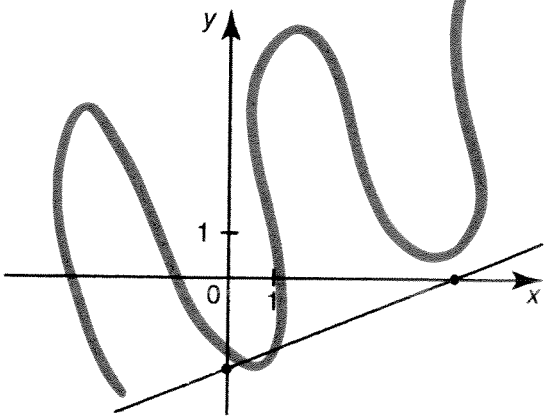
D) $(2x + 1)$ and $(x + 1)$

3. Which shaded region represents the solution set of the inequality: $2x + 5y - 10 < 0$?

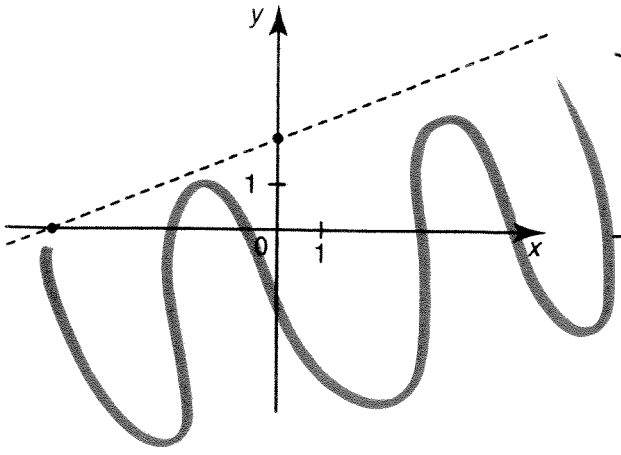
A)



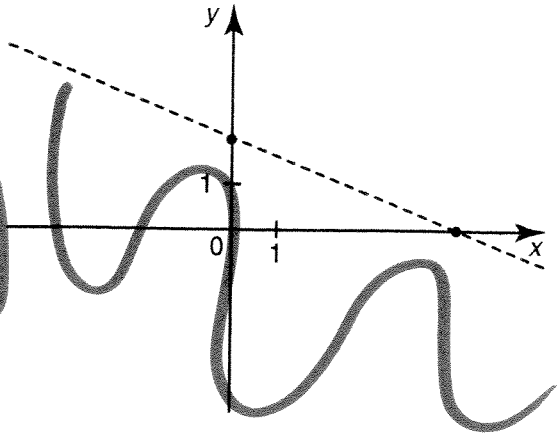
C)



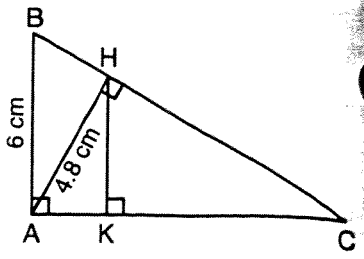
B)



D)



4. In the figure on the right,



- triangle ABC is a right triangle in A.
- $\overline{AH} \perp \overline{BC}$
- $\overline{HK} \perp \overline{AC}$
- m $\overline{AH} = 4.8$ cm
- m $\overline{AB} = 6$ cm

What is, rounded to the nearest tenth, the length of segment HK?

A) 3.8 cm B) 12.2 cm C) 4.8 cm D) 3.6 cm

5. A function f has the following characteristics:

- range $f = [-8, +\infty[$
- the zeros of f are 2 and 6.

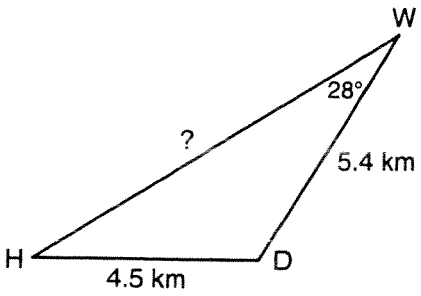
Which one of the following rules corresponds to function f?

A) $y = 2(x + 8)^2 - 6$ C) $y = -2(x - 4)^2 + 8$

B) $y = 2x^2 - 16x + 24$ D) $y = x^2 - 8x + 12$

6. The path travelled by Karen, on a given weekday, to drive her son Raphaël to the daycare and go to work is given by the figure on the right.

