

JUNE EXAM TOPICS CHECKLIST

Secondary 3

Regular

C. NASSIF

Real Numbers

- Pythagorean Theorem
- Types of Numbers
- Exponents
- Scientific Notation
- Convert to fraction

Algebra

- Monomials
- Polynomials
- Adding like terms
- Distributive Property
- FOIL
- Dividing
- Area of Polygons
- Removing common factor
- Finding numerical value

Inequality

- Intervals
- Graphic representation

Linear Equations

- Mathematical Phrases
- Solving x in an equation
- Solving a word problem
- Finding the slope
- Finding the equation of a line
- Graphing an equation
- Direct, Constant, Partial Function
- Comparison Method

Functions

- Rational Function

Surface Area and Volume

- Decomposable Shapes
- Finding the height
- Converting units

Similar Figures

- Scale factor
- Missing measures

Statistics

- Mode, Median, Mean, Range
- Histograms
- Frequency table
- Stem and Leaf
- Weighted Mean
- Stratified Sampling Method
- Box and Whisker

Probability

- Describe extensively
- Describe in words
- Geometric Probability
- Probability tree Diagram
- With and Without Replacement



It was a pleasure!
Have an awesome summer!

MS. Nassif

306 Exam 50

Section 1

Questions 1 to 8

Blacken the letter corresponding to your choice in your answer booklet.

- 1** A teacher penalizes students when they hand in assignments late. The following table represents the relationship between the maximum mark on 25 that a student can obtain and the number of days late that the assignment is handed in to the teacher.

X, number of days late	0	1	2	3	4	5	6	7
Y, maximum mark possible	25	22.5	20	17.5	15	12.5	10	7.5

Which of the following rules corresponds to this relation?

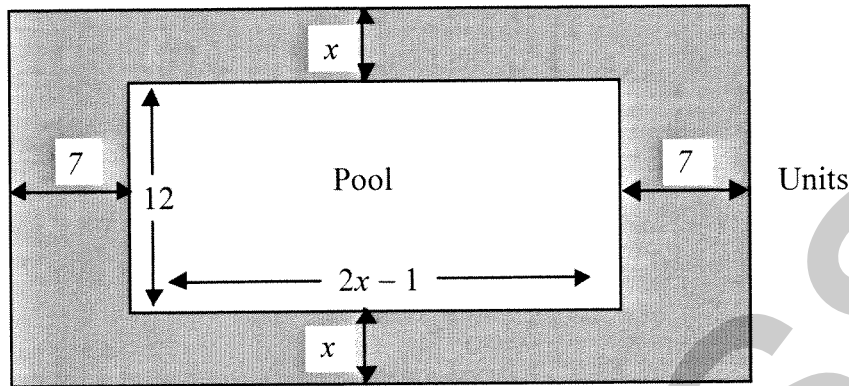
- A) $y = 25 - x$
 - B) $y = 25 - 22.5x$
 - C) $y = 25x - 2.5$
 - D) $y = 25 - 2.5x$
- 2** A photographer charges \$50.00 for an initial consultation fee and \$15.00 per photograph.

How will the Cartesian graph of this situation be affected if the photographer decides to increase the fee per photograph by \$5.00?

- A) The line will start at a higher point on the y-axis.
- B) The line will be less steep.
- C) The line will be steeper.
- D) The line will be longer.

306 Exam 50

- 4 The shaded area represents a cement walkway around a rectangular pool.



Which of the following expressions represents the difference between the outside perimeter of the walkway and the perimeter of the pool?

- A) $x + 4$
B) $x + 7$
C) $2x + 14$
D) $4x + 28$
- 8 The number of minutes of music aired in one hour on nine radio stations is as follows: 41, 44, 44, 45, 45, 49, 49, 49, 49. For this data, the mean is about 46, the median 45, the mode 49, and the range 8. A tenth radio station reports that it airs 41 minutes of music per hour.

What effect will this additional data have?

- A) The mean, median, mode and range will stay the same.
B) The mean will change, but the median, mode and range will stay the same.
C) The mean and median will change, but the mode and range will stay the same.
D) The mean, median and mode will change, but the range will stay the same.

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Section 2

Questions 9 to 16

Show all your work for questions 12 - 16 as part marks will be given.

- 9** a) In scientific notation, the number of hairs on the average human head is 1.5×10^5 and there are approximately 6×10^9 people in the world.

About how many human hairs are there in the world? (Express your answer in scientific notation.)

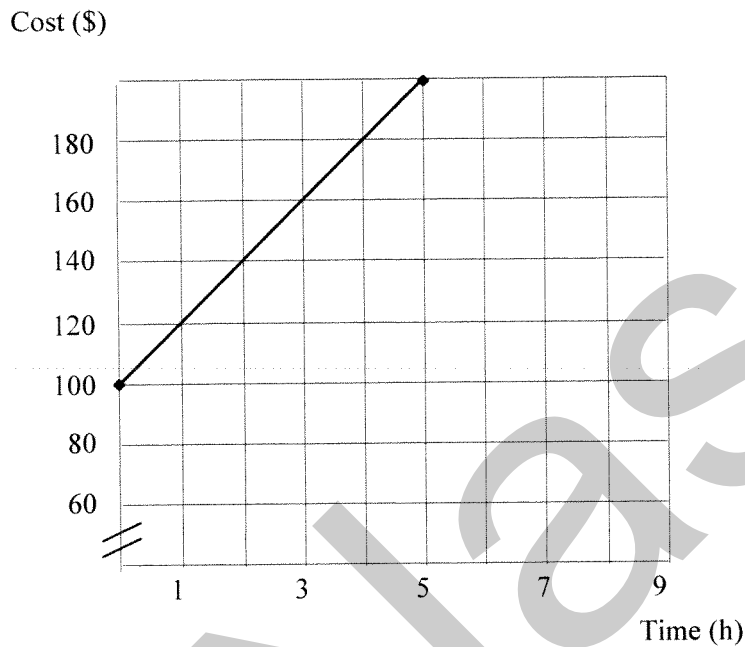
b) Simplify: $\frac{5x^3}{10x^2y}$

10 a) Expand: $(3b - 2)^2$.

b) Find the quotient: $(4m^2 + 6m) \div (2m)$.

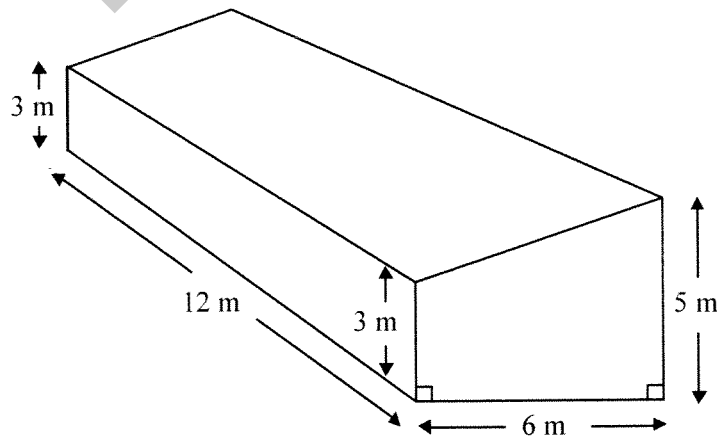
306 Exam 50

- 11 The JEDDS rock climbing club charges an initial membership fee plus an hourly instructional fee. The fees charged by this club are represented by the graph below.



According to this graph, what is the hourly instructional fee?

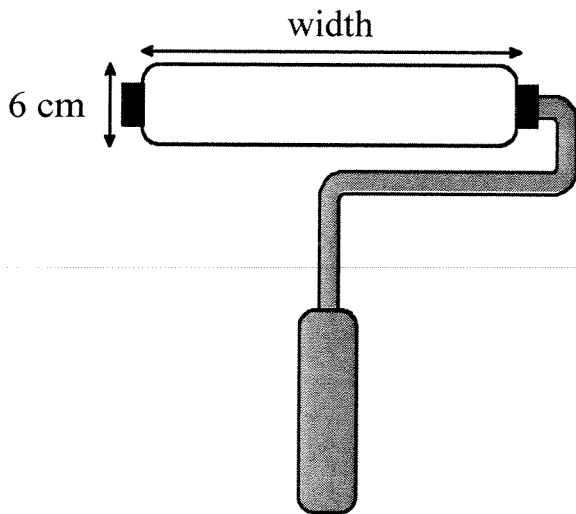
- 12 Given a right prism with dimensions as shown. What is the total volume of this prism?



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- 14** A painter uses a roller with a diameter of 6 cm. Each revolution covers an area of 471 cm^2 .

What is the width of the roller to the nearest cm?



- 16** Ms. Shannon had graded all her third term tests and determined that the mean of the scores was 78. In recording the results in her grade book, she forgot to enter one score. The test results she entered were: 90, 95, 75, 80, 85, 65, 70, 80, and 60.

What was the missing score?

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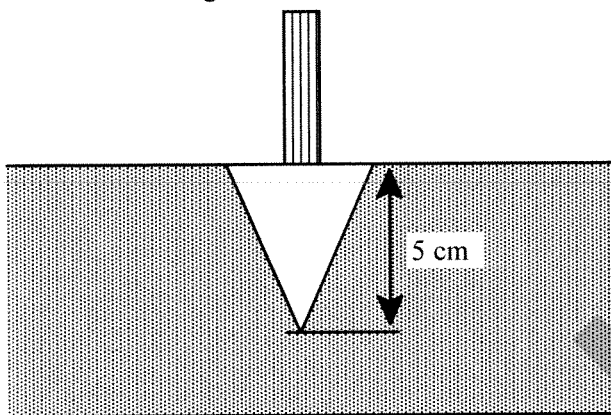
Section 3

Questions 17 to 23

In this section, marks are given for the work you do to arrive at the answer. Show all your work in the answer booklet.

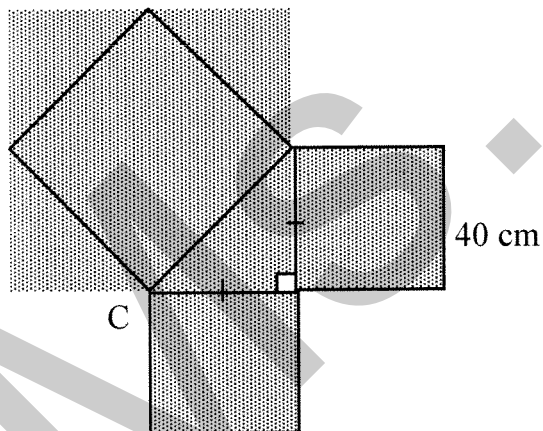
- 17 An industrial drill bit was used to drill a hole, the side view of which is pictured in the diagram below.

It takes 25 mL of glue to fill this hole.



What is the diameter of the hole's opening? (Round to 1 decimal place.)

- 18 Three square patio tiles are positioned so as to form the figure below.



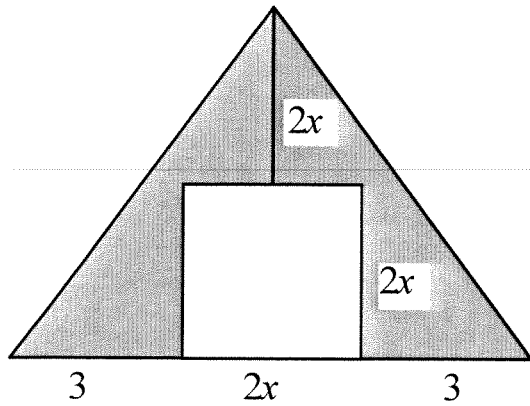
What is the perimeter of this figure?

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20 In the triangle below, the shaded area and the clear area are painted in different colors.

We want to repaint only one of these areas so that the whole triangle will be the same color.

The shaded area measures 48 m^2 .



What area should be repainted to minimize costs?

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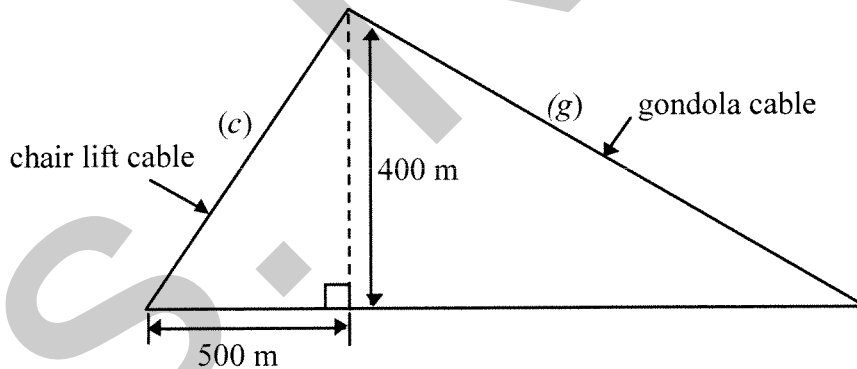
21 Ed and Carla earn extra money taking care of their neighbors' lawns. They offer two plans:

Plan A – \$12.00 to mow the lawn plus \$3.00 for each garbage bag used.

