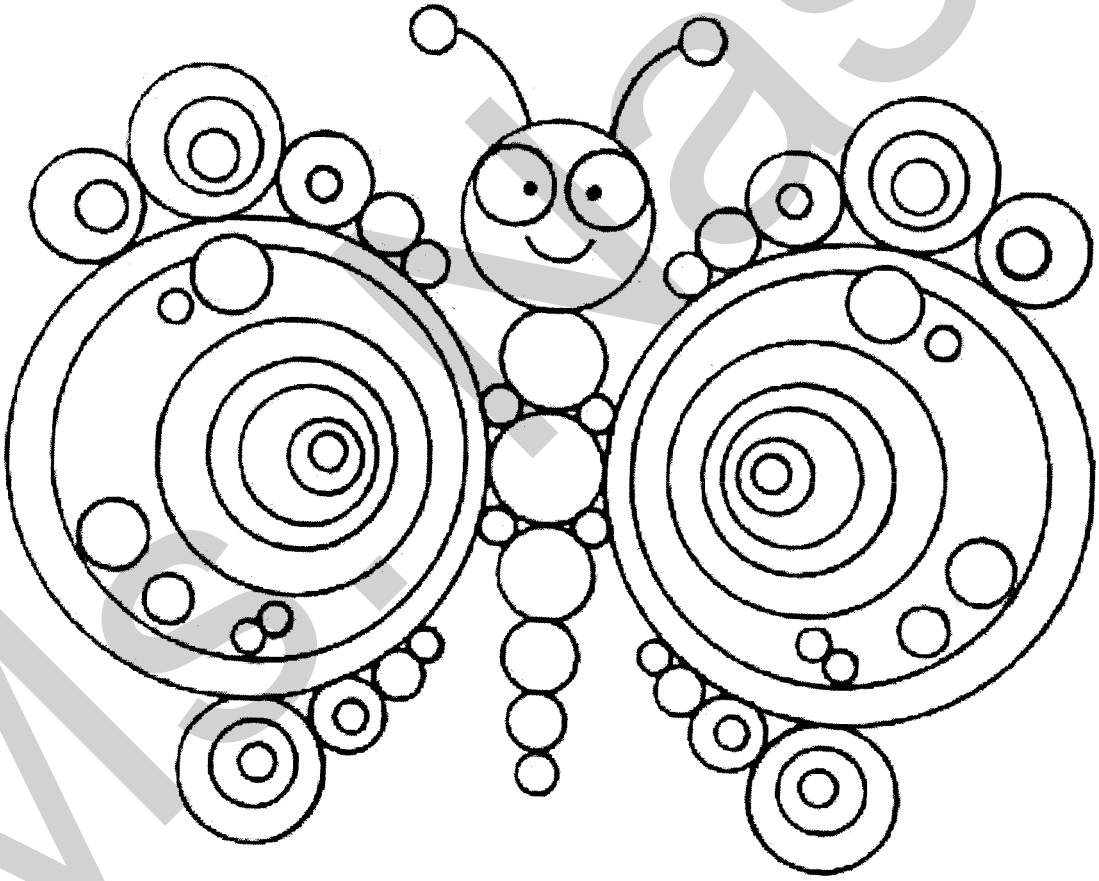
Solution Key

Metric Relations & Similarity Booklet Secondary 3

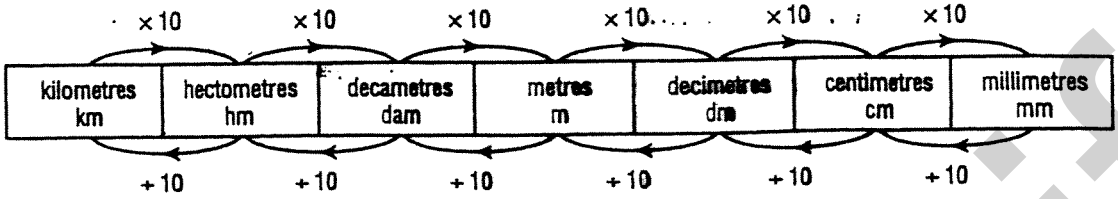


Ms. Nassif

SOLUTIONS

CHANGING UNITS OF LENGTH

The table below shows how to convert one unit of length to another unit of length.



- To convert from one unit of length to the unit of length immediately inferior to it, we multiply by 10.
- To convert from one unit of length to the unit of length immediately superior to it, we divide by 10.

Ex.: 2.4 m = 24 dm = 240 cm = 2400 mm
34.5 m = 3.45 dam = 0.345 hm = 0.0345 km

King Henry Died, Mother Didn't Care Much

km hm dam m dm cm mm

1. *The basic unit for measuring a length in the metric system is the metre (m).*

a) Fill in the blanks.

1. 1 m = 10 dm
2. 1 m = 100 cm
3. 1 m = 1000 mm
4. 1 dam = 10 m
5. 1 hm = 100 m
6. 1 km = 1000 m

2. Convert the following measures to centimetres.

a) 2.4 m 240 b) 0.4 dm 4 c) 0.0048 dam 4.8 d) 24 mm 2.4
e) 0.08 km 8000 f) 0.0025 hm 25 g) 12.7 dm 127 h) 13.47 mm 1.347

3. Convert the following measures to metres.

a) 3 km 3000 b) 5.2 dam 52 c) 478 dm 47.8 d) 72.4 cm 0.724
e) 18 mm 0.018 f) 3.7 hm 370 g) 0.08 km 80 h) 432.8 mm 0.4328

4. Convert the following measures to kilometres.

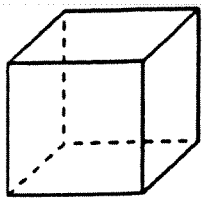
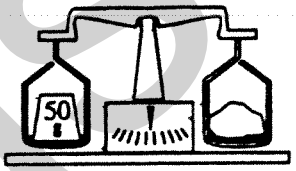
a) 2485 m 2.485 b) 42.8 dam 0.428 c) 3000 cm 0.03 d) 423.8 dm 0.04238
e) 2 400 000 mm 2.4 f) 4 m 0.004 g) 718.2 hm 71.82 h) 4.2 m 0.0042

5. Convert the following measures to the units of length indicated.

a) 748 mm = 0.748 m b) 2480 mm = 248 cm c) 72.5 dm = 0.725 dam
6.18 dam = 6180 mm e) 2450 m = 2.450 km f) 3.4 hm = 3400 dm
g) 7.6 km = 7600 m h) 32.4 cm = 0.324 m i) 624 mm = 0.0624 dam
j) 340 000 cm = 3.4 km k) 218 dm = 0.0218 km l) 62.4 m = 62400 mm
m) 2.432 hm = 24320 cm n) 318.2 dam = 3.182 km o) 1.24 km = 1240 m

6.2 VOLUME, CAPACITY & MASS

Volume and **capacity** are similar. We usually measure solids in cubic units and liquids using units of capacity. Because water changes state as it is frozen or heated, when it is in a liquid form we would measure it in units of capacity but as it freezes and changes into a solid we would then measure it in units of volume. To find the **mass** of any object we use a double-arm balance. (At 4°C, one mL of water has a mass of one gram and it has a volume of one centimetre cubed.)

VOLUME	CAPACITY	MASS
Volume is the amount of space that an object occupies. The most common units we use to measure volume are: m^3, cm^3, & dm^3.  $1000\ cm^3 = 1\ dm^3$ $1000\ dm^3 = 1\ m^3$	**Capacity** is the measure of the amount that an object will hold. The most common units are: mL, L, & kL.  $1000\ mL = 1\ L$ $1000\ L = 1\ kL$	**Mass** is the amount of matter contained in an object and this remains constant. The most common units are: mg, g, kg, & t.  $1000\ mg = 1\ g$ $1000\ g = 1\ kg$ $1000\ kg = 1\ t$

A. Convert the following to the units indicated.

1. 6 g = 0.006 kg
2. 5 m^3 = 5000 dm^3
3. 34 L = 34000 mL
4. 750 kL = 750 000 L
5. ~~9 t = _____ kg~~
6. 4.4 dm^3 = 4400 cm^3
7. 0.4 kL = 400000 mL
8. ~~0.3 t = _____ kg~~
9. 6000 mg = 6 g
10. 0.2 L = 200 mL
11. 17.8 kL= 17800 L
12. 0.007 kg = 7 g
13. 30 000 cm^3 = 30 dm^3
14. 2400 g = 2.4 kg
15. 1 mL = 0.001 L
16. 8577 g = 8.577 kg
17. 2.8 kg = 2800 g
18. 1.7 kg = 1700000 mg
19. 448 kg = 448000000 mg
20. 43 L = 43000 mL
21. 0.055 kg = 55 g
22. 3578 g = 3.578 kg
23. 6.5 L = 6500 mL
24. 0.007 m^3 = 7 dm^3
25. 4.5 m^3 = 4500 dm^3
26. 7 g = 7000 mg
27. 0.007 mg = 0.000007 g
28. 0.076 kL = 76 L