What distance, represented by the segment HW and rounded to the nearest kilometer, separates Karen's house and her work?



A) 7 km B) 3 km C) 6 km D) 8 km

SECTION B

7. Consider the following expression, given that the denominators are non-zero:

$$\frac{x^2+x-12}{x^2-9} \times \frac{2x^2+6x}{2x^2+3x-20}$$

What simplified expression corresponds to the given expression?

The simplified expression that corresponds to the given expression is: _____

8. Consider the following greatest integer function:

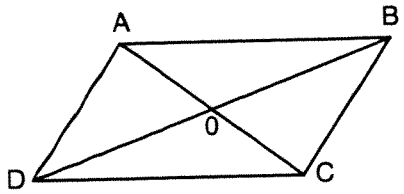
$$f(x) = 2\left[\frac{1}{4}(x-5)\right] + 12$$

Determine the interval corresponding to the set of all the zeros of this function.

The interval corresponding to the set of zeros is: _____

9. Consider the parallelogram ABCD on the right.

Complete the justifications that enable you to prove that triangles A0B and D0C are congruent.



STATEMENT	JUSTIFICATION
1. $\overline{AB} \cong \overline{DC}$	1. The opposite sides of a parallelogram are congruent.
2. $\angle CAB \cong \angle ACD$	2.
3. $\overline{A0} \cong \overline{0C}$	3.
4. $\triangle A0B \cong \triangle D0C$	4. The triangles have a congruent angle between two congruent sides. (SAS congruence case)

10. Determine the two ordered-pair solutions to the following system:

$$\begin{cases} 3x + y = -7 \\ x^2 + 2y + 6x = -13 \end{cases}$$

The two ordered-pair solutions to the system are: _____

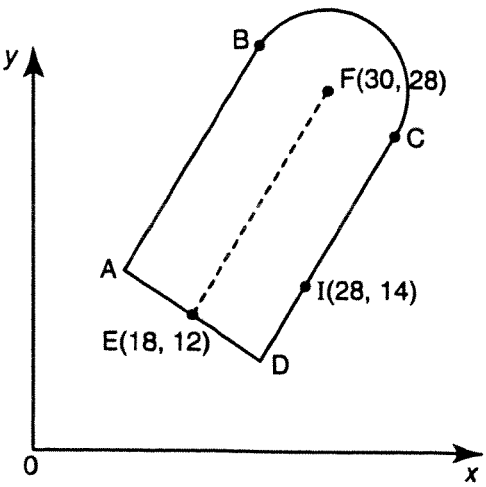
SECTION C

11. THE POOL

We graph on the right represents the aerial view of a pool formed by the rectangle ABCD and the semi-circle with diameter BC. The distances are expressed in metres.

Two posts, represented by points E and F, are placed at the midpoints of segments AD and BC respectively in order to install a net.

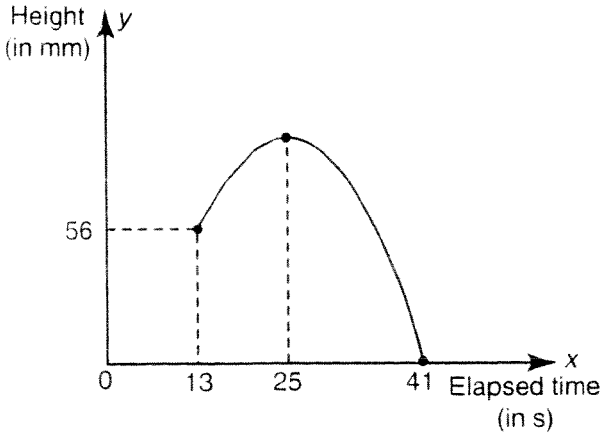
What is, to the nearest square metre, the area of the pool's floor given that it is a flat surface?



12. THE FLASHING POINT

We observe the trajectory of a flashing point on a screen. This trajectory is that of a quadratic function relating the height (in mm) of the flashing point as a function of the elapsed time (in seconds).

After 13 s, the flashing point is at a height of 56 mm. After 25 s, it reaches its maximum height. After 41 s, it is on the x-axis.



The graph, in the Cartesian plane above, represents the trajectory of the flashing point.