Plan B – \$15.00 to mow the lawn plus \$2.00 for each garbage bag used.

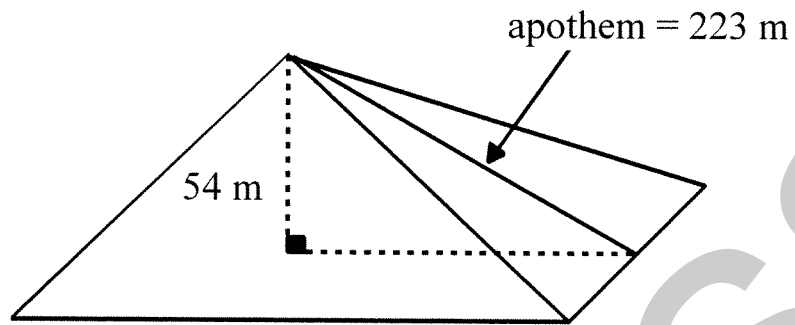
When does plan A become more profitable for Ed and Carla?

22 Both a chair lift and a gondola are used to transport skiers to the top of a ski hill. The length of the gondola cable is twice the length of the chair lift cable. The situation is represented by the model below with the dimensions as shown.



If the gondola travels at 5 m per second, how long will the gondola ride take?

- 23** A model of Quetzalcoatl, Mexico's largest pyramid, is shown below. The pyramid has a square base, a height of 54 m, and an apothem of 223 m.



What is the lateral area of this pyramid?

Ms. Nassif

Answer Key Exam 50

4. CORRECTION KEY

Section 1

Questions 1 to 8 4 marks or 0 marks

- 1 D
2 C
3 A
4 D

- 5 B
6 C
7 A
8 B

Section 2

Questions 9 to 16 4 marks each
2 marks for each of parts a and b in questions 9 and 10

- 9 a) 9×10^{14}
b) $\frac{x}{2y}$

- 10 a) $9b^2 - 12b + 4$
b) $2m + 3$

- 11 Answer The hourly instructional fee is \$20.

2 marks /4
2 marks

2 marks /4
2 marks

Answer Key Exam 50

12 Example of an acceptable solution

The student decomposed the prism, found volumes of the two component prisms, and total volume.

Answer The volume is 288 m³.

Allot 2 marks if the student decomposed the prism into two and found only one of the volumes (216 m³ or 72 m³).

Alternate method: Volume = area of trapezoid multiplied by length of prism

Allot 2 marks if student found area of trapezoid, 24 m².

14 Answer The width of the roller is 25.0 cm.

Allot 2 marks if the student states the equation only:

$$6\pi r = 471.$$

Accept any equivalent equation also.

/4

/4

/4

16 Answer The missing score is 80. [/4]

Section 3

Questions 17 to 23 4 marks each [/4]

17 Example of an appropriate solution [/4]

$$\frac{\pi r^2 (5)}{3} = 25$$

$$\pi r^2 = \frac{75}{5} = 15$$

$$r^2 = 4.77$$

$$r = 2.19$$

$$d = 4.4$$

Answer The diameter of the hole's opening is 4.4 cm.

18 Example of an appropriate solution [/4]

Side S of larger square is the hypotenuse of a right triangle.

$$S^2 = 40^2 + 40^2 = 3200$$

$$S = \sqrt{3200} = 56.6 \text{ cm}$$

The perimeter P of the figure is composed of 3 sides of each of the squares.

$$P = 3(56.6) + 3(40) + 3(40) = 409.8$$

Answer The perimeter of the figure is 409.8 cm (accept ± 1 cm)

19 Example of an appropriate solution [/4]

Country	Rock	Jazz	Classical
3251	2963	945	1307
2586	2723	854	957
1389	2385	683	542
256	2108	298	254
7482	10179	2780	3060

Rock is most popular at 10179.

Jazz is least popular at 2780.

Difference: $10179 - 2780 = 7399$.

Average: $7399 \div 4 = 1849.8$

Answer The average per store is 1849.8 (accept ± 1)

20 Example of an appropriate solution [/4]

The area of the triangle is $\frac{(4x)(2x+6)}{2} = 4x^2 + 12x$

The clear area is $(2x)(2x) = 4x^2$

The shaded area is $4x^2 + 12x - 4x^2 = 12x$

Since $12x = 48$, then $x = 4$

Thus the clear area is $4x^2 = 4(4^2) = 64$

Answer To minimize costs the shaded area should be repainted.

21 Example of an appropriate solution [/4]

Number of bags	0	1	2	3	4
Plan A	12	15	18	21	24
Plan B	15	17	19	21	23

Answer Plan A is more profitable when more than 3 bags are used.

22 Example of an appropriate solution [/4]

$$c^2 = 400^2 + 500^2$$

$$= 410\,000$$

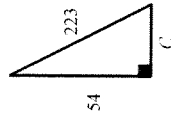
$$c = 640.3 \text{ m}$$

$$g = 2c = 2(640.3) = 1280.6$$

Travel time $1281 \div 5 = 256.2$

Answer The gondola ride will take 256 seconds. (accept ± 1 sec)
 Example of an appropriate solution

23



$$c^2 = 223^2 - 54^2$$

$$c^2 = 46\,813$$

$$c = 216.4$$

Base of triangle forming a lateral face

$$b = 2c$$

$$b = 2(216.4)$$

$$b = 432.8$$

$$\text{Area of 1 face is } \frac{(223)(432.8)}{2} = 48\,249.0 \text{ m}^2$$

$$\text{Lateral area} = 4(48\,249.0) = 192\,996$$

Answer The lateral area is 192 996 m² (accept ± 4 m²).

Section 1

Questions 1 to 8

Blacken the letter corresponding to your choice in your answer booklet.

14

1 A teacher penalizes students when they hand in assignments late. The following table represents the relationship between the maximum mark on 25 that a student can obtain and the number of days late that the assignment is handed in to the teacher.

X, number of days late	0	1	2	3	4	5	6	7
Y, maximum mark possible	25	22.5	20	17.5	15	12.5	10	7.5

Which of the following rules corresponds to this relation?

A) $y = 25 - x$

B) $y = 25 - 22.5x$

C) $y = 25x - 2.5$

D) $y = 25 - 2.5x$

2 A photographer charges \$50.00 for an initial consultation fee and \$15.00 per photograph.

How will the Cartesian graph of this situation be affected if the photographer decides to increase the fee per photograph by \$5.00?

A) The line will start at a higher point on the y-axis.

B) The line will be less steep.

C) The line will be steeper.

D) The line will be longer.

Ms. Nassif

Part A

Questions 1 to 8

Darken the letter that represents your choice on the Multiple-Choice Answer Sheet.

1

The following table gives the cost of renting a paintball facility.

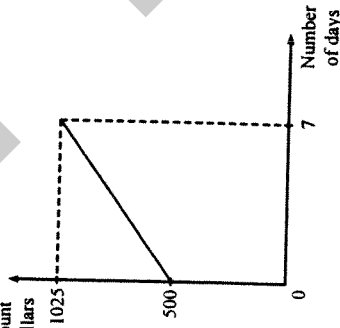
x (number of participants)	0	2	4	6
y (cost in dollars)	200	224	248	272

What is the rule that describes the cost of renting the facility?

- A) $y = 200$
- B) $y = 200x + 24$
- C) $y = 24x + 200$
- D) $y = 12x + 200$

2

A school collected donations for a disaster relief fund over a 7-day period, as shown in the accompanying graph.



They started with an initial amount of \$500 and ended with \$1025.

What is the rate of change?

- A) \$75.00 per day
- B) \$146.43 per day
- C) \$217.86 per day
- D) \$71.43 per day

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3

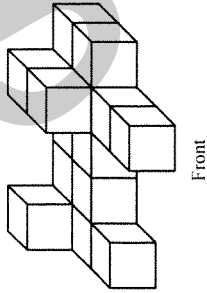
The volume, in km^3 , of the sun is given by the formula $V = \frac{4\pi}{3}(700\,000)^3$.

Which of the following best approximates the volume of the sun in scientific notation?

- A) $1.44 \times 10^{11} \text{ km}^3$
- B) $1.44 \times 10^{18} \text{ km}^3$
- C) $8.79 \times 10^9 \text{ km}^3$
- D) $4.19 \times 10^{18} \text{ km}^3$

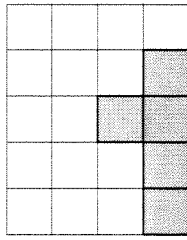
6

Consider the following solid.

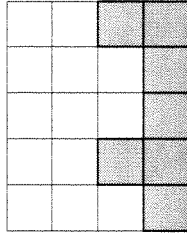


Which view is correctly represented?

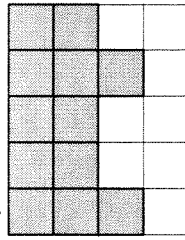
A) Left



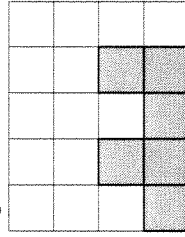
C) Back



B) Top



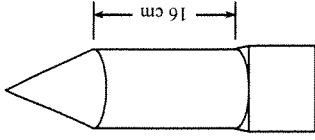
D) Right



7

A model is made by connecting a cone, a cylinder, and a cube.

- The height of the cylinder is 16 cm.
- The height of the cone is half the height of the cylinder.
- The cube and the cone have the same height.



What is the total volume of the model?
(Round to the nearest whole number.)

- A) 981 cm^3
- B) 1450 cm^3
- C) 1718 cm^3
- D) 4265 cm^3

8

A group of students had the following scores on a math quiz:

55, 60, 68, 75, 79, 80, 82, 83

What number can be added to these scores in order to keep the median the same?

- A) 75
- B) 76
- C) 77
- D) 79

Part B Questions 9 to 15

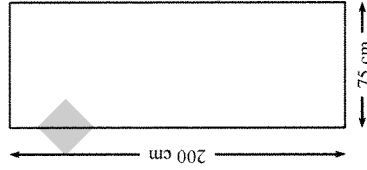
Show all appropriate work.
Write your answer in the space provided.

9 The relation between the number of people (n) it takes to paint the walls of a cafeteria and the time required in hours (T) is given by the following table:

Number of People vs. Time	
n (number of people)	2 3 4 6
T (time in hours)	18 12 9 6

In your Answer Booklet, graph the relation. Label your graph.

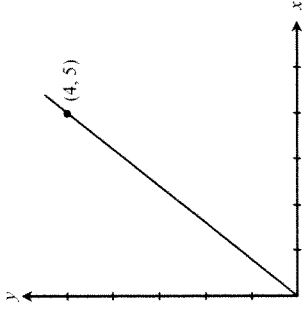
10 A rectangular doorway measures 200 cm by 75 cm.



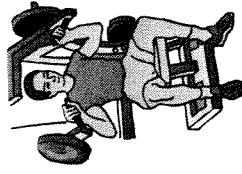
Could a circular mirror measuring 220 cm in diameter be passed sideways through this opening?

Calculation:

11 a) What is the rate of change for the graph below?



b) Pat joined a gym. It costs him \$30 per month to be a member.



What rule could be used to represent the relation between months and total cost?

12 Simplify

a)
$$\frac{12a^4 - 6a^2 + 9a}{3a}$$

Expand

b) $(3x - 4)^2$

13 Convert the measures below into the indicated units:

- a) $14.5 \text{ dam}^3 = \dots \text{ m}^3$
 b) $150 \text{ dl} = \dots \text{ cm}^3$

14 The vertices of quadrilateral ABCD on a Cartesian plane have the following coordinates:
 A(-6, 8), B(-9, 5), C(-3, 4), D(-3, 6)

The following composite transformation is used to produce quadrilateral A'B'C'D':
 $T_{(0, 5)} \circ R_{90^\circ}$

In your Answer Booklet, give the coordinates of the following vertices. Construct the image of quadrilateral ABCD by applying the given composite transformation.

15 The table below lists the heights, in cm, of the 20 students in Ms. Saha's math class.

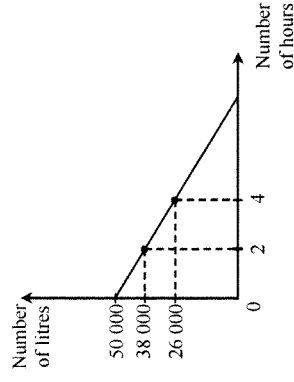
180	167	170	168	160	178	171	169
167	169	163	173	171	168	175	165
166	172	170	169				

- a) In your Answer Booklet, fill out the frequency table using the data above.
 b) In your Answer Booklet, construct a histogram to represent the data. Title and label your histogram.