NAME Solution Key GROUP _____

A. Convert each of the following linear measurements to the units indicated.

1. 42.3 m = 4230 cm
2. 18 km = 18 000 000 mm
3. 46 cm = 460 mm
4. 5 dm = 0.05 dam
5. 3.28 cm = 32.8 mm
6. 24 m = 2400 cm
7. 45 cm = 0.0045 hm
8. 83 hm = 83 000 dm
9. 18 dm = 0.0018 km
10. 86 m = 860 dm
11. 12 km = 12 000 000 mm
12. 3 cm = 0.003 dam
13. 18 km = 18 000 m
14. 327 mm = 0.327 m

K H Da M D C M x

B. Convert each of the following area measurements to the units indicated.

1. 23 m^2 = 230 000 cm^2
2. 9 m^2 = X ha
3. 35 cm^2 = 3500 mm^2
4. 6 m^2 = 600 dm^2
5. 0.1 cm^2 = 10 mm^2
6. 5 m^2 = 50000 cm^2
7. 65 m^2 = 6500000 mm^2
8. 6 dm^2 = 600 cm^2
9. 16 ha = X m^2
10. 35 a = X ha
11. 25 mm^2 = 0.25 cm^2
12. 4 m^2 = X a
13. 1 m^2 = 10000 cm^2
14. 13 km^2 = 13000000000 cm^2

C. Convert each of the following volume measurements to the units indicated.

1. 6 cm^3 = 6000 mm^3
2. 200 m^3 = 0.000002 km^3
3. 200 m^3 = 200 000 000 cm^3
4. 200 m^3 = 0.0002 hm^3
5. 200 m^3 = 200 000 000 000 mm^3
6. 42 m^3 = 0.042 dam^3
7. 1 km^3 = 1 000 000 000 m^3
8. 46 hm^3 = 0.046 km^3
9. 4 dam^3 = 4000 m^3
10. 0.2 m^3 = 200 000 cm^3
11. 42.3 m^3 = 42 300 000 cm^3
12. 3 km^3 = 3 000 000 000 m^3
13. 945 cm^3 = 945 000 mm^3
14. 82 hm^3 = 82 000 dam^3

D. Convert each to the indicated linear unit before adding or subtracting.

1. 6.4 cm + 7 cm = 13.4 cm
2. 13.5 m − 3.62 m = 9.88 m
3. 80 cm + 1.2 m = 200 cm
4. 0.09 km + 20 dam = 290 m
5. 4 m − 19 cm = 381 cm
6. 2 dm + 6 cm = 260 mm
7. 13 m − 0.6 dam = 7 m
8. 2.9 m + 1.6 cm = 2.916 m
9. 3 m + 0.03 km = 33 m
10. 90 mm + 12 cm = 21 cm
11. 12 dam − 20 m = 100 m
12. 105 cm + 3682 mm = 4.732
13. 1.6 hm + 200 m = [illegible] km
14. 216 dm − 1839 mm = 177.61 cm
15. 65 m + 83 cm = 65.83 m
16. 41 cm + 16 mm = 426 mm

E. Convert each to the indicated area unit before adding or subtracting.

1. 6.4 m^2 − 2.9 cm^2 = 63997.1 cm^2
2. 8 m^2 − 400 cm^2 = 7.96 m^2
3. 6 a − 2.3 m^2 = X m^2
4. 0.3 ha − 410 m^2 = X m^2
5. 9 km^2 − 14 ha = X ha
6. 16 cm^2 + 4 m^2 = 40016 cm^2
7. 88 cm^2 − 920 mm^2 = 78.8 cm^2
8. 16.2 dm^2 − 145 cm^2 = 1475 cm^2
9. 192 cm^2 + 8 dm^2 = 0.0092 m^2
10. 14 cm^2 + 3 m^2 = 3.0014 m^2
11. 18 ha − 120 a = X m^2
12. 1400 m^2 + 1850 m^2 = X ha
13. 0.8 m^2 + 780 dm^2 = 86000 cm^2
14. 0.1 ha + 14 a = X m^2
15. 121 cm^2 − 0.02 dm^2 = 119 cm^2
16. 0.07 a + 4632 dm^2 = X m^2

F. Convert each to the indicated volume unit before adding or subtracting.

1. 7.2 dm^3 + 5 dm^3 = 0.012 m^3
2. 7 dm^3 − 300 cm^3 = 6700 cm^3
3. 5978 mm^3 + 2 cm^3 = 7978 mm^3
4. 1 m^3 + 5 dam^3 = 5001000 dm^3
5. 36 m^3 − 36 dm^3 = 35964000 cm^3
6. 1000 km^3 + 1 km^3 = 1001 km^3
7. 3.4 m^3 − 0.002 dm^3 = 3399998 cm^3
8. 4000 mm^3 − 1 cm^3 = 3 cm^3
9. 83 hm^3 − 0.009 km^3 = 74000000 m^3
10. 48 hm^3 + 0.005 km^3 = 53000000 m^3
11. 6.2 dm^3 + 5 cm^3 = 6205 cm^3
12. 8 dm^3 − 400 cm^3 = 7600 cm^3
13. 1 cm^3 − 1 mm^3 = 999 mm^3
14. 5 m^3 + 4.2 m^3 = 0.0092 dam^3
15. 0.001 cm^3 + 1 mm^3 = 2 mm^3
16. 2000 m^3 − 2 dam^3 = [illegible] km^3

Name: _____

Group: _____ Date: _____

Objective 5.2 To convert a measure of volume from one unit to another

5

Remember: 1 cubic decimetre = 1 litre KH Da M D CM

1 litre ≙ 1 kilogram of water

1. Convert each volume measure into the indicated unit.

a) 1 cm^3 = 1000 mm^3

b) 1 dm^3 = 1000 cm^3

c) 1 m^3 = 1000 dm^3

d) 1 dam^3 = 1000 m^3

e) 1 hm^3 = 1000 dam^3

f) 1 km^3 = 1000 hm^3

g) 20 dam^3 = 20000 m^3

h) 500 cm^3 = 500000 mm^3

i) 100 hm^3 = 100 000 dam^3

j) 40 m^3 = 40 000 dm^3

k) 90 km^3 = 90 000 hm^3

l) 200 dm^3 = 200 000 cm^3

2. Express each volume in cubic centimetres. cm^3

a) 242 dm^3 = 242 000

b) 1 m^3 = 1 000 000

c) 74 mm^3 = 0.074

d) 3.25 dam^3 = 3 250 000 000

e) 1 mm^3 = 0.001

f) 0.25 dm^3 = 250

3. Express each volume in cubic metres. m^3

a) 100 000 cm^3 = 0.1

b) 1000 dm^3 = 1.0 or 1

c) 5650 dam^3 = 5 650 000

d) 100 mm^3 = 0.000 000 1

e) 0.632 km^3 = 632 000 000

f) 42 dam^3 = 42 000

4. Convert each volume measure to the indicated unit.

a) 3 mm^3 = 0.000 003 dm^3

b) 0.3765 m^3 = 376 500 cm^3

c) 2.35 km^3 = 2350 hm^3

d) 700 dam^3 = 700 000 m^3

e) 0.1 m^3 = 0.000 000 1 hm^3

f) 2.135 hm^3 = 2 135 000 m^3

5. Convert each volume into litres.

a) 1 m^3 = 1000 L

b) 562 dm^3 = 562 L

c) 3000 dm^3 = 3000 L

d) 0.3 m^3 = 300 L

e) 3200 mm^3 = .0032 L

f) 600 cm^3 = 0.6 L

PROPORTIONALITY

SHORT

Solve for the value of x in each equation.