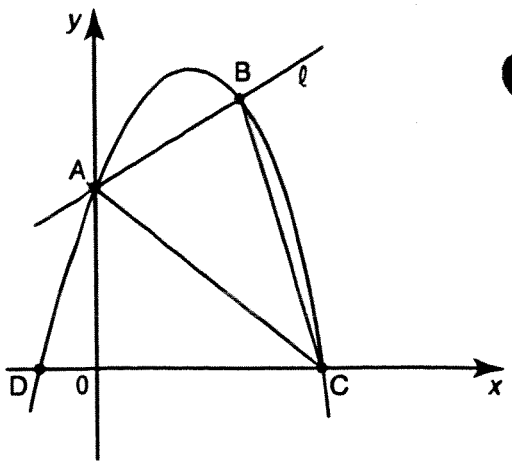
Determine the maximum height reached by the flashing point.

13. AREA OF A TRIANGLE

In the Cartesian plane on the right, the line ℓ with equation: $x - y + 5 = 0$ and the parabola with equation: $y = -x^2 + 4x + 5$ intersect at points A and B.

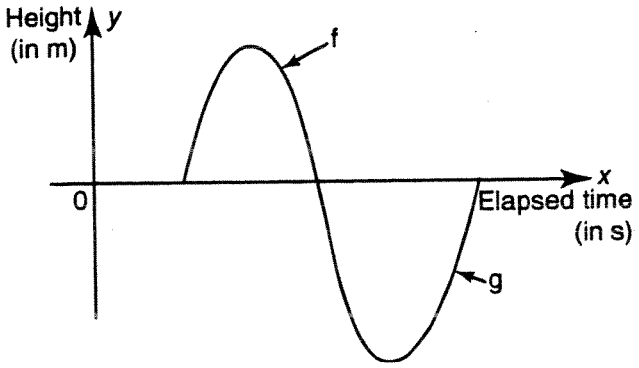
Points C and D are the points where the parabola intersects the x-axis.

Determine the area of triangle ABC.



15. A DOLPHIN

A marine biologist has represented, in the Cartesian plane below, the trajectory of a dolphin. The variable x represents the elapsed time, in seconds, and y represents the dolphin's height, expressed in metres. Two portions of parabolas represent this trajectory: one that is above the surface of the water, given by function f, and one that is below the surface, given by function g.



The marine biologist recorded the following information:

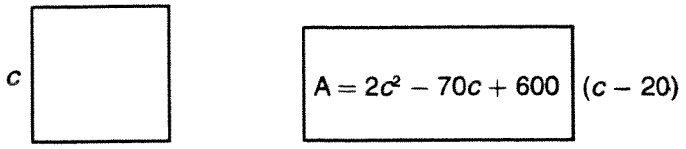
- The rule of function f is: $f(x) = -(x - 6)^2 + 4$.
- 2 seconds after entering the water, the dolphin is at a depth of 6 m.
- 5 seconds after entering the water, the dolphin reaches its maximum depth.

At what depth will the dolphin be 6 seconds after entering the water?

16. A LANDSCAPING ARRANGEMENT

↑ same area

A landscaping architect wants to create two equivalent flowerbeds, one rectangular and one square, as represented in the following figure.

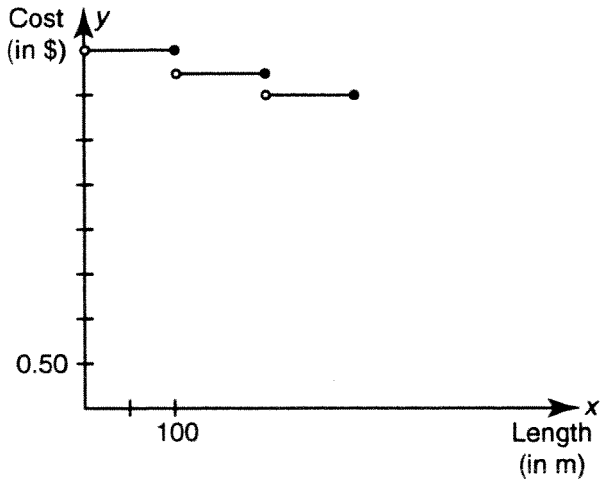


The square flowerbed, to be planted with shrubs, has a side length of c metres.

The rectangular flowerbed, to be planted with flowers, has an area represented by the polynomial $2c^2 - 70c + 600$ and a width represented by the binomial $(c - 20)$.

The architect wants to surround both flowerbeds with a fence.

The graph below gives the cost of the fence according to the length, in metres, of the fence.