Part C Questions 16 to 23

- ▶ Answer each question in the space provided below the question. Each question is worth 4 marks.
 - ▶ Show all your work as well as your answer. The work shown is taken into consideration when marks are awarded.
 - ▶ Your written information must be legible, complete, and clearly stated in correct language so the marker understands exactly what you have done.
- Even if your answer is correct, no marks will be given unless acceptable work is shown.

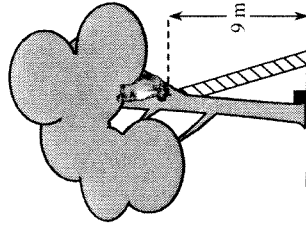
16 Jossia's aboveground pool holds 50 000 litres of water. The pool developed a leak. The accompanying linear graph shows that after 2 hours, 38 000 litres of water remain in the pool, while after 4 hours, 26 000 litres remain.



How many litres of water remain in the pool after 7 hours?

Show all your work.

17 Calvin's cat is stuck in a tree. The cat is 9 m above the ground. A person trying to rescue the cat places an 8 m ladder 3.5 m from the base of the tree.



To the nearest tenth of a metre, how far is the cat from the top of the ladder?

Show all your work.

18

The perimeter of the rectangular room on the right is $(30x + 18)$ metres.
The width is $(3x - 2)$ metres.
Logan wants to install a carpet that covers the entire floor.



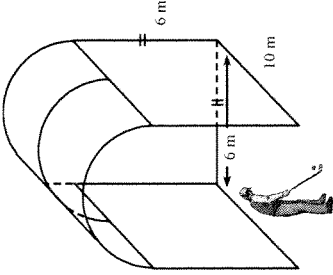
How many square metres of carpet are needed to cover the floor?

Express your answer as a simplified algebraic expression.

21

Netting is required to enclose a cage for golfers to practice at the GOLF DOME.

The cage is in the form of a rectangular prism with a half-cylindrical top. No netting is required for the front opening or the floor.



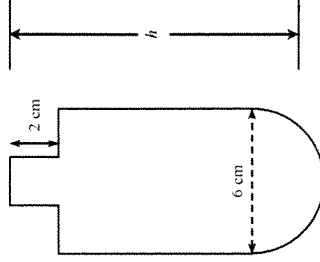
Using the dimensions shown on the diagram, how much netting, to the nearest square metre, is required to enclose the cage?

Show all your work.

22

The view on the right represents a cross-section of a plastic bottle composed of a hemisphere, a large cylinder, and a small cylinder.

When only the hemisphere and large cylinder are filled, the bottle holds 500 mL of water.



To the nearest tenth of a centimetre, what is the total height, h , of the bottle?

Show all your work.

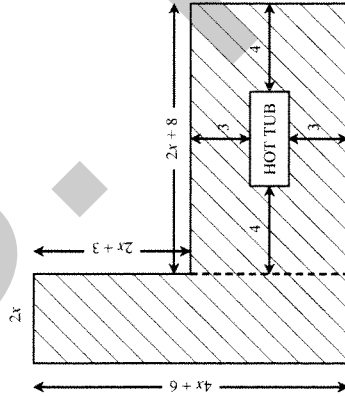
19

The floor of a wooden deck is designed such that there is a space set aside for a HOT TUB.

What is the area of the wooden portion of the deck?

Give your answer in simplified algebraic form.

Show all your work.





23

The Montreal Expos Baseball Club played their final season in 2004. The number of wins in 18 of their last 20 seasons is listed, in descending order, in the following chart.



94	91	88	87	85
84	83	83	81	81
74	71	68	68	67
67	66	65		

In each of the two seasons that are missing, the team won the **same** number of games. The mean number of wins for all 20 seasons was 77.95.

What is the median number of wins for the 20 seasons?

Show all your work.

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4. CORRECTION KEY

Questions 1 to 8 4 marks or 0 marks

- 1
- 2
- 3
- 4

- Part A
- D 5
 - C 6
 - B 7
 - A 8

Questions 9 to 15 4 marks each

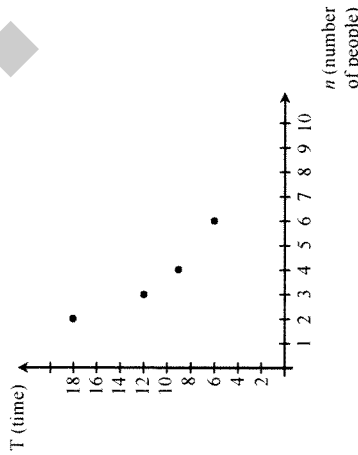
- 9

An example of an appropriate graph.

4, 3 or 0 marks

1/4

People vs. Time



- 4 marks Graph drawn and labelled correctly
 - 3 marks Graph drawn correctly but incorrectly labelled or not labelled
 - 0 marks Any other case
- Note:** Do not penalize students who connected the points or extended the graph.

10

Calculation
 $d^2 = 200^2 + 75^2$
 $d \approx 213.6$

3 or 0 marks

1/4

Justification:

The diagonal of the doorway measures 213.6 cm. Placing the mirror sideways will NOT allow it to pass through the opening.

1 or 0 marks

11

a) The rate of change is 1.25.
Note: Accept also $1\frac{1}{4}$ or $\frac{5}{4}$ as an answer.
 Allot 1 mark if student writes $y = 1.25x$.
 Allot 0 marks if student writes $1.25x$.

2, 1 or 0 marks

1/4

b)

The rule is $C = 30n$.
 Do not penalize students who use different variables.
 Accept any rule of the form $C = 30n + b$ provided that the student identifies b as an initial fee.

2 or 0 marks

12

a) $4a^3 - 2a + 3$
 b) $9x^2 - 24x + 16$

2 or 0 marks
 2 or 0 marks

1/4

13

a) $14.5 \text{ dm}^3 = 14\,500 \text{ m}^3$
 b) $150 \text{ dL} = 15\,000 \text{ cm}^3$

2 or 0 marks
 2 or 0 marks

1/4

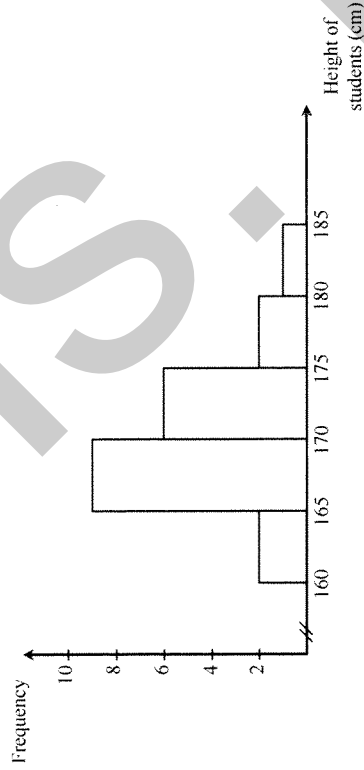
15

Height	Frequency
[160, 165]	2
[165, 170]	9
[170, 175]	6
[175, 180]	2
[180, 185]	1

a)

b)

Height of Students



- 4 marks
- 3 marks
- 2 marks
- 0 marks

Graph drawn and labelled correctly: table correct
 Graph drawn correctly but not labelled, table correct
 Table alone correct or graph alone correct or correct graph based on incorrect table
 Any other case

Note: The tally column is optional; no marks are allotted.

16

Example of an appropriate solution

Initial value is 50 000

$$\text{Rate of change} = \frac{26\,000 - 38\,000}{4 - 2} = -6000$$

Let x : number of hours that the pool has been leaking
 y : number of litres of water remaining in the pool.

Rule $y = -6000x + 50\,000$

For $x = 7$
 $y = -6000(7) + 50\,000$
 $= 8000$

Answer After 7 hours, 8000 litres of water remain in the pool.

Note: Students who have correctly calculated the rate of change have shown that they have a partial understanding of the problem.

17

Example of an appropriate solution

$$a^2 + 3.5^2 = 8^2$$

$$a = 7.2$$

$$\text{and } 9 - a = 1.8$$

Answer: To the nearest length of a metre, the cat is 1.8 m above the top of the ladder.

Note: Students who have correctly calculated the missing leg of the triangle have shown that they have a partial understanding and should receive 3 marks.

Do not penalize students who have rounded incorrectly or have not rounded their answer.

18

Example of an appropriate solution

$$p = 2L + 2w$$

$$21 = p - 2w$$

$$L = \frac{(p - 2w)}{2}$$

Length of room

$$\frac{30x + 18 - 2(3x + 32)}{2} + 2$$

$$\frac{24x + 22}{2} + 2$$

$$12x + 11$$

Area of room

$$(12x + 11)(3x - 2)$$

$$36x^2 + 9x - 22$$

Answer: As a simplified algebraic expression, $36x^2 + 9x - 22$ square metres of carpet are needed.

Note: Students who correctly determined the length of the room have shown that they have a partial understanding of the problem.

19

Example of an appropriate solution

Area of "tail" rectangle

$$(4x + 6)(2x)$$

$$8x^2 + 12x$$

Area of remaining deck

$$(2x + 8)(4x + 6 - (2x + 3))$$

$$(2x + 8)(2x + 3)$$

$$4x^2 + 22x + 24$$

Dimensions of hot tub

$$2x + 8 - 4$$
 by $2x + 3 - 3 - 3$

$$2x$$
 by $2x - 3$

Area of hot tub

$$2x(2x - 3)$$

$$4x^2 - 6x$$

Total area of wooden portion

$$(8x^2 + 12x) + (4x^2 + 22x + 24) - (4x^2 - 6x)$$

$$8x^2 + 40x + 24$$

Answer: The area of the wooden portion of the deck, in simplified algebraic form, is $8x^2 + 40x + 24 \text{ m}^2$.

Note: Students who have correctly calculated any two of the three rectangular areas correctly have shown that they have a partial understanding of the problem.

20

Example of an appropriate solution

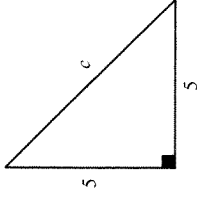
Volume = 125 dm^3

\therefore Each side of cube is 5 dm

Calculate the length of the hypotenuse (On the front face)

$$c^2 = 5^2 + 5^2$$

$$c = \sqrt{50} \approx 7.071$$



The shaded triangular face is equilateral, with sides 7.071

Calculate its height

$$h^2 + 3.535^2 = 7.071^2$$

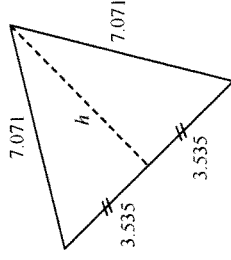
$$h^2 = 37.5028$$

$$h = 6.124 \text{ dm}$$

Area of triangle

$$A = \frac{1}{2}(7.071)(6.124)$$

$$A = 21.6512 \text{ dm}^2$$



Answer: To the nearest hundredth dm^2 , the area of the shaded triangular face is 21.65 .

Note: Students who have correctly calculated the length of the diagonal (hypotenuse) have shown that they have a partial understanding of the problem.

Do not penalize students who have rounded incorrectly or have not rounded their answer.

Ms. Nassif

23

Example of an appropriate solution

Total wins for the 20 seasons
(20)(77.95) = 1559

Total for 18 seasons in chart
1403

Total for two missing seasons
156

∴ The Expos won $\frac{156}{2} = 78$ games in each of these seasons

Calculation of median

The two 78's become the 11th and 12th entries in descending order.

Averaging the 10th and 11th entries, gives the median for the distribution.

Therefore the median is

$$\frac{81 + 78}{2} = 79.5$$

Answer: The median number of wins for the 20 seasons is 79.5.

Note: Students who have correctly calculated the total wins for 18 and 20 seasons have shown that they have a partial understanding of the problem.

**21**

Example of an appropriate solution

Prism

$$\text{Area of 2 sides: } 2 \times 10 \times 6 = 120 \text{ m}^2$$

$$\text{Area of back: } 6 \times 6 = 36 \text{ m}^2$$

$$\text{Area of Prism: } 156 \text{ m}^2$$

Half cylinder

$$\text{Lateral area: } \frac{1}{2} \bullet 2\pi \bullet r \bullet h$$

$$= \pi(3)0$$

$$= 30\pi$$

$$\approx 94.25 \text{ m}^2$$

$$\text{Area of back face: } \frac{1}{2} (\text{area of base})$$

$$= \frac{1}{2} \bullet \pi^2$$

$$= \frac{1}{2} \bullet \pi \bullet 3^2$$

$$= 4.5\pi$$

$$\approx 14.14 \text{ m}^2$$

Area of cylindrical section

$$108.39 \text{ m}^2$$

Total area

$$156 \text{ m}^2 + 108.39 \text{ m}^2 = 264.39 \text{ m}^2$$

Answer: The total amount of netting required to enclose the cage is **264 m²**.**Note:** Students who have correctly calculated the area of either the rectangular prism or the half cylindrical region correctly have shown that they have a partial understanding of the problem.