1. $\frac{12}{x}=\frac{24}{x+5}$

$12(x+5)=24x$
$60=12x$
$x=5$

2. $\frac{x-3}{5}=\frac{x+5}{15}$

$15x-45=5x+25$
$10x=70$
$x=7$

3. $\frac{2}{3x-4}=\frac{1}{4}$

$8=3x-4$
$12=3x$
$x=4$

4. $\frac{5x}{x+1}=4$

$4x+4=5x$
$4=1x$

5. $\frac{3}{5-3x}=\frac{1}{2}$

$6=5-3x$
$1=-3x$
$x=-1/3$

6. $\frac{3}{x}=\frac{2}{5-x}$

$15-3x=2x$
$15=5x$
$x=3$

7. $\frac{2}{x}=\frac{5}{3x-1}$

$6x-2=5x$
$-2=-1x$
$x=2$

8. $\frac{12}{2-x}=\frac{15}{7+x}$

$84+12x=30-15x$
$54=-27x$
$x=-2$

9) $\frac{3x+1}{x-1}=5$

$5x-5=3x+1$
$2x=6$
$x=3$

10) $\frac{x-6}{3}=\frac{-2x-2}{15}$

$15x-90=-6x-6$
$21x=84$
$x=4$

11) $\frac{6}{3x-1}=\frac{3}{4}$

$24=3(3x-1)$
$24=9x-3$
$27=9x$
$x=3$

12) $\frac{5}{x}=\frac{7}{x-4}$

$5x-20=7x$
$-2x=+20$
$x=-10$

Solving Proportions

Solve each proportion.

1) $\frac{10}{8} = \frac{n}{10}$ $n = 12.5$

2) $\frac{7}{5} = \frac{x}{3}$ $x = 4.2$

3) $\frac{9}{6} = \frac{x}{10}$ $x = 15$

4) $\frac{7}{n} = \frac{8}{7}$ $n = 6.1$

5) $\frac{4}{3} = \frac{8}{x}$ $x = 6$

6) $\frac{7}{b+5} = \frac{10}{5}$ $b = -1.4$

7) $\frac{6}{b-1} = \frac{9}{7}$ $b = 5.6$

8) $\frac{4}{m-8} = \frac{8}{2}$ $m = 9$

9) $\frac{5}{6} = \frac{7n+9}{9}$ $n = -0.21$

10) $\frac{4}{9} = \frac{r-3}{6}$ $r = 5.7$

11) $\frac{7}{9} = \frac{b}{b-10}$ $b=-35$

12) $\frac{9}{k-7} = \frac{6}{k}$ $K=-14$

13) $\frac{4}{n+2} = \frac{7}{n}$ $n=-4.7$

14) $\frac{n}{n-3} = \frac{2}{3}$ $n=-6$

15) $\frac{x-3}{x} = \frac{9}{10}$ $x=30$

16) $\frac{5}{r-9} = \frac{8}{r+5}$ $r=32.3$

17) $\frac{p+10}{p-7} = \frac{8}{9}$ $p=-146$

18) $\frac{2}{8} = \frac{n+4}{n-4}$ $n=-6.7$

19) $\frac{n-5}{n+8} = \frac{2}{7}$ $n=10.2$

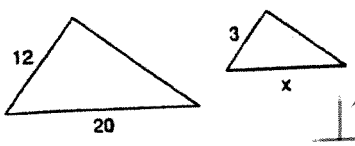
20) $\frac{n-6}{n-7} = \frac{9}{2}$ $n=7.3$

SIMILAR FIGURES

Similar Figures

Each pair of figures is similar. Find the missing side.

1)



$\frac{12}{3} = \frac{20}{x}$

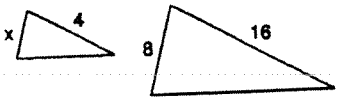
$x = 5$

2)

$\frac{x}{9} = \frac{1}{3}$

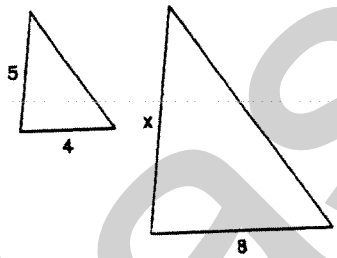
$x = 3$

3)



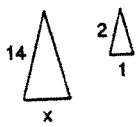
$x = 2$

4)



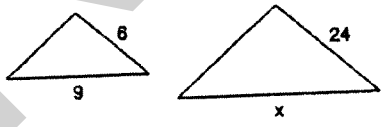
$x = 10$

5)



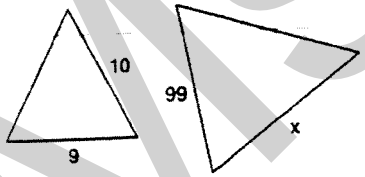
$x = 7$

6)



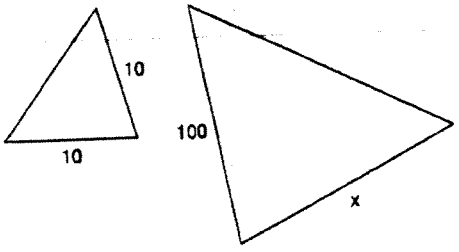
$x = 36$

7)



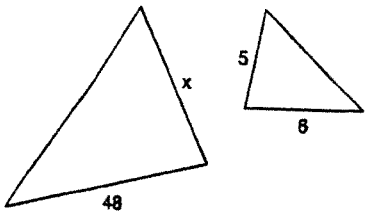
$x = 110$

8)



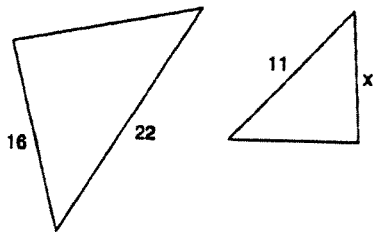
$x = 100$

9)



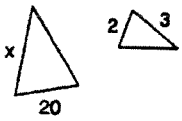
$x = 40$

10)



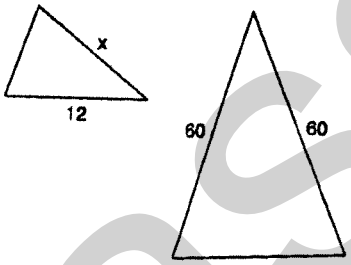
$x = 8$

11)



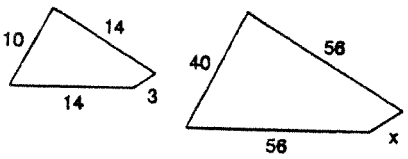
$x = 30$

12)



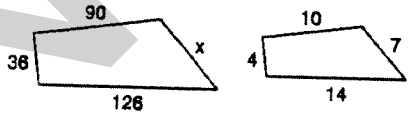
$x = 12$

13)



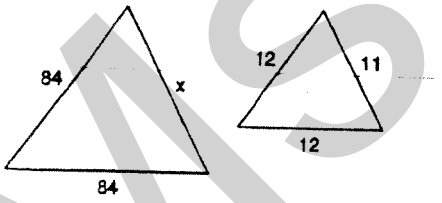
$x = 12$

14)



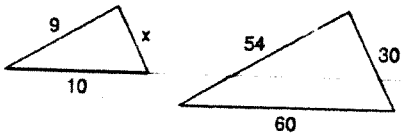
$x = 63$

15)



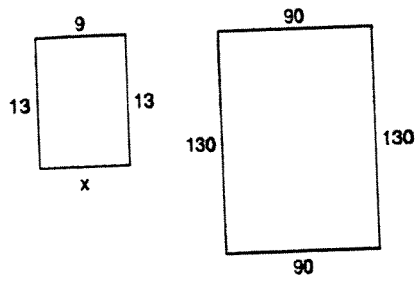
$x = 77$

16)



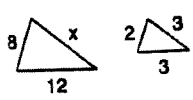
$x = 5$

17)



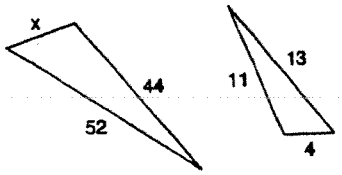
$x = 9$

18)



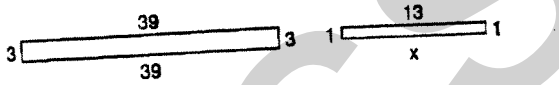
$x = 12$

19)



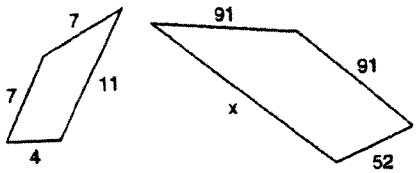
$x = 16$

20)



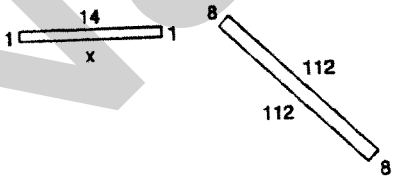
$x = 13$

21)



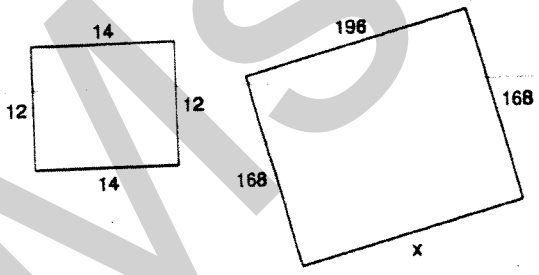
$x = 143$

22)



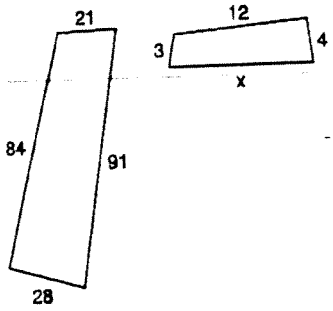
$x = 14$

23)



$x = 196$

24)



$x = 13$

SIMILARITY

Fill in the blanks.