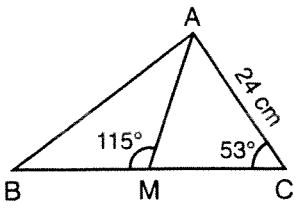


What is the total cost of the fence surrounding the flowerbed of flowers?

EXAM 3

SECTION A

1. The median AM is drawn in the triangle ABC on the right.



We have:

$m\angle AMB = 115°$

$m\angle ACM = 53°$

$m\,\overline{AC} = 24$ cm

What is, to the nearest tenth, the length of the side BC?

A) 53.1 cm

B) 23.4 cm

C) 46.8 cm

D) 26.5 cm

2. The equation of a line ℓ is given by: $\frac{x}{a} + \frac{y}{b} = 1$ where $a \neq 0$ and $b \neq 0$.

Which one of the following equations represents a line perpendicular to ℓ?

A) $y = \frac{b}{a}x$

B) $y = \frac{a}{b}x$

C) $y = \frac{-b}{a}x$

D) $y = \frac{-a}{b}x$

3. Given that the denominator is non-zero, what is the remainder of the following division?

$$(4x^3 - 8x^2 - x + 2) \div (x - 1)$$

A) 0

B) −9

C) −5

D) −3

4. Which one of the following polynomials is equal to the square of a binomial?

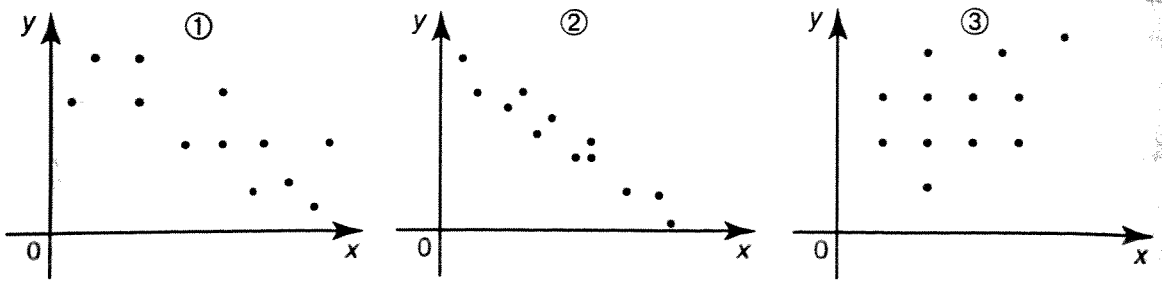
A) $4x^2 - 9$

B) $4x^2 - 6x + 9$

C) $4x^2 - 12x - 9$

D) $4x^2 - 12x + 9$

5. The following scatter plots represent three 2-variable distributions.



Which one of the following sequences represents the ranking of these distributions by the strength of their linear correlations, from weakest to strongest?

A) ③, ①, ②

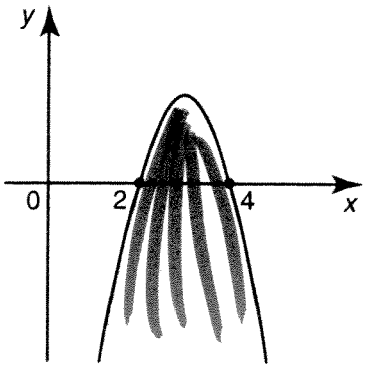
B) ③, ②, ①

C) ②, ①, ③

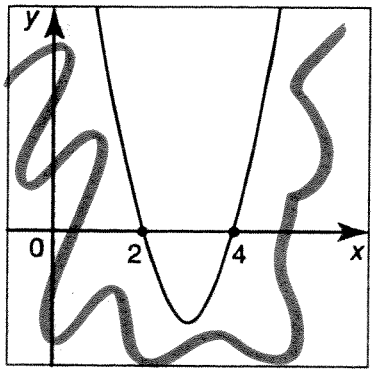
D) ①, ②, ③

6. Which shaded region represents the solution set to the inequality: $y - 2 \geq -2(x - 3)^2$?

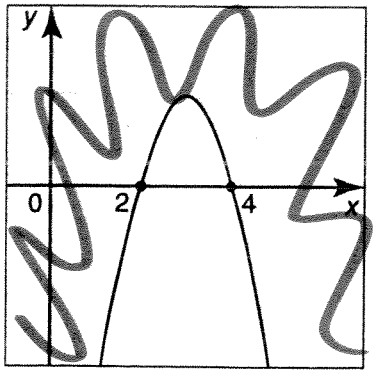
A)