Do not penalize students who have rounded incorrectly or have not rounded their answer.

508-3744.E55

**22**

Example of an appropriate solution

Hemisphere

Radius

$$\frac{6}{2} = 3 \text{ cm}$$

Volume

$$\frac{4\pi}{3} r^3 \bullet \frac{1}{2} = \frac{4\pi}{3} \bullet 3^3 \bullet \frac{1}{2}$$

$$\approx 56.55 \text{ cm}^3$$

Cylinder

Let x be the height of the cylinder

Volume

$$500 \text{ cm}^3 - 56.55 \text{ cm}^3 = 443.45 \text{ cm}^3$$

$$\pi r^2 x = 443.45$$

$$\pi(3)^2 x = 443.45$$

$$x = \frac{443.45}{9\pi}$$

$$\approx 15.68$$

Total height

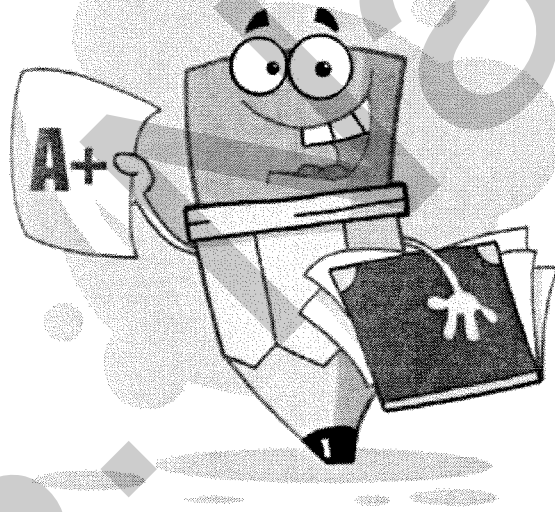
$$3 + 15.68 + 2 \approx 20.68 \text{ cm}$$

Answer: The total height of the bottle, to the nearest tenth of a centimetre, is **20.7 cm**.**Note:** Students who have correctly calculated the volume of the cylinder have shown that they have a partial understanding of the problem.

Do not penalize students who have rounded incorrectly or have not rounded their answer.

508-3744.E55

RECENT EXAMS



MS. NASSIF

Ms. Nassif

June 2012

Part A Questions 1 to 6

In the Student Booklet, darken the letter that corresponds to your answer.

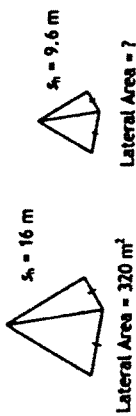
Each question is worth 4 marks.

1. Which interval notation represents the solution set of the inequality below?

$$-2x + 5 \leq 11$$

- A) $[3, +\infty[$ C) $] -\infty, -3]$
 B) $] -\infty, 3]$ D) $] -3, +\infty [$

2. Two square based pyramids are similar. The lateral area of the larger pyramid is 320 m^2 . The slant height of the larger pyramid is 16 m . The slant height of the smaller pyramid is 9.6 m .



What is the lateral area of the smaller pyramid?

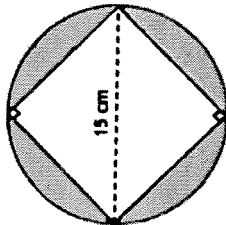
- A) 69.12 m^2 C) 160 m^2
 B) 115.2 m^2 D) 192

3. What is the simplified algebraic expression?

$$\frac{(4x - 8)(3x + 6)}{6}$$

- A) $2x^2 + 8x - 8$ C) $2x^2 - 8$
 B) $2x^2 + 8$ D) $6x^2 - 42$

4. On the disk-shaped target below, the square is inside the circle. The diameter of the circle is 15 cm . A point is chosen randomly from this target.



What is the probability, to the nearest percentage, that the chosen point is in the shaded area?

- A) 32% C) 64%
 B) 36% D) 68%

5. What is the result of the following operation in proper scientific notation?
 $(0.95 \times 10^3) \cdot (25.6 \times 10^2)$

- A) 2.432×10^{-1}
- B) 2.432×10^{-14}
- C) 2.432×10^1
- D) 24.32×10^{-2}

6. In Brenda's Science class, the calculations of her term mark are shown in the table below. However, her Test 1 mark is missing.

Term Content (Weighting)	Brenda's Marks
Project (40%)	70
Test 1 (15%)	?
Test 2 (25%)	73
Quiz 1 (5%)	40
Quiz 2 (5%)	80
Assignment (10%)	90
TERM MARK	71

What is Brenda's mark for Test 1?

- A) 65%
- B) 55%
- C) 71%
- D) 73%

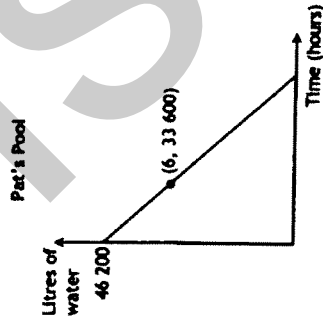
Part B Questions 7 to 10
 In the Student Booklet, write your answer in the space provided.
 Each question is worth 4 marks.

7. There are 26 students in Christian's class.
 Christian's term mark for math is 81%.
 The marks of the entire class, including Christian's, are below:

- | | | | | | | |
|----|----|----|----|----|----|----|
| 45 | 50 | 50 | 55 | 60 | 62 | 64 |
| 65 | 68 | 70 | 70 | 74 | 74 | 76 |
| 80 | 80 | 80 | 81 | 82 | 84 | 86 |
| 86 | 88 | 90 | 92 | 92 | 98 | |

- a) What is the inter-quartile range (IQR)?
- b) In which quarter is Christian's mark located?

8. In the Fall, Pat needs to drain her swimming pool using a small garden hose. The pool holds 46 200 litres of water, the pool drains at a constant rate. After 6 hours, 33 600 litres of water remain in the pool.



Over what interval of time will the pool contain water?
Give your answer in interval notation.

9. A company makes 2 sizes of spherical ornaments: large and small. The ornaments are sold in similar cylindrical containers.



- The area of 1 small ornament is 78.5 cm^2 .
- The volume of the small cylinder is 490.625 cm^3 .
- The volume of the large cylinder is 3925 cm^3 .

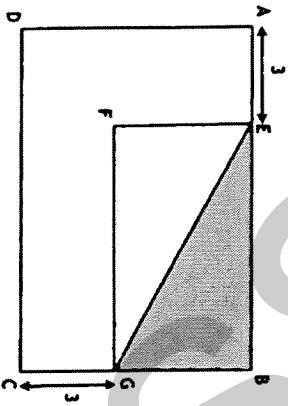
$$V = 490.625 \text{ cm}^3 \quad V = 3925 \text{ cm}^3$$

What is the radius of the large cylinder?

Note: Use $\pi = 3.14$

10. The area of rectangle ABCD is $12x^2 + 9x$.

Side \overline{AD} is the GCF (greatest common factor) of $12x^2 + 9x$.



What algebraic expression represents the area of triangle BEG?

Part A Questions 1 to 6

Darken the letter that corresponds to your answer.

Each question is worth 4 marks.

1. (A) (B) (C) (D)
2. (A) (B) (C) (D)
3. (A) (B) (C) (D)
4. (A) (B) (C) (D)
5. (A) (B) (C) (D)
6. (A) (B) (C) (D)

Part B Questions 7 to 10

Write your answer in the space provided.

Each question is worth 4 marks.

7. a) The interquartile range (IR) is

b) Christian's mark is located in the quarter.

8. The pool contains water over the interval hours.

9. The radius of the large cylinder is cm.

10. The algebraic expression representing triangle BEG is

Part C

This part of the examination consists of Questions 11 to 16.

For each question, you must show all your work to justify your answer. The work you show on all the pages of Part C will be considered.

Your work must be organized and clearly presented and cannot simply involve listing the calculator applications or programs used to obtain results or information.

Each question in Part C is worth 10 marks.

11. Wakeboarding

Amy and Jimmy were talking about wakeboarding in the summer.

Amy likes to go to Lake Brunet Boating Club. The Club charges a certain amount per hour to wakeboard and a fixed amount for equipment rental.

Three hours of wakeboarding costs \$205.

Five hours of wakeboarding costs \$325.

Jimmy wakeboards at the Centre de Plein Air in the Laurentians.

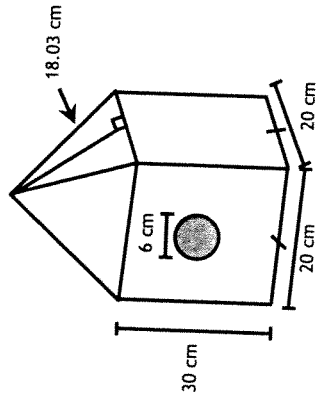
The cost of wakeboarding at Centre de Plein Air is \$50 per hour plus \$45 to rent the equipment.

After how many hours of wakeboarding would it cost the same at both wakeboarding facilities and what would that cost be?

12. Birdhouse

Students in a woodworking class made birdhouses. Karen is painting the outside of her birdhouse red. She needs to calculate the surface area of the birdhouse to know how much paint is needed. The birdhouse has a square base.

- The shaded hole has a diameter of 6 cm.
- The sloped edge of the roof is 18.03 cm as indicated below.



What is the total surface area of her birdhouse?

13. Popular Apps

A survey was done across Quebec to see which cell phone apps (applications) were the most popular among teenagers and adults to download on their cell phones. The mean is used to analyse the results of the survey.



There are 27 more men(18+) than women(18+) who downloaded a News/Weather app. Below is the incomplete chart displaying the results.

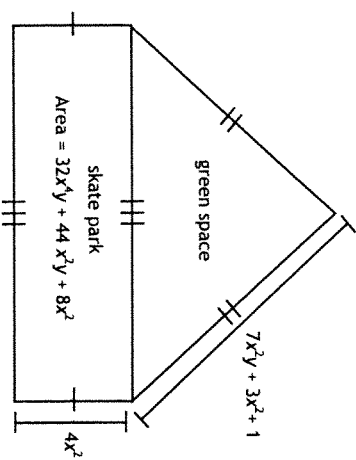
Age APPS Category	Boys 12-17	Girls 12-17	Men (18+)	Women (18+)	Mean
Games	1042	934	828	664	
Social Networks	688	675	613	480	
News/Weather	347	258			
Music	982	867	749	602	
Average of the means					666

How many men(18+) subscribe to a News/Weather app?

14. Skate Park

Below is a rectangle representing the skate park. Connected to the skate park is a triangular green space.

- The area of the skate park is $32x^2y + 44x^2y + 8x^2$
- The width of the rectangle is $4x^2$
- One of the sides of the green space is $7x^2y + 3x^2 + 1$



How much larger is the perimeter of the green space compared to the perimeter of the skate park?

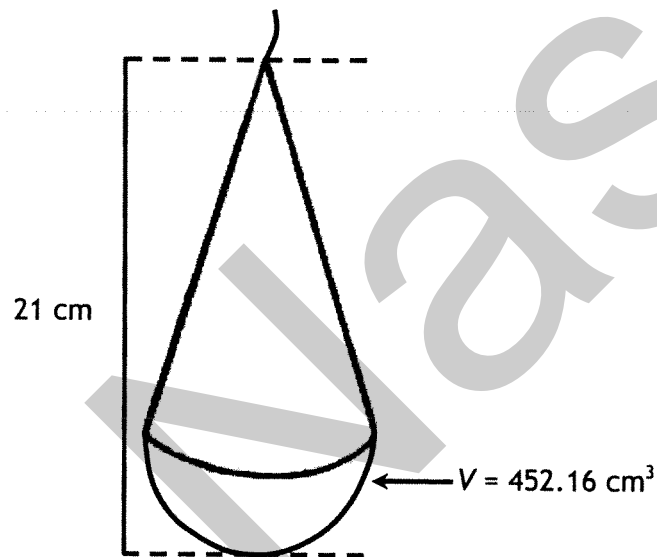
15. Candle Making

Students are making candles as part of their entrepreneurship project.

Each candle consists of a cone and a hemisphere.

- The volume of the hemisphere is 452.16 cm^3 .
- The total height of the candle is 21 cm.

The students must make 25 candles.