	SIDES RATIO	AREA RATIO	VOLUME RATIO
1	2/3	4/9	8/27
2	7/8	49/64	343/512
3	7/8	49/64	343/512
4	4/9	16/81	64/729 = 64/125
5	4/5	16/25	
6	3	9	27
7	5/6	100/144	1000/1728 = 125/216
8	2	4	8
9	10	100	1000
10	23/30	5.29/9	12.2/27 = 12167/27000
11	7/8	49/64	343/512
12	7/8	49/64	343/512
13	5/7	25/49	125/343
14	5/6	25/36	1000/1728
15	2/2.45	4/6	8/14.7
16	1/2	1/4	1/8
17	3/4	9/16	27/64
18	7/10	49/100	343/1000
19	2/6 = 1/3	4/36 = 1/9	8/216
20	8/14 = 4/7	64/196 = 16/49	512/2744

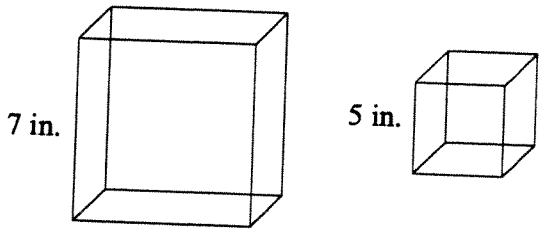
Similarity

21. The ratio of heights of 2 similar figures is 2/3, find the ratio of areas. 4/9
22. The ratio of areas of 2 similar figures is 4/16, find the ratio of perimeters. 1/2
23. The ratio of volumes of 2 similar figures is 8/125, find the scale factor. 2/5
24. The ratio of sides of 2 similar figures is 0.2, find the ratio of volumes. 0.008
25. The ratio of volumes of 2 similar figures is 125/216, find the ratio of areas. 25/36
26. The ratio of areas of 2 similar figures is 49/81, find the ratio of volumes. 343/729
27. The ratio of volumes of 2 similar figures is 27/1000, find the ratio of areas. 9/100
28. The ratio of radii of 2 similar figures is 7/4, find the ratio of volumes. 343/64
29. The ratio of perimeters of 2 similar figures is 5, find the ratio of areas. 25
30. The ratio of areas of 2 similar figures is 100/125, find the ratio of volumes. 1000
31. The ratio of volumes of 2 similar figures is 64/729, find the ratio of areas 16/81
32. The ratio of similarity of 2 similar figures is 9, find the ratio of volumes. 729
33. The ratio of volumes of 2 similar figures is 8000, find the ratio of areas. 400
34. The ratio of areas of 2 similar figures is 36, find the ratio of perimeters. 6
35. The ratio of volumes of 2 similar figures is 125, find the ratio of radii. 5
36. The ratio of perimeters of 2 similar figures is 9/10, find the ratio of volumes. 729/1000
37. The ratio of heights of 2 similar figures is 1/2, find the ratio of areas. 1/4
38. The ratio of areas of 2 similar figures is 4/9, find the ratio of similarity. 2/3
39. The ratio of volumes of 2 similar figures is 729/1000, find the ratio of areas. 81/100
40. The ratio of areas of 2 similar figures is 4, find the ratio of volumes. 8
41. The ratio of volumes of 2 similar figures is 216, find the ratio of heights. 6
42. The scale factor of 2 similar figures is 9/8, find the ratio of volumes. 729/512
43. The ratio of similarity of 2 similar figures is 1.25, find the ratio of areas. 1.5625

Area and Volume of Similar Figures

For each pair of similar solids, find the scale factor of the solid on the left to the solid on the right. Then find the ratios of the surface areas and the ratio of the volumes.

1.

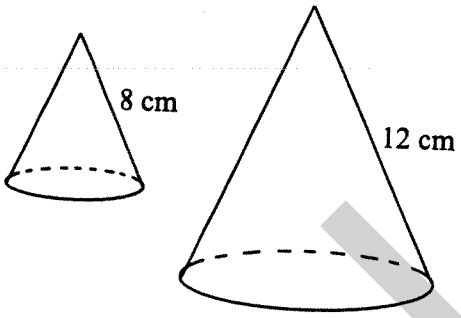


scale factor: 7/5

ratio of surface areas: 49/25

ratio of volumes: 343/125

2.

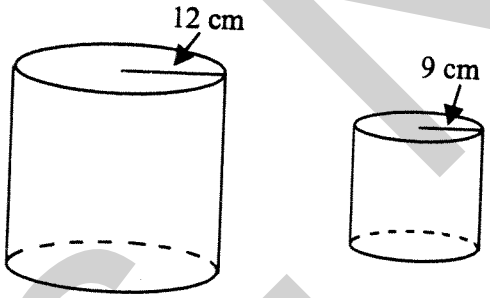


scale factor: 8/12 = 2/3

ratio of surface areas: 4/9

ratio of volumes: 8/27

3.

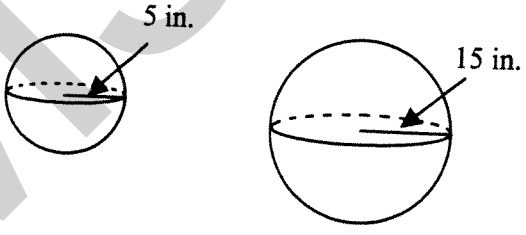


scale factor: 12/9 = 4/3

ratio of surface areas: 16/9

ratio of volumes: 64/27

4.



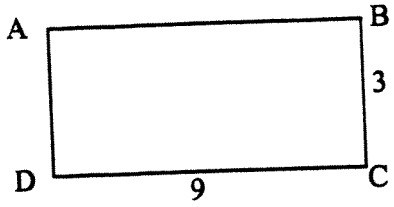
scale factor: 5/15 = 1/3

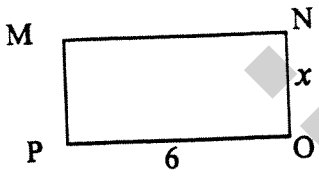
ratio of surface areas: 1/9

ratio of volumes: 1/27

Area and Volume of Similar Figures

1. The rectangles below are similar.





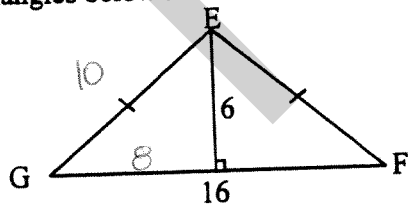
a. What is the similarity ratio of rectangle ABCD to rectangle MNOP? Simplify your ratio. 9/6 = 3/2

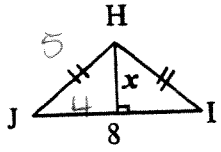
b. What is the value of x? Use mathematics to explain how you determined your answer. 3/2 = 3/x x = 2

c. What is the perimeter of each rectangle? 24, 16

d. What is $\frac{\text{Perimeter of rectABCD}}{\text{Perimeter of rectMNOP}}$? Simplify your ratio. $\frac{24}{16} = \frac{3}{2}$

2. The isosceles triangles below are similar.





a. What is the similarity ratio of triangle EFG to triangle HIJ? Simplify your ratio. 16/8 = 2

b. What is the value of x? $\frac{16}{8} = \frac{6}{x}$ $x = 3$

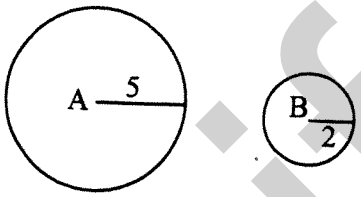
c. What is the perimeter of each triangle? (Hint: use Pythagorean Theorem) 36, 18

d. What is $\frac{\text{Perimeter of } \Delta EFG}{\text{Perimeter of } \Delta HIJ}$? Simplify your ratio. $\frac{36}{18} = 2$

Area and Volume of Similar Figures

$C = \pi d$

3. Circle A and circle B are similar.



   a. What is the similarity ratio of circle A to circle B? $5/2$

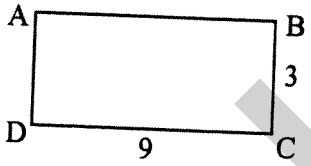
   b. What is the circumference of each circle? Leave answers in terms of π.