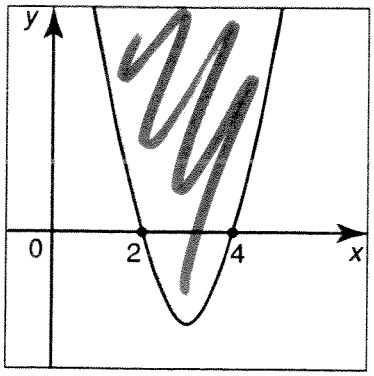
C)



B)

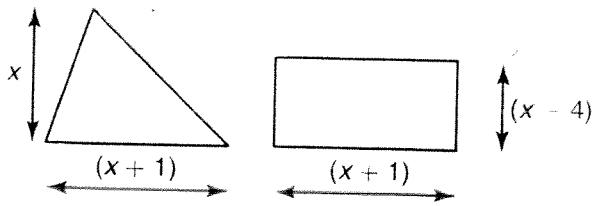


D)



SECTION B

7. The triangle and rectangle on the right are equivalent. What is the numerical value of the rectangle's perimeter?

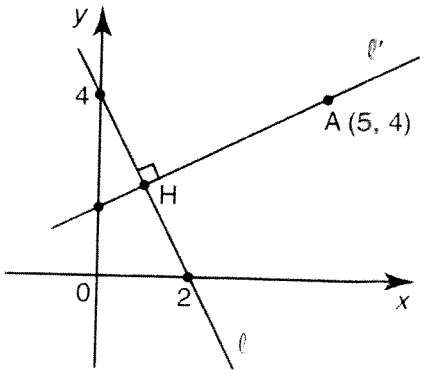


The perimeter of the rectangle is equal to: _____

8. The lines ℓ and ℓ' on the right are perpendicular at point H.

The x-intercept of line ℓ is equal to 2, and its y-intercept is equal to 4. The line ℓ' passes through the point A(5, 4).

What are the coordinates of point H?



The coordinates of point H are: _____

9. The cost of mailing a package depends on its mass.

This cost is calculated using the following greatest integer function.

$$f(x) = -0.5\left\lfloor \frac{-x}{100} \right\rfloor + 2$$

where x represents the mass of the package, in grams, and $f(x)$ represents the mailing cost, in dollars.

In what interval is the mass of a package if the cost of mailing this package is equal to \$12?

The mass of the package is in the interval: _____

10. In the Cartesian plane on the right, functions f and g are represented by parabolas.

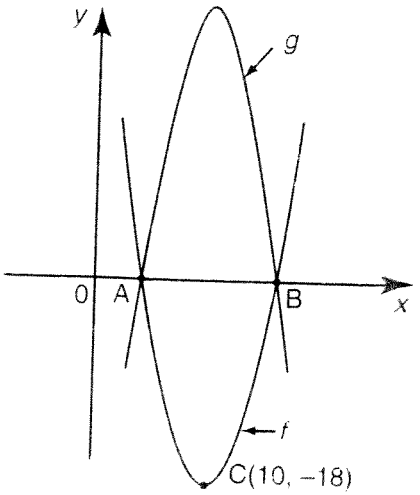
The intersection points A and B of the parabolas are located on the x-axis.

The rule of function g is $g(x) = \frac{-2}{3}(x - 10)^2 + 24$

Point C(10, −18) is the vertex of the parabola representing function f.

What is the initial value of function f?

The initial value of function f equals: _____



SECTION C

11. A MAGIC NUMBER

Raphael thinks he has invented a magic trick.

Nathalie tests this magic trick by picking the number 10 in the first step.

The following table describes the steps of this magic trick.

	STEPS	TEST
1.	Choose a natural number.	10
2.	Add 6 to the number chosen in step 1.	
3.	Multiply the sum obtained in step 2 by the number chosen in step 1.	
4.	Add 5 to the product obtained in step 3.	
5.	Divide the result obtained in step 4 by the sum of 5 and the number chosen in step 1.	
6.	Subtract 1 from the quotient obtained in step 5.	

If Nathalie has made no calculation mistakes, the result obtained in step 6 is the number chosen in step 1.

Show that this magic trick works for any natural number.

13. A DISTANCE BETWEEN TWO POINTS

The line ℓ, given by the equation: $2x - y - 3 = 0$, intersects the parabola given by the equation: $y = -(x - 2)^2 + 4$ at points A and B.

What distance, to the nearest tenth, separates the parabola's vertex V from the midpoint M of segment AB?