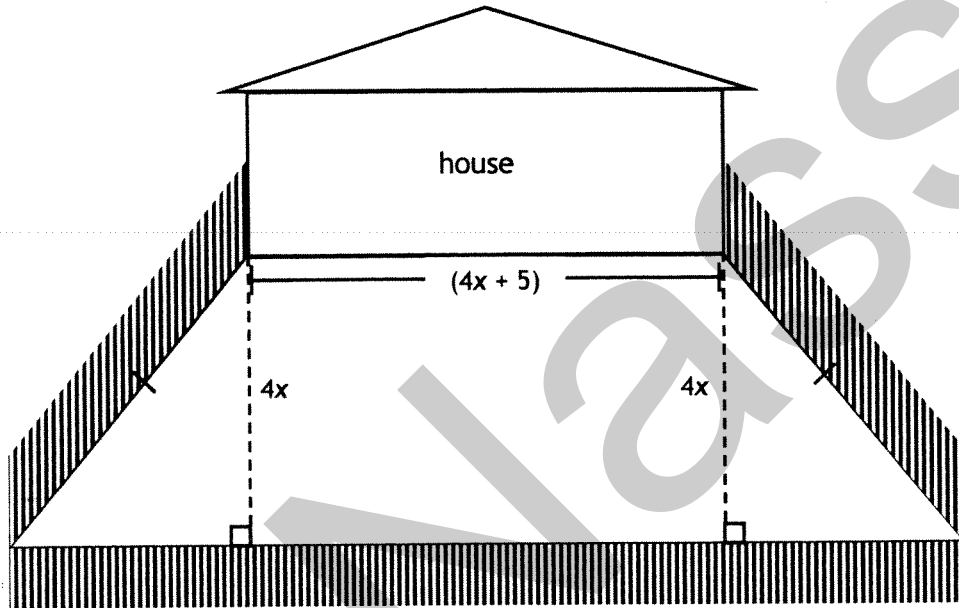


How many litres of wax will it take to make 25 candles?

Note: Use $\pi = 3.14$

16. Fenced In

The backyard below is in the shape of an isosceles trapezoid and its area is $(28x^2 + 20x)$ square meters. The yard is fenced in on three sides and the house forms the small base of the trapezoid.



- The height of the trapezoid in meters is $4x$.
- The width of the house in meters is $(4x + 5)$.

What is the binomial that represents the length of the fence needed to surround the backyard?

JUNE 2011

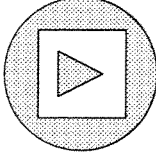
C. NASSIF

PART A

This part of the examination consists of Questions 1 to 6.

Each question in this part of the examination is worth 4 marks.

On page 5 of your *Student Booklet*, fill in the box under the letter that corresponds to your answer.



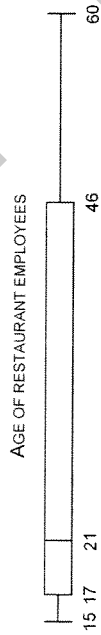
A game of chance involves throwing a dart at a circular target on which a triangle and a square are drawn.

If the dart lands in a shaded area, the player will win a prize.

Which expression below must be used to calculate the probability of winning a prize in this game?

- A) $1 - \frac{\text{Area of the square}}{\text{Area of the circle}}$
- B) $\frac{\text{Area of the circle} + \text{Area of the triangle}}{\text{Area of the circle}}$
- C) $\frac{\text{Area of the circle} - \text{Area of the triangle} + \text{Area of the square}}{\text{Area of the circle}}$
- D) $\frac{\text{Area of the circle} - \text{Area of the square} + \text{Area of the triangle}}{\text{Area of the circle}}$

1. The following box-and-whisker plot is based on the ages of the 9 employees in a restaurant. The 9 employees are all of different ages.



Which of the following statements is necessarily true?

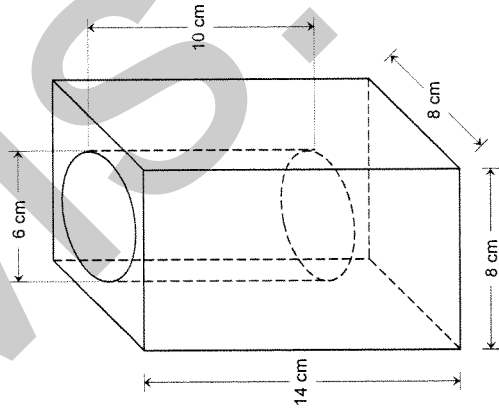
- A) One of the employees is 17 years old.
- B) One of the employees is 21 years old.
- C) None of the employees is 60 years old.
- D) There are fewer employees under the age of 17 than employees over the age of 46.

3. Consider the following five numbers:
- | | | | | |
|------------|------------|------------|------------|------------|
| $\sqrt{0}$ | $\sqrt{1}$ | $\sqrt{2}$ | $\sqrt{3}$ | $\sqrt{4}$ |
|------------|------------|------------|------------|------------|

How many of these numbers are irrational?

- A) 2
- B) 3
- C) 4
- D) 5

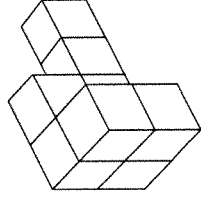
4. A pencil holder was made by hollowing out a right circular cylinder in a block of wood in the shape of a right prism with a square base.



What is the volume of this pencil holder to the nearest cm^3 ?

- A) 165 cm^3 C) 613 cm^3
 B) 519 cm^3 D) 802 cm^3

5. Below is an arrangement of 8 congruent cubes using axonometric perspective.



Which of the following diagrams shows the top view of this arrangement?



6. Which of the following four containers has the greatest capacity?
 A) A 0.03 m^3 container C) A 27.5 L container
 B) A 0.4 dal container D) A $28\,000 \text{ cm}^3$ container

PART B

This part of the examination consists of Questions 7 to 10.

Each question in this part of the examination is worth 4 marks.

Write each of your answers in the space provided on page 5 of your *Student Booklet*.

7. The following table shows the distribution of the 90 000 adults in a town according to different strata.

	Borough			Total
	Lakeland	Bluefield	Hilltop	
Men	15 000	13 500	16 000	44 500
Women	18 000	13 500	14 000	45 500
Total	33 000	27 000	30 000	

A sample of 1 350 adults from this town is required. This sample must be representative of the strata mentioned in the table.

How many men from the Hilltop borough should be in this sample?

8. A group of people rented a skating rink for an evening. The amount of money each person paid is represented by function f described below.

$$f(x) = \frac{2500}{x}$$

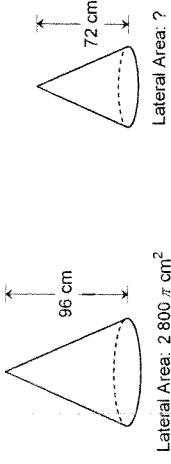
x : number of people in the group

$f(x)$: amount of money paid by each person, in dollars

Each person paid \$12.50 to rent the skating rink.

How many people were in the group?

9. Two right circular cones are similar. The lateral area of the bigger cone is $2800\pi \text{ cm}^2$. The height of the bigger cone is 96 cm. The height of the smaller cone is 72 cm.



What is the lateral area of the smaller cone to the nearest cm^2 ?

10. The following frequency table indicates the finishing times of the 100 runners in a marathon.

Finishing Time (minutes)	Number of Runners
[120, 130[4
[130, 140[20
[140, 150[36
[150, 160[24
[160, 170[16
Total	100

What was the mean finishing time of the runners in this marathon?

Ms. Nassif

11. RAYMOND'S DRIVEWAY

Raymond wants to have his driveway paved. He contacted two paving companies to find out how much it would cost. He was given the following information:

COMPANY A

The cost of paving a driveway is represented by function f described below.

$$f(x) = 35x + 25 \quad \text{where } x : \text{area of the driveway, in m}^2$$

$f(x)$: price, in \$, charged by Company A to pave the driveway

COMPANY B

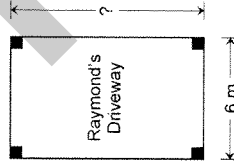
The paving cost includes a base price of \$145 plus an amount proportional to the area of the driveway.

For example, Raymond's neighbour paid Company B \$1 696 to pave his 47 m² driveway.

Raymond has a rectangular driveway that is 6 m wide.

Both Company A and Company B would charge Raymond the same price to pave his driveway.

How long is Raymond's driveway?



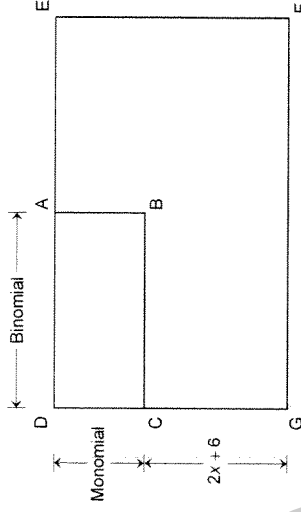
12. THE PERIMETER OF RECTANGLE DEFG

In the figure below, quadrilaterals ABCD and DEFG are rectangles.

The lengths of the sides of rectangles ABCD and DEFG can be represented by polynomials in which the coefficients of the terms and the constant terms are integers.

Note that:

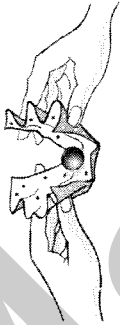
- the length of segment DA is represented by a binomial
- the length of segment DC is represented by a monomial
- the area of rectangle ABCD is represented by the binomial $21x^2 + 15x$
- the perimeter of rectangle ABCD is represented by the binomial $20x + 10$
- $m \overline{DA} = m \overline{AE}$
- $m \overline{CG} = 2x + 6$



What binomial represents the perimeter of rectangle DEFG?

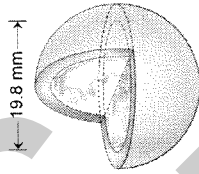
13. IT'S A WRAP!

A piece of candy comes in a wrapper.



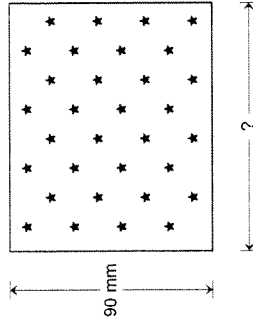
The piece of candy consists of a spherical piece of caramel covered with an even layer of chocolate.

The diameter of the spherical piece of caramel is 19.8 mm.



In this piece of candy, the volume of the piece of caramel is equal to the volume of the chocolate.

The candy wrapper is rectangular and 90 mm wide.



The area of the candy wrapper is 5 times the area of the piece of candy.

What is the length of the candy wrapper to the nearest tenth of a millimetre?

14. THE MINIATURE BUILDING EXHIBIT

Gabriel will be renting space to exhibit miniature buildings.

Gabriel's profit is represented by function f described below.

$$f(x) = ax - 1000 \quad \text{where } a \neq 0$$

$f(x)$: Gabriel's profit, in dollars

x : number of visitors to the exhibit

a : admission fee, in dollars

Gabriel wants to know how many visitors he needs to break even.

Formulate a conjecture describing the change in the number of visitors Gabriel needs to break even after the admission fee is doubled.

15. SARAH'S TRIANGLES

Sarah studied three triangles with the following characteristics:

- They are right triangles.
- In each triangle, the lengths of the two longest sides are two consecutive integers.

The following table indicates the lengths of the sides of the three triangles Sarah studied.

	Lengths of the Two Longest Sides	Length of the Shortest Side
Triangle 1	13 m and 12 m	5 m
Triangle 2	25 dm and 24 dm	7 dm
Triangle 3	41 cm and 40 cm	9 cm

After calculating the length of the shortest side of each of the three triangles she studied, Sarah drew the following conclusion:

"If the lengths of the two longest sides of a right triangle are consecutive integers, then the length of the shortest side of the triangle will also be an integer."

Is Sarah's conclusion true or false? Explain why.

16. A SQUARE TARGET

A square target is divided into three sections: one black, one white and one grey.

Each side of the target measures 28 cm.

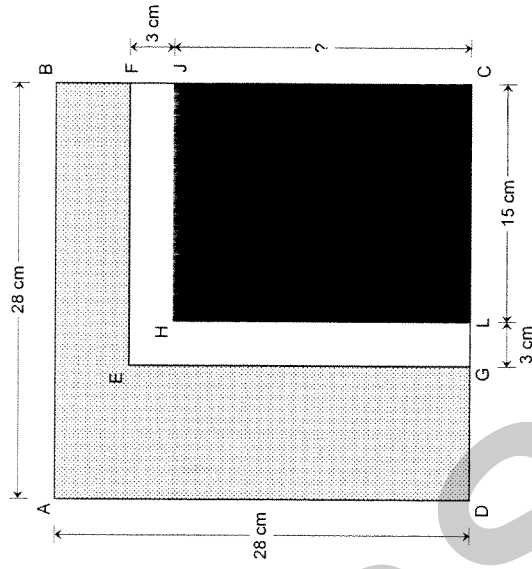
This target is represented by square ABCD in the diagram below.

Quadrilaterals EFCG and HJCL are rectangles.

In addition:

$$m\overline{LC} = 15 \text{ cm}$$

$$m\overline{FJ} = m\overline{GL} = 3 \text{ cm}$$



A randomly thrown dart hit the target. The probability that the dart landed in the white section is less than the probability that it landed in the black section.

What are the possible lengths of segment JC?

Ms. Nassif

ANSWERS JUNE 2011

7. Answer Key for the Prototype Examination

PART A

QUESTIONS 1 TO 6: Give 4 marks for each correct answer.