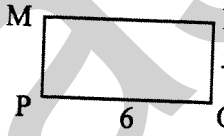
   $C = 10\pi$ $\quad C = 4\pi$

   c. What is $\frac{\text{Circumference of circle A}}{\text{Circumference of circle B}}$? Simplify your ratio. $\frac{10\pi}{4\pi} = \frac{5}{2}$

4. What is the relationship between the ratio of the sides of the similar figures and the ratio of their perimeters or circumferences? same ratio.

5. These rectangles (from problem 1) are similar.





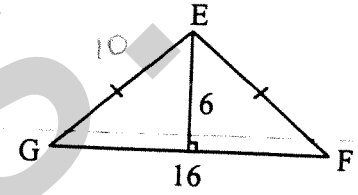
$x = 2$.

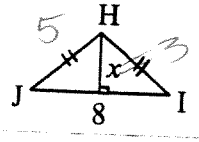
   a. What is the similarity ratio of rectangle ABCD to rectangle MNOP? $9/6 = 3/2$

   b. What is the area of each rectangle? 27 , 12

   c. What is $\frac{\text{Area of rectABCD}}{\text{Area of rectMNOP}}$? Simplify your ratio. $\frac{27}{12} = \frac{9}{4}$

6. These isosceles triangles (from problem 2) are similar.





   a. What is the similarity ratio of triangle EFG to triangle HIJ? 2

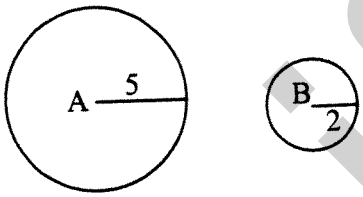
   b. What is the area of each triangle? 48 , 12

   c. What is $\frac{\text{Area of } \Delta EFG}{\text{Area of } \Delta HIJ}$? Simplify your ratio. $\frac{48}{12} = 4$

Area and Volume of Similar Figures

$A = \pi r^2$

7. Circle A and circle B (from problem 3) are similar.



   a. What is the similarity ratio of circle A to circle B? 5/2

   b. What is the area of each circle? Leave the answers in terms of π. 25π, 4π

   c. What is $\frac{\text{Area of circle A}}{\text{Area of circle B}}$? Simplify your ratio. $\frac{25\pi}{4\pi} = \frac{25}{4}$

8. What is the relationship between the ratio of the sides of the similar figures and the ratio of their areas? (Ratio of sides)^2 = Ratio of Area

9. Fill in the blank.

> If two similar polygons or circles have lengths of corresponding sides (or radii) in the ratio of $\frac{a}{b}$, then their areas are in the ratio of $\frac{a^2}{b^2}$.

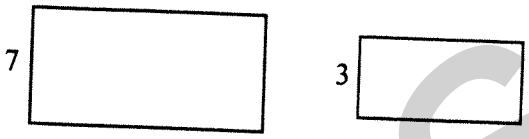
Area and Volume of Similar Figures

1. a. On a piece of graph paper draw a rectangle and determine its area.

   b. Draw another rectangle by doubling the length of each side of the first rectangle. Determine the area of this rectangle.

   c. Explain the relationship between the areas.

2. The rectangles drawn are similar. The lengths of a pair of corresponding sides are given.



   a. What is the similarity ratio of the rectangle on the left to the rectangle on the right?

   7/3

   b. What is the ratio of the perimeter of the rectangle on the right to the perimeter of the rectangle on the left?

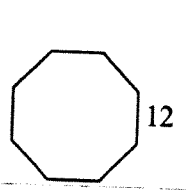
   7/3

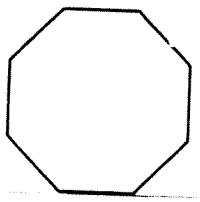
   c. What is the ratio of the area of the figure on the right to the area of the figure on the left?

   49/9

3. Find the ratio of the perimeter of the figure on the left to the figure on the right. Then find the ratio of the area of the figure on the left to the figure on the right. Use mathematics to justify your answers.

   a. Regular Octagons

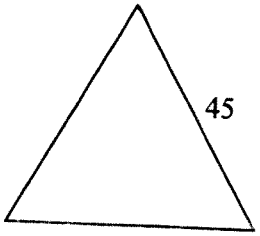


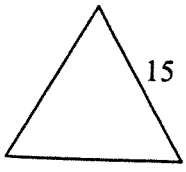


$\frac{A_1}{A_2} = \frac{16}{25}$

$\frac{P_1}{P_2} = \frac{12}{15} = \frac{4}{5}$

   b. Equilateral Triangles





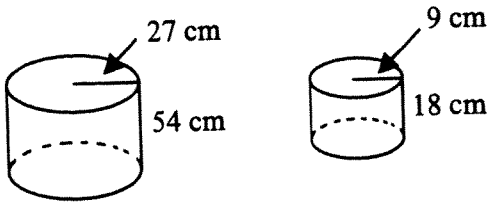
$\frac{A_1}{A_2} = 9$

$\frac{P_1}{P_2} = \frac{45}{15} = 3$

Area and Volume of Similar Figures

3 The cylinders below are similar.

Note: Figures are not drawn to scale.



a. What is the similarity ratio of the dimensions of the cylinder on the left to the cylinder on the right? $\frac{27}{9} = 3$

b. Determine the volume of each cylinder.

left cylinder: $V = A_b \times h$
$= \pi(27)^2 \times 54$
$= 39366\pi$

right cylinder: $V = \pi r^2 \times h$
$= \pi(9)^2 \times 18$
$= 1458\pi$

c. What is the ratio of the volume of the left cylinder to the volume of the right cylinder? $39366\pi / 1458\pi = \frac{19682}{729} = 27$

d. How does the ratio of volumes compare to the similarity ratio of the dimensions of the cylinders?

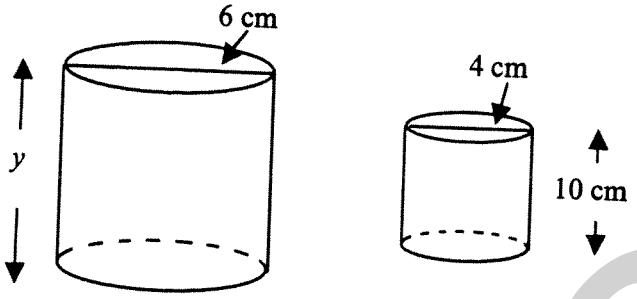
(Ratio of Dimensions)^3 = Ratio of Volume

4. Fill-in the blanks:

If two similar solids have corresponding dimensions in the ratio of $\frac{a}{b}$, then their **volumes** are in the ratio of $\frac{a^3}{b^3}$.

Area and Volume of Similar Figures

5. Look at the two cylinders shown below. The ratio of corresponding diameters is equal to the ratio of the corresponding heights.



a. What is the height of the large cylinder? $\frac{6}{4} = \frac{y}{10}$ $y = 15$

b. Find the ratio of the volumes of the cylinders. Use mathematics to justify your answer. $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$

6. The ratio of the volumes of two tetrahedrons is 1000:1. The smaller tetrahedron has a side length of 8 centimeters. What is the side length of the larger tetrahedron?

$\frac{V_1}{V_2} = \frac{1}{1000}$ $\frac{S_1}{S_2} = \frac{1}{10} = \frac{8}{x}$ $x = 80$ cm

7. Suppose that all pizzas have the same thickness and the cost and number of servings both depend only on the surface area. A pizza 10 inches in diameter costs \$8.12 and serves 2 people.

a. How much should a 14-inch pizza cost? $\frac{S_1}{S_2} = \frac{10}{14} = \frac{5}{7}$ $\frac{A_1}{A_2} = \frac{25}{49} = \frac{8.12}{x}$ $x = 15.92$\$

b. How many people would the 14-inch pizza serve?

8. A trophy that is 8 inches tall weighs 4 pounds. A trophy of similar shape is 12 inches tall. How much does the larger trophy weigh? Assume that the weight is proportional to the volume in any solid.

