$$\frac{4^{\times 2}}{5_{\times 2}} + \frac{1^{\times 5}}{2^{\times 5}} \qquad \frac{4^{\times 4}}{9_{\times 4}} + \frac{1^{\times 3}}{12^{\times 3}} \qquad \frac{1^{\times 3}}{20_{\times 3}} + \frac{2^{\times 4}}{15_{\times 4}}$$

$3 \cdot 3$ $\quad 2 \cdot 2 \cdot 3$ $\quad 2 \cdot 2 \cdot 5$ $\quad 5 \cdot 3$

$\times 4$ $\quad \times 3$

$$\frac{8}{10} + \frac{5}{10} \qquad 3 \cdot 3 \cdot 2 \cdot 2 \qquad 5 \cdot 2 \cdot 2 \cdot 3$$

$$\frac{13}{10} \qquad \frac{16}{36} + \frac{3}{36} \qquad \frac{3}{60} + \frac{8}{60}$$

$$\frac{19}{36} \qquad 11/60$$

Oct 16-2:44 PM

Examples: Simplify and state restrictions.

1) $\frac{x}{x}\frac{(x-2)}{(x+1)} + \frac{3}{x}\frac{(x+1)}{(x+1)}$

* Do not ever ever ever cancel sideways.

$$\frac{x(x-2)}{x(x+1)} + \frac{3(x+1)}{x(x+1)}$$

$$\frac{x^2-2x+3x+3}{x(x+1)}$$

middle of order

Do not cancel at this stage

$$\frac{x^2+1x+3}{x(x+1)}$$

Last step: can this be factored?

No. its the end

2) $\frac{3}{x^2-16}+\frac{x}{x+4}$

Step1: Factor if poss.

$\frac{3}{(x-4)(x+4)}+\frac{x}{(x+4)}\frac{(x-4)}{(x-4)}$

Step2: Figure out the denom...

$\frac{3}{(x+4)(x-4)}+\frac{x(x-4)}{(x+4)(x-4)}$

Step3: Write down the common deno. once and then write the rest

$\frac{x^2-4x+3}{(x+4)(x-4)}$

Write denom... once at this point

OR

at the end, try factoring the numerator.

$\frac{(x-1)(x-3)}{(x+4)(x-4)}$

3) $\frac{2}{(x-3)}-\frac{5}{(x+3)}$

$\frac{2(x+3)}{(x-3)(x+3)}$ $-$ EVIL $\frac{5(x-3)}{(x-3)(x+3)}$

$=\frac{2(x+3)\;-5(x-3)}{(x-3)(x+3)}$

$=\frac{2x+6-5x+15}{(x-3)(x+3)}$

$=\frac{-3x+21}{(x-3)(x+3)}$ Factor

$=\frac{-3(x-7)}{(x-3)(x+3)}$

4) $\frac{a-2}{4a} - \frac{\boxed{4-6a}}{8a}$

FACTOR EVERYTHING & SIMPLIFY BEFORE FINDING COMMON DENOMINATOR.

$\frac{a-2}{4a} - \frac{2(2-3a)}{8a}$

$\frac{a-2}{4a} - \frac{(2-3a)}{4a}$

They are together b/cuz they are separated by −/+.

$\frac{a-2-2+3a}{4a} = \frac{4a-4}{4a} = \frac{\cancel{4}(a-1)}{\cancel{4}a}$

$= \frac{a-1}{a}$

$\frac{(\cancel{4a}-4)}{\cancel{4a}}$

5) $\frac{\cancel{x+2}}{x^2+4x+4} - \frac{\cancel{x-4}}{x^2-2x-8}$

$(x+2)\cancel{(x+2)}$ $\quad$ $\cancel{(x-4)}(x+2)$

$\frac{1}{x+2} - \frac{1}{x+2}$

0

You may cancel at the beginning

$$6)\ \frac{(x+2)(5x+1)}{x^2-2x-3} - \frac{(5x-3)(x+1)}{x^2-x-6}$$

$$(x+2)(x-3)(x+1) \qquad (x-3)(x+2)(x+1)$$

$$\frac{(x+2)(5x+1)}{(x-3)(x+1)(x+2)} - \frac{(5x-3)(x+1)}{(x-3)(x+1)(x+2)}$$

$$\frac{9x+5}{(x-3)(x+1)(x+2)}$$

1.6 Solving a second degree equation by factoring pg. 29-32 #1-8

FIRST DEGREE EQUATIONS

Let's start by reviewing the 3 cases for first degree equations with form: $bx^1 + c = 0$.