- | | A | B | C | D |
|----|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| 1. | <input type="checkbox"/> | <input checked="" type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 2. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input checked="" type="checkbox"/> |
| 3. | <input checked="" type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4. | <input type="checkbox"/> | <input type="checkbox"/> | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| 5. | <input type="checkbox"/> | <input type="checkbox"/> | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| 6. | <input checked="" type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |

PART B

QUESTIONS 7 TO 10: Give 4 marks for each correct answer.

- There should be **240** men from the Hilltop borough in this sample.
- There were **200** people in the group.
- To the nearest cm^2 , the lateral area of the smaller cone is **4 948** cm^2 .
Note: Accept $1575\pi \text{ cm}^2$.
 Do not penalize students who did not round off their answer to the nearest cm^2 .
- The mean finishing time of the runners in this marathon was **147.8** minutes.

PART C

11. RAYMOND'S DRIVEWAY

A. EXAMPLE OF APPROPRIATE REASONING

RULE OF THE FUNCTION REPRESENTING THE PRICE CHARGED BY COMPANY B TO PAVE A DRIVEWAY

x : area of the driveway, in m^2
 $g(x)$: price, in \$, charged by Company B to pave a driveway
 $g(x) = ax + b$ where $b = 145$ and $g(47) = 1696$
 $1696 = a(47) + 145$
 $1551 = a(47)$
 $33 = a$
 Rule: $g(x) = 33x + 145$

A. AREA OF RAYMOND'S DRIVEWAY

The area of Raymond's driveway is equal to the value of x for which $g(x) = f(x)$.
 $33x + 145 = 35x + 25$
 $-2x = -120$
 $x = 60$
 Area of Raymond's driveway: 60 m^2

A. LENGTH OF RAYMOND'S DRIVEWAY

Width of the driveway \times Length of the driveway = Area of the driveway
 $6 \text{ m} \times$ Length of the driveway = 60 m^2
 Length of the driveway = 10 m

A. CONCLUSION

Raymond's driveway is 10 m long.

12. THE PERIMETER OF RECTANGLE DEFG

A. EXAMPLE OF APPROPRIATE REASONING

➤ LENGTHS OF THE SIDES OF RECTANGLE ABCD

Area of rectangle ABCD = $m\overline{DC} \times m\overline{DA}$

$$21x^2 + 15x = \text{Monomial} \times \text{Binomial}$$

$$3x(7x + 5) = \text{Monomial} \times \text{Binomial}$$

If $m\overline{DC} = 3x$ and $m\overline{DA} = 7x + 5$, let's see whether the perimeter of rectangle ABCD is represented by $20x + 10$.

$$\text{Perimeter of rectangle ABCD: } 2(m\overline{DA} + m\overline{DC}) = 2(7x + 5 + 3x) = 2(10x + 5) = 20x + 10$$

Thus, $m\overline{DA} = 7x + 5$ and $m\overline{DC} = 3x$.

➤ LENGTHS OF THE SIDES OF RECTANGLE DEFG

$$m\overline{DG} = m\overline{DC} + m\overline{CG} = 3x + 2x + 6 = 5x + 6$$

$$m\overline{DE} = m\overline{DA} + m\overline{AE} = 2 \times m\overline{DA} = 2(7x + 5) = 14x + 10$$

➤ PERIMETER OF RECTANGLE DEFG

$$\begin{aligned} \text{Perimeter of rectangle DEFG} &= 2(m\overline{DG} + m\overline{DE}) \\ &= 2(5x + 6 + 14x + 10) \\ &= 2(19x + 16) \\ &= 38x + 32 \end{aligned}$$

➤ CONCLUSION

The binomial $38x + 32$ represents the perimeter of rectangle DEFG.

13. IT'S A WRAP!

A. EXAMPLE OF APPROPRIATE REASONING

➤ RADIUS OF THE PIECE OF CANDY

Since all spheres are similar, the piece of candy as a whole is similar to the piece of caramel. Consequently, the ratio of their volumes is equal to the cube of the ratio of their radii (k).

$$\text{Volume of the candy} = k^3$$

$$\text{Volume of the caramel} = k^3$$

$$\text{Volume of the caramel} + \text{Volume of the chocolate} = k^3$$

$$\text{Volume of the caramel} = k^3$$

$$2 \times \text{Volume of the caramel} = k^3$$

$$\text{Volume of the caramel} = k^3$$

$$2 = k^3$$

$$\text{Radius of the candy} = k$$

$$\text{Radius of the caramel} = k$$

$$\text{Radius of the candy} = \sqrt[3]{2}$$

$$\frac{19.8 \text{ mm} + 2}{9.9 \text{ mm}} = 1.2599 \dots$$

$$\text{Radius of the candy} = 1.2599 \dots$$

$$\text{Radius of the candy} = 12.4732 \dots \text{ mm}$$

➤ AREA OF THE PIECE OF CANDY

$$\text{Area of the piece of candy} = 4\pi(12.4732 \dots \text{ mm})^2 = 1955.0907 \dots \text{ mm}^2$$

➤ AREA OF THE CANDY WRAPPER

$$\text{Area of the candy wrapper} = 5 \times \text{Area of the piece of candy} = 5 \times 1955.0907 \dots \text{ mm}^2 = 9775.4536 \dots \text{ mm}^2$$

➤ LENGTH OF THE CANDY WRAPPER

$$\text{Length of the candy wrapper} \times \text{Width of the candy wrapper} = \text{Area of the candy wrapper}$$

$$\text{Length of the candy wrapper} \times 90 \text{ mm} = 9775.4536 \dots \text{ mm}^2$$

$$\text{Length of the candy wrapper} = 108.6161 \dots \text{ mm}$$

➤ CONCLUSION

To the nearest tenth of a millimetre, the length of the candy wrapper is 108.6 mm.

Note: To determine the radius of the piece of candy, students could use the following procedure:

- Volume of the caramel = $\frac{4\pi(19.8 \text{ mm} + 2)^3}{3} = 4064.3789 \dots \text{ mm}^3$
- Volume of the candy = $2 \times \text{Volume of the caramel} = 8128.7578 \dots \text{ mm}^3$
- Radius of the piece of candy: $\frac{4\pi(\text{Radius of the candy})^3}{3} = 8128.7578 \dots \text{ mm}^3$
Radius of the candy = 12.4732... mm

14. THE MINIATURE BUILDING EXHIBIT

A. EXAMPLE OF APPROPRIATE REASONING

FIRST EXAMPLE

Suppose that the admission fee is \$5. Thus, $f(x) = 5x - 1000$.
 Number of visitors needed to break even:

$$\begin{array}{r} 0 = 5x - 1000 \\ 1000 = 5x \\ 200 = x \end{array}$$

} 200 visitors

The admission fee is doubled to \$10. Thus, $f(x) = 10x - 1000$.
 Number of visitors needed to break even:

$$\begin{array}{r} 0 = 10x - 1000 \\ 1000 = 10x \\ 100 = x \end{array}$$

} 100 visitors

The number of visitors needed decreases by 100.

SECOND EXAMPLE

Suppose that the admission fee is \$1.25. Thus, $f(x) = 1.25x - 1000$.
 Number of visitors needed to break even:

$$\begin{array}{r} 0 = 1.25x - 1000 \\ 1000 = 1.25x \\ 800 = x \end{array}$$

} 800 visitors

The admission fee is doubled to \$2.50. Thus, $f(x) = 2.50x - 1000$.
 Number of visitors needed to break even:

$$\begin{array}{r} 0 = 2.50x - 1000 \\ 1000 = 2.50x \\ 400 = x \end{array}$$

} 400 visitors

In the first two examples, the number of visitors needed to break even decreases by half when the admission fee is doubled. Let's see if this is true with a third example.

THIRD EXAMPLE

Suppose that the admission fee is \$2. Thus, $f(x) = 2x - 1000$.
 Number of visitors needed to break even:

$$\begin{array}{r} 0 = 2x - 1000 \\ 1000 = 2x \\ 500 = x \end{array}$$

} 500 visitors

The admission fee is doubled to \$4. Thus, $f(x) = 4x - 1000$.
 Number of visitors needed to break even:

$$\begin{array}{r} 0 = 4x - 1000 \\ 1000 = 4x \\ 250 = x \end{array}$$

} 250 visitors

The number of visitors needed to break even decreases by half.

CONJECTURE

After the admission fee is doubled, the number of visitors Gabriel needs to break even decreases by half.

15. SARAH'S TRIANGLES

A. EXAMPLE OF APPROPRIATE REASONING

USING EXAMPLES TO VERIFY THE CONCLUSION

Lengths of the Two Longest Sides	Length of the Shortest Side
5 cm and 4 cm	The Pythagorean theorem applies to any right triangle. (Length of the shortest side) ² + (4 cm) ² = (5 cm) ² (Length of the shortest side) ² + 16 cm ² = 25 cm ² (Length of the shortest side) ² = 9 cm ² The shortest side is 3 cm long. The length of the shortest side of the triangle is an integer.
10 cm and 9 cm	(Length of the shortest side) ² + (9 cm) ² = (10 cm) ² (Pythagorean theorem) (Length of the shortest side) ² + 81 cm ² = 100 cm ² (Length of the shortest side) ² = 19 cm ² The shortest side is 4.3588... cm. Since the length of the shortest side of the triangle is not an integer, this example contradicts Sarah's conclusion.

CONCLUSION

- Sarah's conclusion is true.
 Sarah's conclusion is false.

Explanation

There is at least one example that contradicts Sarah's conclusion.

16. A SQUARE TARGET

A. EXAMPLE OF APPROPRIATE REASONING

➤ MINIMUM LENGTH OF SEGMENT JC

y : length of segment JC, in cm

The probability that the dart landed in the white section is less than the probability that it landed in the black section.

Area of the white section $<$ Area of the black section
Area of the target $<$ Area of the target

Area of rectangle EFCG – Area of rectangle HJCL $<$ Area of rectangle HJCL

Area of square ABCD $<$ Area of square ABCD

$$m\overline{CG} \times m\overline{CF} - m\overline{CL} \times m\overline{CJ} < m\overline{CL} \times m\overline{CJ}$$
$$\frac{(m\overline{AB})^2}{28^2} < \frac{(m\overline{AB})^2}{28^2}$$

$$18(y+3) - 15y < \frac{15y}{28^2}$$

$$18(y+3) - 15y < 15y$$

$$18y + 54 - 15y < 15y$$

$$3y + 54 < 15y$$

$$54 < 12y$$

$$4.5 < y$$

The length of segment JC is greater than 4.5 cm.

➤ MAXIMUM LENGTH OF SEGMENT JC

$$m\overline{JC} + m\overline{FJ} \leq m\overline{CB}$$

$$m\overline{JC} + 3 \text{ cm} \leq 28 \text{ cm}$$

$$m\overline{JC} \leq 25 \text{ cm}$$

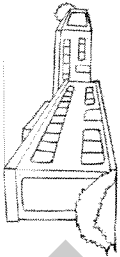
The length of segment JC is 25 cm at most.

➤ CONCLUSION

The possible lengths of segment JC are the values within the interval $]4.5, 25]$ cm or the real-number values of y such that $4.5 < y \leq 25$.

1. GOING TO SCHOOL

One morning, Tina and her brothers, Michael and Manoli, left the house to go to school. Tina was ready first, so she walked to school at a relaxed speed. Michael was ready second, so he jogged to school to avoid being late. Manoli was ready last. In order to get to school on time, he decided to ride his bike.



The table below shows their constant speeds (km/hour) and the duration of their trip (hours).

Sibling	Speed (km/hour)	Duration of Trip (hours)
Tina	5 km/h	0.6 hours
Michael	10 km/h	0.3 hours
Manoli	25 km/h	0.12 hours

The next morning, their father drove them to school.

At what constant speed must the father drive to get to school in exactly five minutes?

2. COMMON FACTOR

Consider the following algebraic expressions:

Algebraic expressions	Simplified monomials
$16x^3y^4 + 3x^3y^4 - 4x^3y^4$	
$\sqrt{25x^8y^6}$	
$(4x^{-3}y^2)(5x^{10}y)$	
$\frac{20x^{-3}y^4}{4x^{-8}y^2}$	

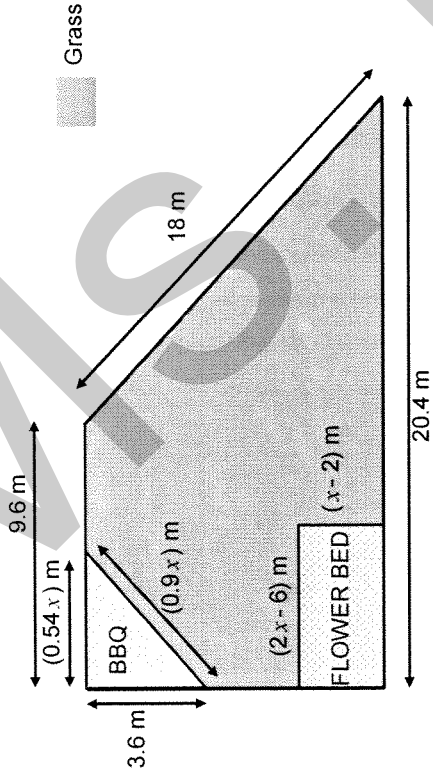
Find the numerical value of the greatest common factor of all the expressions provided in the table above, considering that:

- x is 4
- y is 3

What is the numerical value of the greatest common factor given the conditions that were specified?