$\frac{S_1}{S_2} = \frac{8}{12} = \frac{2}{3}$

$\frac{V_1}{V_2} = \left(\frac{2}{3}\right)^3 = \frac{8}{27} = \frac{4}{x}$

$x = 13.5$ pounds

Area and Volume of Similar Figures

1. The ratio of the areas of two squares is $\frac{16}{25}$.

   a. What is the ratio of their sides? 4/5

   b. The larger square has sides of length 10 centimeters. What is the side length of the smaller square?

$$\frac{4}{5}=\frac{x}{10} \quad x=8$$

2. Two rooms are similar in shape, with corresponding lengths in the ratio of $\frac{2}{3}$. It takes 3 gallons of paint to cover the walls of the larger room. How much paint will be needed to paint the smaller room?

$$\frac{4}{9}=\frac{x}{3} \quad x=1.33$$

3. The ratio of the areas of two circles is $\frac{9}{16}$.

   a. What is the ratio of their radii? 3/4

   b. The smaller circle has a radius of 6 centimeters. What is the radius of the larger circle?

$$3/4=6/x \quad x=8$$

4. A rectangular yard has an area of 180 m^2. A similar yard has an area of 20 m^2. If the length of the larger yard is 20 m, what is the length of the smaller yard?

$$\frac{A_1}{A_2}=\frac{20}{180}=1/9 \qquad \frac{S_1}{S_2}=\frac{1}{3}=\frac{x}{20}$$

$x=6.7$

5. Jose bought carpeting for his living room and for his dining room. His living room is similar to his dining room and 1.5 times as long. If it costs $1000 for the carpet for the dining room, how much should it have cost to buy the carpet for the living room?

$$\frac{S_1}{S_2}=\frac{1}{1.5}=\frac{2}{3}$$

$$\frac{A_1}{A_2}=\frac{4}{9}=\frac{1000}{x}$$

$x=2250$

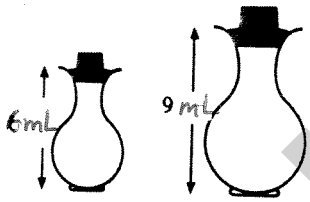
Similar Shapes – Area and Volume Scale Factors

1. Two perfume bottles are mathematically similar in shape.

   The smaller one is 6 millimetres high and holds 30 millilitres of perfume.

   The larger one is 9 millimetres high.

   What volume of perfume will the larger one hold?

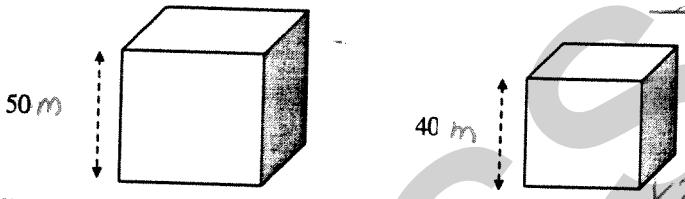


$K = \frac{6}{9} = \frac{2}{3}$

$K^3 = \frac{8}{27} = \frac{30}{x}$

$x = 101.25 m$

2. The two boxes below are mathematically similar and both have to be wrapped with decorative paper.



   If it requires 3.27 m^2 of paper to cover the large box, calculate the amount of paper needed to cover the smaller box.

$K = \frac{40}{50} = \frac{4}{5}$

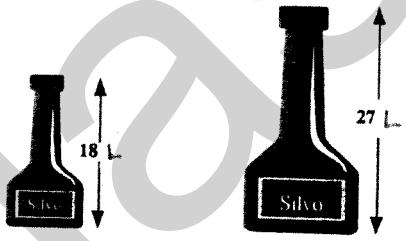
$K^2 = \frac{16}{25} = \frac{x}{3.27}$

$x = 2.09 m^2$

3. The diagram shows two bottles of Silvo Shampoo.

   The two bottles are **mathematically similar**, and the cost of the shampoo depends only on the volume of liquid in the bottle.

   If the small one costs 80$, what should the large one cost ?



$K = \frac{18}{27} = \frac{2}{3}$

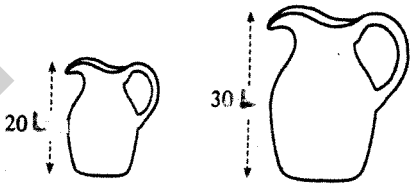
$K^3 = \frac{8}{27} = \frac{80}{x}$

$x = 270\$$

4. The diagram shows two jugs which are mathematically similar.

   The volume of the smaller jug is 0.8 litres.

   Find the volume of the larger jug.

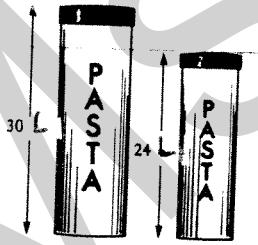


$K = \frac{20}{30} = \frac{2}{3}$

$K^3 = \frac{8}{27} = \frac{0.8}{x}$

$x = 2.7 L$

5. The diagram shows two storage jars which are mathematically similar.



   The volume of the large jar is 1.2 litres.

   Find the volume of the smaller jar.

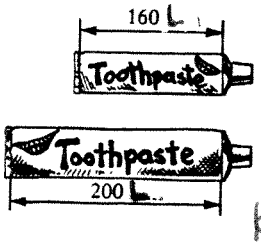
$K = \frac{24}{30} = \frac{4}{5}$

$K^3 = \frac{64}{125} = \frac{x}{1.2}$

$x = 0.611 L$

6. The diagram shows two tubes of toothpaste.

   Assuming that the tubes are mathematically similar, and that the price of toothpaste depends only on the volume of toothpaste in the tube, what would be the cost of the large tube when the small one costs $1.12?



$K = \frac{160}{200} = \frac{4}{5}$

$K^3 = \frac{64}{125} = \frac{1.12}{x}$

$x = 2.19\$$

22

Area and Volume of Similar Figures

6. It costs $440 to carpet a room that measures 16 feet by 24 feet. How much would it cost to carpet a similar room that measures 12 feet by 18 feet?

$ 247.50

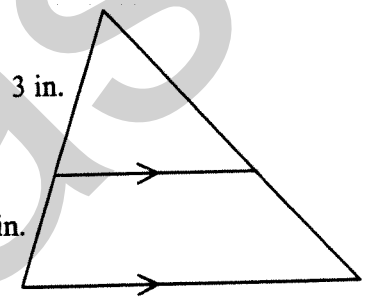
7. The lengths of corresponding altitudes in two similar triangles are in the ratio of $\frac{1}{5}$. What is the ratio of their areas?

1/25

8. The areas in two similar parallelograms are in the ratio of $\frac{4}{25}$. What is the ratio of the lengths of corresponding altitudes?

2/5

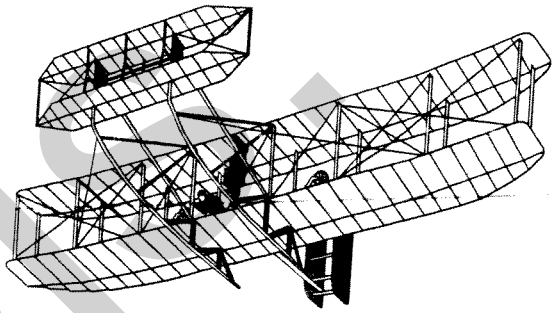
9. Given the figure at the right, what is the ratio of the areas of the small triangle to the large triangle?

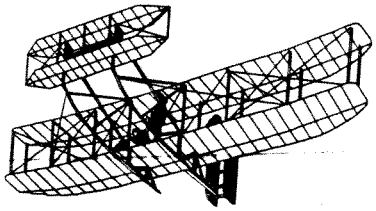


9/25

$\frac{9}{25} = \frac{3}{5}$

10. Two scale models of the Wright brothers plane the Kittyhawk are shown below. The two models are similar to each other because they are both similar to the original plane. The larger model is 25 cm long, and the smaller model is 15 cm long.





15/25 = 3/5

a. What is the ratio of the wingspan of the larger model to the wingspan of the smaller model?

b. What is the ratio of the area of the top wing of the larger model to the area of the top wing of the smaller model?

9/25

Area and Volume of Similar Figures

1. The ratio of the areas of two squares is $\frac{16}{25}$.

   a. What is the ratio of their sides?

   b. The larger square has sides of length 10 centimeters. What is the side length of the smaller square?

2. Two rooms are similar in shape, with corresponding lengths in the ratio of $\frac{2}{3}$. It takes 3 gallons of paint to cover the walls of the larger room. How much paint will be needed to paint the smaller room?

3. The ratio of the areas of two circles is $\frac{9}{16}$.

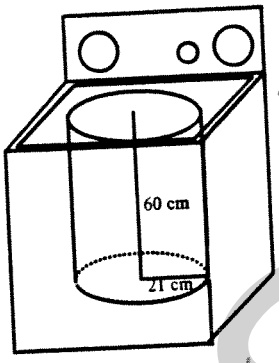
   a. What is the ratio of their radii?

   b. The smaller circle has a radius of 6 centimeters. What is the radius of the larger circle?

4. A rectangular yard has an area of 180 m^2. A similar yard has an area of 20 m^2. If the length of the larger yard is 20 m, what is the length of the smaller yard?

5. Jose bought carpeting for his living room and for his dining room. His living room is similar to his dining room and 1.5 times as long. If it costs $1000 for the carpet for the dining room, how much should it have cost to buy the carpet for the living room?

5 A factory makes a certain washing machine model that has a cylindrical tub with a radius of 21 cm and a height of 60 cm, as shown in the adjacent diagram. There is a demand for a similar smaller model. What must the dimensions of the tub of the smaller model be if its volume is to be 24 630.09 cm^3?



step1
$V = A_b \times h$
$\pi(21)^2 \times 60$
83084.4

step2
$K^3 = \frac{24630.09}{83084.4}$
$\frac{29.09}{43.64} = K$

step3
$\frac{29.09}{43.64} = \frac{x}{21} = \frac{y}{60}$

radius $x = 14$
$y = 40$ height

6 A right triangle has a height of 15 cm and a base of 20 cm. A second triangle, similar to the first, has an area of 600 cm^2.

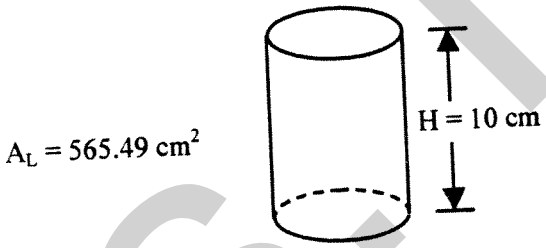
$A = \frac{15 \times 20}{2} = 150$

What are the measures of base b and height h of the second triangle?

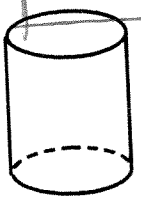
$K^2 = \frac{150}{600} = \frac{1}{4}$
$K = \frac{1}{2} = \frac{15}{x} = \frac{20}{y}$

Height $x = 30$
Base $y = 40$

11 The two cylinders, shown below, are similar.



$r = 9$



The lateral area of the large cylinder is 565.49 cm^2 and its height is 10 cm.

If the radius of the smaller cylinder is 3 cm, what is its volume?

step1
$A_L = P_B \times h$
$= 2\pi rh$
$565.49 = 2\pi r(10)$
$r = 9\ cm$

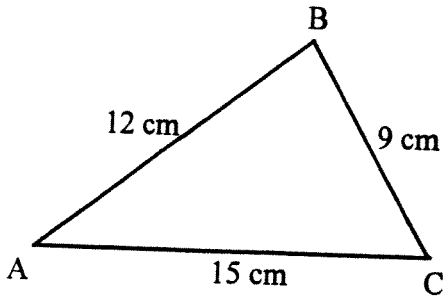
step2
$V = \pi r^2 \times h$
$= \pi(9)^2 \times 10$
$= 2544.69\ (BIG)$

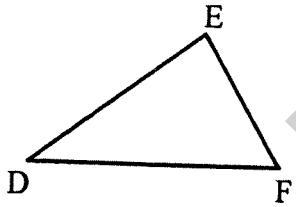
step3
$K^3 = \left(\frac{3}{9}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

step4
$\frac{1}{27} = \frac{x}{2544.69}$

$x = 94.25\ cm^3$

13 Triangle DEF and triangle ABC are similar.





If the ratio of the areas of ΔABC: ΔDEF is 9 : 1, what is the length of segment DE?

A) 36 cm

B) 5 cm

C) 4 cm

D) $1.\overline{3}$ cm

$K^2 = \frac{1}{9}$ $K = 1/3 = \frac{x}{12}$

14 The dimensions of two similar prisms are in the ratio of $\frac{3}{2}$. The volume of the smaller prism is 250 cm^3.

What is the volume of the larger prism?

$K = \frac{3}{2}$ $K^3 = \frac{27}{8} = \frac{x}{250}$

$x = 8437.5\ cm^3$

15 Two cylindrical water glasses are of different sizes.

One glass, whose radius is 3 cm, has a maximum capacity of 135π cm^3.
The other glass, 12 cm high, has a maximum capacity of 147π cm^3.

Are these glasses similar solids?

$V = \pi r^2 \times h$

$135\pi = \pi(3)^2 \times h$

$h = 15$

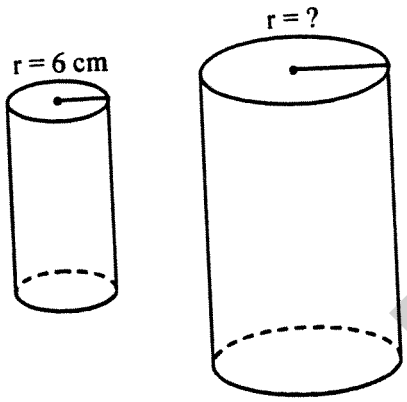
$K = \frac{12}{15} = \frac{4}{5}$ $K^3 = \frac{64}{125}$

$K^3 = \frac{135\pi}{147\pi} = \frac{45}{49}$

NOT similar

16 The two cylinders on the right are similar.

The ratio of their areas is $\frac{1}{25}$.



The radius of the small cylinder is 6 cm.

$k=1/25$ $k=1/5=6/x$

What is the radius of the large cylinder?

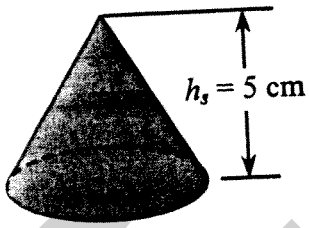
A) 300 cm

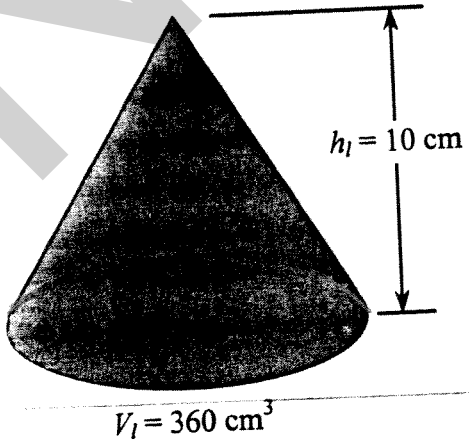
B) 150 cm

C) 60 cm

D) 30 cm

17 The two right circular cones below are similar. The height of the smaller cone is 5 cm and its volume is 45 cm^3. The height of the larger cone is 10 cm and its volume is 360 cm^3. What is the ratio of the areas of the two cones?





$k^3=\frac{45}{360}=1/8$

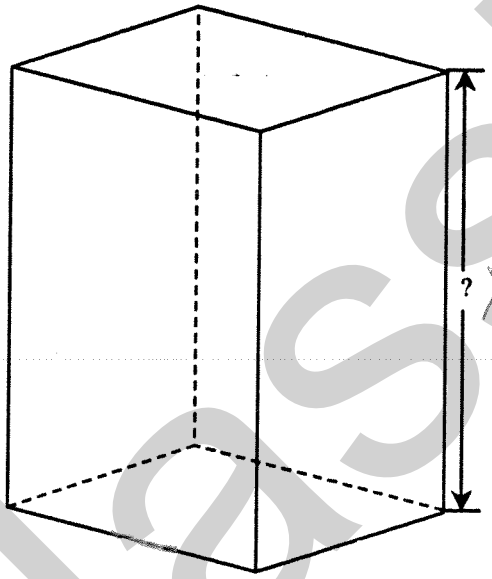
$k=1/2=$

$k^2=1/4$

18 The two rectangular right prisms shown below are similar. The dimensions of the smaller prism are given in the diagram below. The volume of the larger prism is 1728 cm^3. What is the height of the larger prism?

$K^3 = 64/1728 = 1/27$

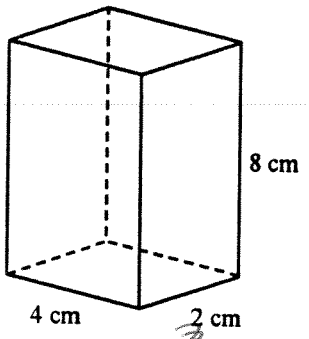
Larger prism



$K = 1/3 = 8/x$

$x = 24$

Smaller prism



$V = 64\text{cm}^3$

The prisms are **not** drawn to scale.

20 The square-based right prisms given below are similar.

The volume of the bigger prism is 1080 cm^3. The edges of its base each measure 6 cm.

Step 2

$K = 12/30 = 2/5$

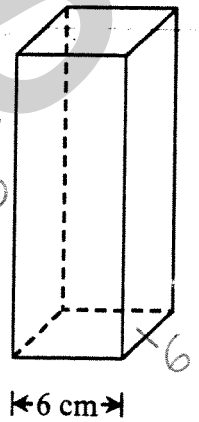
Step 1 The height of the smaller prism is 12 cm.