3. LAWN CARE

David wants to fertilize the grass of his backyard with a new ecological brand of fertilizer that costs $\$0.80/\text{m}^2$. His backyard is in the shape of a right trapezoid as shown on the illustration below. The figure is not drawn to scale.



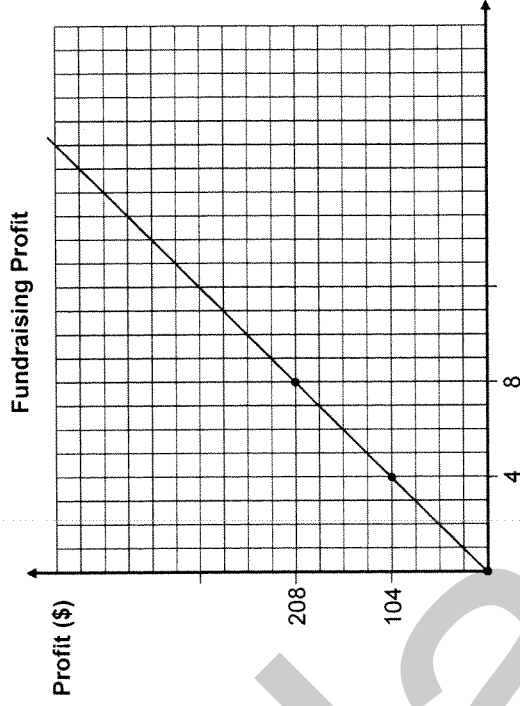
How much will it cost David to fertilize the grass in his backyard?

4. FUNDRAISING

In a high school, two groups of secondary students decide to raise money to help buy mosquito nets to send to Africa.

Group A invests money to buy pieces of jewellery which they will resell for profit. Each piece will be sold for the same price. They know that if they sell 85 pieces, they will make a profit of $\$480$. If they sell 120 pieces, they will make a profit of $\$760$. At the end of the campaign, they sold 90 pieces of jewellery.

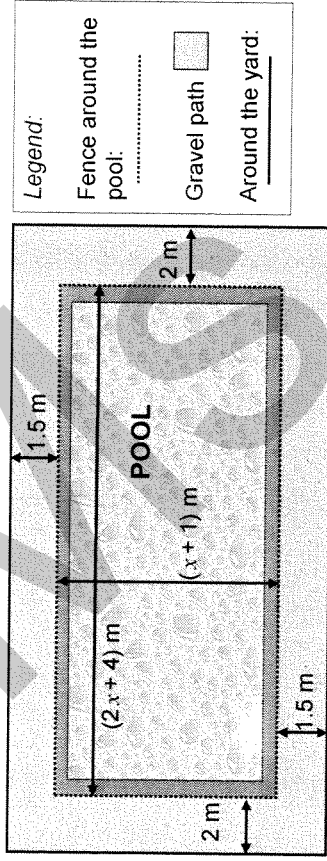
Group B wants to collect used bicycles and sell them. The following graph shows the profit according to the number of bicycles sold.



Considering that Group B earned a higher profit than Group A, what was the minimum number of bicycles sold?

5. MAKING THE POOL SAFE

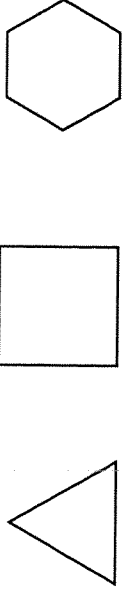
Raphael has a beautiful pool in his backyard. He recently put a path made of gravel around the pool as shown in the picture below. The area of the gravel path is 48 m^2 . The figure is not drawn to scale.



As a safety precaution, Raphael would like to put a fence around the pool. The fence costs \$45/m (taxes included).

How much will Raphael have to pay to put a fence around his pool?

6. THE AREA OF REGULAR POLYGONS

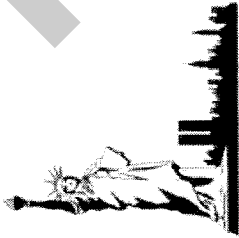


Consider a group of regular polygons whose perimeters are equal.

Formulate a conjecture comparing the number of sides of a regular polygon and its area.

Show or explain how you found your answer.

7. NEW YORK CITY



Luka is planning a trip to New York City. In order to find the best price for his trip, he researches different options offered online. In doing his research, he finds two different packages. The following is the information that he found:

Option 1: Travel Ticks

Transportation (round trip): \$250

Hotel:

Number of nights	Total cost of hotel
2	250
4	500
6	750

Luka is not sure how many days he will be staying in New York City.

Prepare a summary that outlines which option is the least expensive, depending on the number of nights that he will be staying in New York City.

Option 2: Dream Tours

Transportation (round trip): \$150

Hotel: membership fee of \$25 and \$150 per night

8. CHARITY REPORT

Jenny, the Director of the "Soleil de Vie" charity, has asked you to help prepare a report for the Board of Directors summarizing the donations they received in 2008 and 2009.



The following table gives the number of donations made in 2008:

Amount of donation (\$)	Frequency
[0, 250[5249
[250, 500[2500
[500, 750[300
[750, 1000[500
[1000, 1250[0
[1250, 1500[400

Amount of donation (\$)	Frequency
[1500, 1750[0
[1750, 2000[50
[2000, 2250[0
[2250, 2500[0
[2500, 2750[0
[2750, 3000[1

In 2009, they received 10 000 donations. The box-and-whisker plot below summarizes the data according to the amount of each donation (in dollars).



The Director would like to make the following statements to the Board of Directors comparing the donations from 2008 and 2009.

"The mean donation in 2008 was \$300, but in 2009, the mean donation was \$100."

"In 2009, we had more people who donated between \$300 and \$5000 than the number of people who donated between \$5 and \$75."

"For both years, however, our maximum donation was no more than \$3000."

Explain why these statements are not correct and rewrite them so that Jenny can present them correctly.

1. GOING TO SCHOOL

<p>Description of the task: Use a rational function to determine the speed needed to get to school in five (5) minutes.</p>	<p>Theme: Arithmetic & Algebra Concepts and processes:</p> <ul style="list-style-type: none"> • Rational function of the form $y = \frac{k}{x}$ • Representing and interpreting the inverse function • Observing patterns • Finding the rules
--	--

A. Example of an appropriate solution

Rule of the function representing the duration of the trip according to the speed:

x: Speed (km/h)

y: Duration of Trip (hours)

The product of the variables is constant, therefore it is a rational function of the form $y = \frac{k}{x}$

$$0.6 = \frac{k}{5}$$

$$k = 3$$

The distance from the house to the school is 3 km.

The rule of the function is therefore: $y = \frac{3}{x}$

Check if this rule is valid for the other siblings' values:

Michael (x = 10): Manoli (x = 25):

$$y = \frac{3}{10}$$

$$y = \frac{3}{25}$$

$$y = 0.3 \text{ hours}$$

$$y = 0.12 \text{ hours}$$

Speed for a 5 minute journey:

$$5 \text{ minutes} = \frac{1}{12} \text{ hour}$$

$$y = \frac{3}{\frac{1}{12}}$$

$$x = 36 \text{ km/hour}$$

Conclusion:

The father must drive at a speed of 36 km/hour to get to school in 5 minutes.

2. COMMON FACTOR

<p>Description of the task: Simplifying algebraic expressions to find a common factor and evaluating the resulting monomial.</p>	<p>Theme: Arithmetic & Algebra Concepts and processes</p> <ul style="list-style-type: none"> • manipulating numerical and algebraic expressions • performing context-related calculations with integral exponents (rational base) and fractional exponents • finding the common factor
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A. Example of an appropriate solution

The following expressions are simplified as follows:

$16x^3y^9 + 3x^2y^4 - 4x^3y^4$	$15x^3y^4$
$\sqrt{25x^8y^6}$	$5x^4y^3$
$(4x^{-3}y^2)(5x^{10}y)$	$20x^7y^3$
$\frac{20x^{-3}y^4}{4x^{-8}y^2}$	$5x^5y^2$

The greatest common factor that can be factored from these four resulting monomials is: $5x^3y^2$

Given that $x = 4$.

Given that $y = 3$.

The numerical value of the greatest common factor is:

$$5x^3y^2 = 5(4)^3(3)^2 = 2880$$

3. LAWN CARE

<p>Description of the task: Find the area of grass to calculate the cost of fertilizer.</p>	<p>Theme: Geometry & Algebra Concepts and processes</p> <ul style="list-style-type: none"> • Multiplying/dividing algebraic expression • Pythagorean theorem • Area of decomposable solid
<p>A. Example of an appropriate solution</p> <p>Step 1: Height of trapezoid Find the base of the right-angled triangle: 20.4 m - 9.6 m = 10.8 m Apply Pythagorean Theorem to find height of the trapezoid. $a^2 + b^2 = c^2$ $(10.8)^2 + h^2 = 182$ $116.64 + h^2 = 324$ $h^2 = 324 - 116.64$ $h^2 = 207.36$ $h = 14.4$ m</p>	
<p>Step 2: Area of trapezoid $A = \frac{(B + b) \times h}{2}$ $A = \frac{(20.4 + 9.6) \times 14.4}{2}$ $A = 216 \text{ m}^2$</p>	
<p>Step 3: Solving for the value of x (use information from BBQ): Apply Pythagorean Theorem to find the value of x. $a^2 + b^2 = c^2$ $(0.54x)^2 + (3.6)^2 = (0.9x)^2$ $0.2916x^2 + 12.96 = 0.81x^2$ $0.5184x^2 = 12.96$ $x^2 = 25$ $x = 5$</p>	
<p>Step 4: Area of BBQ (Area of triangle) $A = \frac{b \times h}{2}$ $A = \frac{(0.54 \times 5) \times 3.6}{2}$ $A = \frac{9.72}{2}$ $A = 4.86 \text{ m}^2$</p> <p>Step 5: Area of Flower Bed (Area of rectangle) $A = \ell \times w$ $A = (2(5) - 6)(5 - 2)$ $A = (4)(3)$ $A = 12 \text{ m}^2$</p> <p>Step 6: Area of grass Area of trapezoid - Area BBQ - Area Flower Bed $216 - 12 - 4.86 = 199.14 \text{ m}^2$</p> <p>Step 7: Cost to fertilize grass $199.14 \text{ m}^2 \times \frac{\\$0.80}{\text{m}^2} = \\$159.31$ Note: Accept \$159.32 because some students will argue that businesses always round up.</p>	

4. FUNDRAISING

<p>Description of the task: Determine number of bicycles to sell in order to make a greater profit than the girls.</p>	<p>Theme: Algebra Concepts and processes</p> <ul style="list-style-type: none"> • Polynomial function degree 1 <ul style="list-style-type: none"> ◦ partial variation ◦ direct variation • System of first degree equation
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A. Example of an appropriate solution

Step 1: Determine the amount of money raised by Group A

Let x_1 be the number of pieces of jewellery sold and y_1 be the net profit from the jewellery sales

We have points given: (85,480) and (120,760)

Use the two data points to calculate the rate of change: $a = \frac{\Delta y}{\Delta x} = \frac{760 - 480}{120 - 85} = \frac{280}{35} = 8$

Since each piece of jewellery is sold at a fixed price the relation can be expressed as a linear function in the for $y = ax + b$

Using the slope and one other point, solve for b.

$$y = ax + b$$

$$760 = (8)(120) + b$$

$$760 = 960 + b$$

$$760 - 960 = b$$

$$-200 = b$$

Therefore the rule of this relation is: $y = 8x - 200$

Use the function rule to determine money raised when 90 pieces of jewellery are sold:

$$y = 8x - 200$$

$$y = (8)(90) - 200$$

$$y = 720 - 200$$

$$y = \$520$$

Step 2: Minimum number of bicycles sold

Let x_2 be the number of bicycles sold and y_2 be the net profit from bicycle sales

From the graph we have two points: (0,0) and (4,104)

Use the two data points to calculate the rate of change: $a = \frac{\Delta y}{\Delta x} = \frac{104 - 0}{4 - 0} = \frac{104}{4} = 26$

The graph illustrates a direct linear function therefore the rule can be expressed as $y = ax$

Therefore the rule of this relation is: $y = 26x$

Use the function rule to determine the number of bicycles sold if the money raised exceeds \$520:

$$520 = 26x$$

$$\frac{520}{26} = x$$

$$20 = x$$

Group B must sell more than 20 bicycles.