Find height

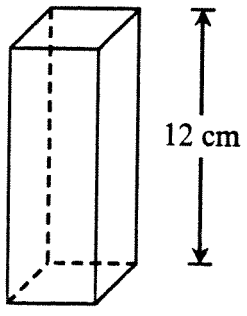
$V = A_B \times h$

$1080 = 36 \times h$

$h = 30$



Volume: 1080 cm^3



Volume: ?

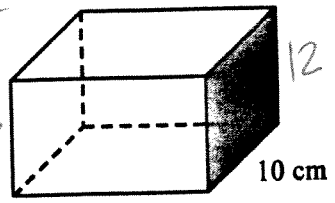
$K^3 = 8/125 = x/1080$

$x = 69.12$

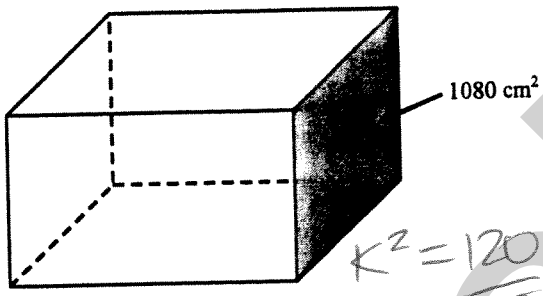
What is the volume of the smaller prism?

22 The rectangular prisms below are similar. The dimensions of the base of the smaller prism are 20 cm by 10 cm. Its volume is 2400 cm^3. The area of the shaded face of the larger prism is 1080 cm^2.

Step 1
Find height
$2400 = 20 \times 10 \times h$
$h = 12$



$V = 2400\ cm^3$



$k^2 = \frac{120}{1080} = \frac{1}{9} \Rightarrow k = \frac{1}{3}$

What is the volume of the larger prism?

$k^3 = \frac{1}{27} = \frac{2400}{x}$

$x = 64800\ cm^3$

24 George has determined that two solids are similar. One solid is 9 cm in height and has a volume of 810 cm^3. The second solid has a volume of 240 cm^3.

What is the height of the second solid?

$k^3 = \frac{240}{810} = \frac{8}{27}$

$k = \frac{2}{3} = \frac{x}{9}$ $\quad x = 6\ cm$

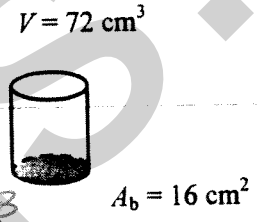
25 The cylinders below are similar solids.

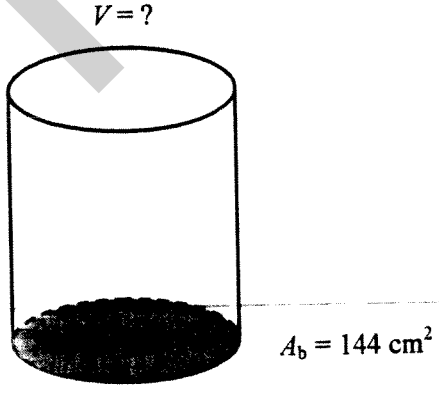
Step 1 $k^2 = \frac{16}{144} = \frac{1}{9}$

Step 2 $k^3 = \frac{1}{27}$

Step 3 $\frac{1}{27} = \frac{72}{x}$

$x = 1944\ cm^3$





The area of the base of the smaller cylinder is 16 cm^2, and its volume is 72 cm^3.

The area of the base of the larger cylinder is 144 cm^2.

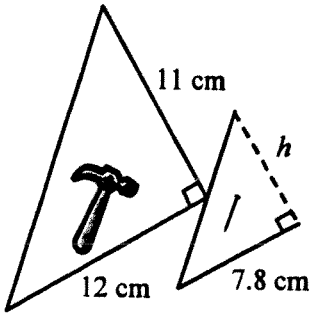
What is the volume of the larger cylinder to the nearest cubic centimetre?

26 A construction company uses two similar triangles to create its logo, as shown below.

The height of the large triangle is 11 cm and the base is 12 cm.

The base of the small triangle is 7.8 cm.

$\frac{h}{11} = \frac{7.8}{12}$



What is the height (h) of the small triangle?

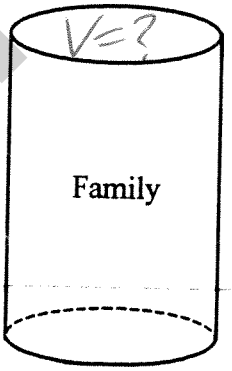
A) 6.8 cm

B) 7.15 cm

C) 8.51 cm

D) 16.9 cm

The *Chunky Meal in a Can* soup is available in two sizes: the single serving size and the family size. The cans are similar solids.



$V = 240$
$A_B = 16$

$A_B = 25$
$V = ?$

Step 1 $k^2 = \frac{16}{25}$

$k = \frac{4}{5}$

Step 2 $k^3 = \frac{64}{125}$

Step 3 $\frac{64}{125} = \frac{240}{x}$

The area of the base of the smaller can is 16 cm^2 and its volume is 240 cm^3.
The area of the base of the larger can is 25 cm^2.

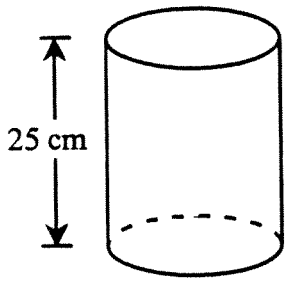
$x = 468.75$

What is the volume of the larger can to the nearest cubic centimetre?

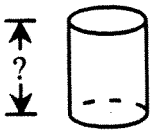
30 The two right cylinders shown below are similar. The height of the bigger cylinder is 25 cm, and its volume is 6250 cm^3. The volume of the smaller cylinder is 400 cm^3. What is the height of the smaller cylinder?

step1 $K^3 = \frac{400}{6250} = \frac{8}{125}$

step2 $K = \frac{2}{5}$



Volume: 6250 cm^3



Volume: 400 cm^3

step3 proportion

$\frac{2}{5} = \frac{x}{25}$

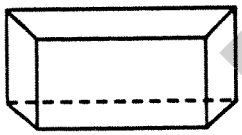
$x = 10$ cm

33 Two right prisms with trapezoidal bases are similar. The ratio of their heights is $\frac{3}{2}$.

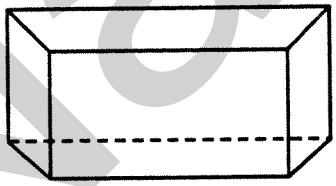
The volume of the smaller prism is 40 cm^3.

step1 $K = \frac{2}{3}$

step2 $K^3 = \frac{8}{27}$



Volume: 40 cm^3



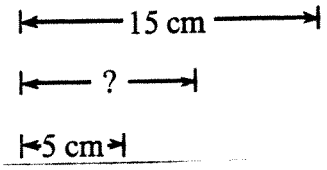
Volume: ?

step3 $\frac{8}{27} = \frac{40}{x}$

$x = 135$ cm^3

What is the volume of the larger prism?

34 A layer cake is made up of three similar cylinders. The total height of the cake is 22 cm.



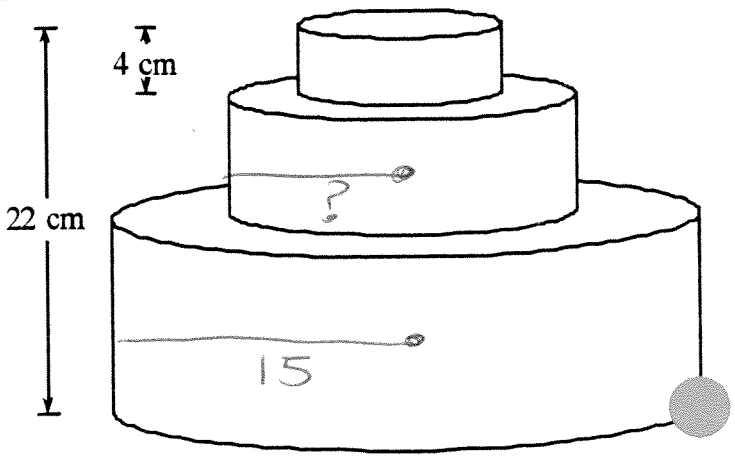
The height of the top layer is 4 cm. Its radius is 5 cm.

The radius of the bottom layer is 15 cm.

step1: find height bottom layer.

$K = \frac{5}{15} = \frac{4}{x}$ $x = 12$

What is the radius of the cake's middle layer?



step2 find h of middle layer

$22 - 4 - 12 = 6$ cm

step3: find r of middle

$\frac{5}{\quad} = \frac{4}{\quad}$ $x = 7.5$ cm