OR Group B must sell at least 21 bicycles

5. MAKING THE POOL SAFE

Description of the task: Determining the cost of gating a pool	Theme: Geometry & Algebra Concepts and processes <ul style="list-style-type: none"> • Area of figures that can be split into sectors • Direct variation • Multiplying/dividing polynomials
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A. Example of an appropriate solution

Algebraic area of the pool

$$(2x + 4)(x + 1) = 2x^2 + 6x + 4$$

Dimension of exterior rectangle

$$L: 2x + 4 + 4 = (2x + 8)$$

$$W: x + 1 + 3 = (x + 4)$$

Total area with gravel path

$$(2x + 8)(x + 4) = 2x^2 + 16x + 32$$

Area of gravel

$$(2x^2 + 16x + 32) - (2x^2 + 6x + 4) = 10x + 28$$

Value of x

$$10x + 28 = 48$$

$$x = 2$$

Perimeter of the pool

$$2(2x + 4) + 2(x + 1)$$

$$= 6x + 10$$

$$= 6(2) + 10$$

$$= 22 \text{ m}$$

Cost of the fence

$$22 \times 45 = \$990$$

6. THE AREA OF REGULAR POLYGONS

Description of the task:

Make a conjecture comparing the number of sides of a regular polygon and its area

Theme: Geometry

Concepts and processes

- Pythagorean theorem
- Area of regular polygons
- Perimeter of regular polygons

A. Example of an appropriate solution

If the perimeter is set to 24 units then:

For the triangle:

Let x be the length of the sides:

$$P = 3x$$

$$24 = 3x$$

$$\frac{24}{3} = \frac{x}{1}$$

$$8 = x$$

$$c^2 = a^2 + b^2$$

$$b^2 = 8^2 - 4^2$$

$$b^2 = 64 - 16$$

$$b^2 = \sqrt{48}$$

$$b \approx 6.93 \text{ units}$$

$$A = \frac{\text{base} \times \text{height}}{2}$$

$$A = \frac{8 \times 6.93}{2}$$

$$A \approx 27.71 \text{ units}^2$$

For the square or diamond:

Let x be the length of the sides:

$$P = 4x$$

$$24 = 4x$$

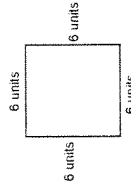
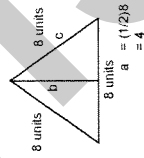
$$\frac{24}{4} = \frac{x}{1}$$

$$6 = x$$

$$A = \text{base} \times \text{height}$$

$$A = 6 \times 6$$

$$A = 36 \text{ units}^2$$



For the hexagon:

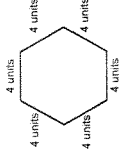
Let x be the length of the sides:

$$P = 6x$$

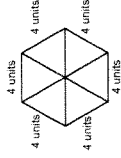
$$24 = 6x$$

$$\frac{24}{6} = \frac{x}{1}$$

$$4 = x$$



A hexagon is made up of six (6) equilateral triangles:



There the apothem can be calculated using the Pythagorean Theorem:

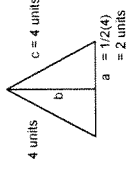
$$c^2 = a^2 + b^2$$

$$b^2 = 4^2 - 2^2$$

$$b^2 = 16 - 4$$

$$b^2 = \sqrt{12}$$

$$b \approx 3.46 \text{ units}$$



$$A_{\text{triangle}} = \frac{\text{base} \times \text{height}}{2}$$

$$A_{\text{triangle}} = \frac{4 \times 3.46}{2}$$

$$A_{\text{triangle}} \approx 6.92 \text{ units}^2$$

$$A_{\text{Hex}} = A_{\text{triangle}} \times 6$$

$$A_{\text{Hex}} \approx 6.92 \times 6$$

$$A_{\text{Hex}} \approx 41.52$$

Summary:

Shape	Perimeter	Number of sides	Area
Triangle	24 units	3	27.71 units ²
Square	24 units	4	36 units ²
Hexagon	24 units	6	41.52 units ²

Conjecture: For regular polygons of equal perimeter, as you increase the number of sides the area will also increase.

7. NEW YORK CITY

Description of the task: Comparing two linear functions	Theme: Arithmetic & Algebra Concepts and processes <ul style="list-style-type: none"> comparing situations
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A. Example of an appropriate solution

Step 1: Find the rule for option 1

Let x be the number of nights in NYC
Let y be the total cost of the trip

Using two points from the table of values, (6, 750) and (4, 500), calculate the slope:

$$a = \frac{\Delta y}{\Delta x}$$

$$y = 125x + 250$$

$$= \frac{750 - 500}{6 - 4}$$

$$= \frac{250}{2}$$

$$= 125$$

Use the slope and one point to solve for b

$$y = ax + b$$

$$y = 125x + b$$

$$750 = 125(6) + b$$

$$750 - 750 = b$$

$$b = 0$$

Add \$250 for transportation and option 1 can be represented by the equation:
 $y = 125x + 250$

Step 2: Find the rule for option 2

The transportation and membership fees are the fixed costs(b).

The \$150 per night is the rate therefore option 2 can be represented by the equation $y = 150x + 175$

Number of nights	Total cost for option 1 (\$)	Total cost for option 2 (\$)
1	375	325
2	500	475
3	625	625
4	750	775
5	875	925

If Luka stays in NYC for 1 or 2 nights, option 2 is cheaper than option 1

If Luka stays in NYC for 3 nights, both options are equal since the total cost is the same.

If Luka stays in NYC for 4 or more nights, option 1 is cheaper than option 2.

8. CHARITY REPORT

A. Example of an appropriate solution

The following can be deduced from the information provided:

2008:

- Mean was \$318.29.
- The same number of people donated between \$300 and \$5000 as donated between \$5 and \$75.
- The maximum donation was in the range of [2750, 3000], but the exact maximum value is unknown.

2009:

- We do not have enough information from just the box and whisker chart to determine the mean.
- The \$100 indicates the median donation.
- The box and whisker plot is divided in quartiles, therefore the same number of people donated between \$5 and \$75 as the number of people that donated between \$300 and \$5000.
- The maximum donation was \$5000.

The following are examples of correct statements:

- The mean donation from 2008 is \$318.29. We know that the median donation from 2008 is less than \$250. We do not have enough information from the chart given to find the mean donation from 2009, but we know that the median donation in 2009 was \$100. Therefore, we cannot compare means and medians between the two years.
- The same number of people made donations between \$300 and \$5000 as made donations from \$5 to \$75.
- In 2008, the maximum donation was in the range [2750, 3000]. In 2009, the maximum donation was \$5000.

Ms. Nassif

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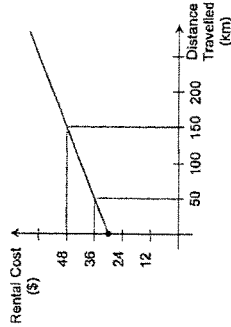
1. GETTING AROUND TOWN

Leo wants to rent a scooter to go sightseeing in Cairo, Egypt. He has rates from two rental companies. Here is the information he was given.

COST OF RENTING A SCOOTER FROM COMPANY A
IN RELATION TO THE DISTANCE TRAVELLED

Distance Travelled (km)	Rental Cost (\$)
25	28.50
40	31.20
50	33.00

COST OF RENTING A SCOOTER FROM COMPANY B
IN RELATION TO THE DISTANCE TRAVELLED



Leo wants to pay as little as possible to rent a scooter. He decides to rent a scooter from Company B. Has Leo made the right choice? Explain your answer.

2. KNOW YOUR DESTINATION

Karim is a tour guide in Egypt. The travel agency he works for is doing a statistical study on tourists' knowledge of Egypt when they first arrive in the country.

The groups Karim guides are usually made up of 20 tourists. Upon their arrival in Egypt, Karim writes each person's name on a piece of paper. Then he picks 5 names at random. He asks the people whose names have been drawn to answer a few questions.

Here is one of the questions, along with the answers given by tourists in the last 6 groups that Karim has been a tour guide for.

"How many Egyptian cities can you name?"

NUMBER OF CITIES IDENTIFIED BY
TOURISTS ARRIVING IN EGYPT

Number of Cities Identified	Number of Tourists
2	1
3	2
4	7
5	8
6	5
7	4
8	1
9	2
Total:	30

Karim must now submit a report giving the results of his statistical study.

This report must:

- indicate the type of statistical study he conducted
- indicate the target population
- include a box-and-whisker plot showing the dispersion of the data he collected

3. GAS CONSUMPTION

The Fletcher and Goodwin families are travelling in Egypt. Each family rents a car to get around in. When they pick up their cars, both gas tanks are full.

CAR RENTED BY THE FLETCHERS

The function f described below represents the amount of fuel in the gas tank of the car rented by the Fletchers in relation to the distance travelled since the tank was filled.

$$f(x) = -0.1x + 60$$

x : distance travelled, in kilometres, since the tank was filled

$f(x)$: amount of fuel in the gas tank, in litres

The domain of function f is $[0, 600]$.

CAR RENTED BY THE GOODWINS

The Goodwins rented a different type of car.

The maximum capacity of the gas tank in the Goodwins' rental car is 10 litres more than the maximum capacity of the gas tank in the Fletchers' rental car.

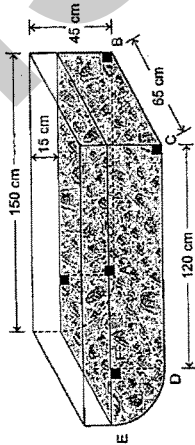
To travel 100 km, the Goodwins' rental car uses 4 more litres of gas than the Fletchers' rental car.

Draw a graph comparing the amount of gas in the tank of each car in relation to the distance travelled, in kilometres, since the tank was filled.

4. CLEOPATRA'S GOATS

Legend has it that Cleopatra, Queen of Ancient Egypt, took baths in goats' milk.

Imagine that her bathtub looks like the one in the diagram below. In this diagram, quadrilateral ABCD is a rectangle. Arc ED is an arc of a circle with centre F.



Before Cleopatra steps into the bathtub, it is filled with milk to a point 15 cm from the top.

Assume that Cleopatra takes a goats' milk bath 4 times a month.

One goat produces approximately 572 litres of milk a year.

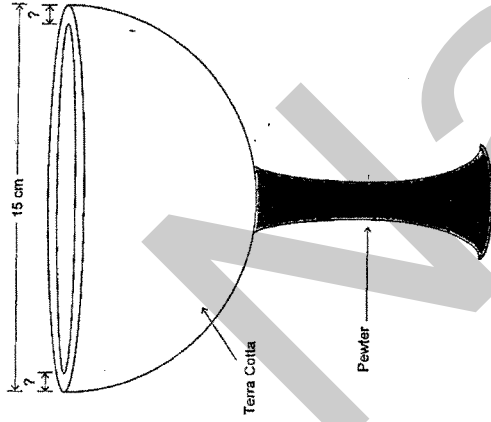
How many goats would Cleopatra have to have in her herd to produce the milk used in her baths in one year?

5. PEWTER STEMWARE

In Ancient Greece, bowls, vases and goblets were usually made out of terra cotta, or baked clay.

A terra cotta goblet with a pewter stem was discovered during an archaeological dig. The terra cotta goblet is in the shape of a half sphere of uniform thickness.

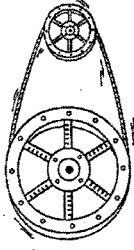
The exterior diameter of the goblet is 15 cm.



To make a reproduction of this goblet, 170 cm³ of terra cotta was used. This reproduction will be used in history class.

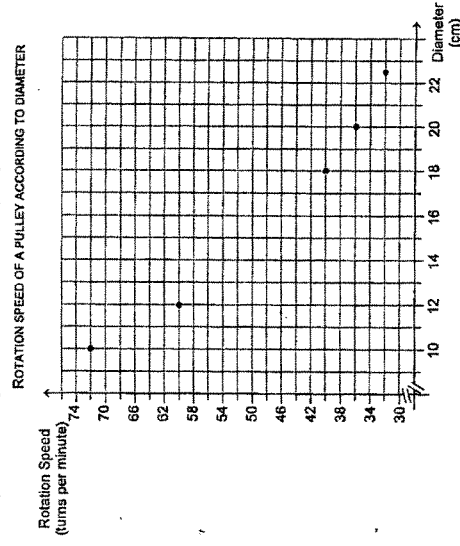
To the nearest hundredth of a centimetre, how thick is the terra cotta goblet?

6. PULLEYS



On a factory assembly line, two pulleys of different diameters are attached by a belt.

The rotation speed of a pulley depends on its diameter. The following graph shows the rotation speed according to diameter for 5 different pulleys.



What is the diameter of the pulley with a rotation speed of 90 turns per minute?