112

Case 1: 1 solution → Ex: $3x - 9 = 0$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$ $S = \{3\}$

Case 2: No solution → Ex: $4x - 6 = 9x - 5x + 20$

$4x - 4x = 20 + 6$

$\boxed{0x = 26}$

Ask Yourself: $0 \times ? = 26$ $S = \{\phi\}$

Case 3: Many Solutions →

Ex: $5x - 1 = 4x + 1x + 9 - 10$

$5x - 5x = -1 + 1$

$\boxed{0x = 0}$

Ask Yourself: $0 \times ? = 0$ $S = \{x \in \mathbb{R}\}$

113

A second degree equation/quadratic equation is any equation written in the form:

$$ax^2 + bx + c = 0 \text{ where } a \neq 0.$$

There are 4 ways of solving quadratic equations.

Case 1: Solving quadratic equations of the form $ax^2 + bx + c = 0$ (Can Be Factored) pg 29#1-3

Ex 1: Solve

$x^2 - 7x = -10$

Put your answers in solution set notation, meaning S = {.....}

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x = 5 \quad x = 2$

in order

$S = \{2, 5\}$

Verify answer

$(2)^2 - 7(2) = -10\ ?$

$-10 = -10\ \checkmark$

$(5)^2 - 7(5) = -10\ ?$

$-10 = -10\ \checkmark$

Ex 2: Solve $x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

$(x-3)(x-3) = 0$

$x - 3 = 0$

$x = 3$

useless

$x - 3 = 0$

$x = 3$

$S = \{3\}$

do not repeat 3.

Ex 3: Solve $x^2 - \cancel{7x} = -\cancel{7x} - 25$

$\sqrt{x^2} = \sqrt{-25}$

undefined

$S = \{\phi\}$

$x^2 + 25 = 0$

not DOS

Ex: 4: Solve $16x^2 = 25$ by factoring

DOS $16x^2 - 25 = 0$

$(4x-5)(4x+5) = 0$

$x = 5/4 \quad x = -5/4$

$S = \{-5/4, 5/4\}$

Ex 5: Solve $x^2 - 25x = 0$

$x(x-25) = 0$

$x = 0 \quad x = 25$

$S = \{0, 25\}$

Ex 6: Solve $2x^2 - 13x - 15 = 0$

$(2x-15)(x+1) = 0$

$2x = 15$

$x = 15/2$

$x = -1$

$S = \{-1, \frac{15}{2}\}$

Case 2: Solving quadratic equations of the form:

$x^2 = k$

If $k < 0$ then $S = \phi$

If $k = 0$ then $S =$ one solution

If $k > 0$ then $S =$ 2 solution

* careful: square rooting gives $\pm$ Result.

Ex 1: Solve $x^2 + \cancel{3x} = \cancel{3x} + 100$

$\sqrt{x^2} = \sqrt{100}$

$x = \pm 10$

$S = \{-10, 10\}$

Ex 2: Solve $4x^2 = 49$

$\frac{4x^2}{4} = \frac{49}{4}$

$\sqrt{x^2} = \sqrt{\frac{49}{4}}$

$x = \pm 7/2$

$S = \{-7/2, 7/2\}$

Ex 3: Solve $x^2 - 13 = 0$

$x^2 = 13$

$x = \pm\sqrt{13}$

$S = \{-\sqrt{13}, \sqrt{13}\}$

Ex 4: Solve $x^2 + 9 = 0$

$x^2 = -9$

$S = \{\phi\}$

Ex 5: Solve $\frac{4x^2}{4} = \frac{64}{4}$

Reduce FIRST

$x^2 = 16$

$x = \pm 4$

$S = \{-4, 4\}$

Case 3: Solving quadratic equations of the form
$a(x-h)^2 + k = 0$

Ex 1: Solve $\frac{3(x-4)^2}{3} = \frac{0}{3}$

Method 1

$(x-4)(x-4)=0$

$x-4=0 \quad x-4=0$

$(x-4)^2 = 0$

$x-4=0$

$x=4$

$S = \{4\}$

Method 2

$\sqrt{(x-4)^2} = \sqrt{0}$

$(x-4) = 0$

$x = 4$

Ex 2: Solve $\frac{-0.4(x-4)^2}{-0.4} = \frac{-40}{-0.4}$

$\sqrt{(x-4)^2} = \sqrt{100}$

$x-4 = 10 \quad | \quad x-4 = -10$

$x = 14 \quad | \quad x = -6$

$S = \{-6, 14\}$

Fact: Square Rooting gives $\pm$ Result

NOTE: cannot do zero product principle because $a \cdot b \neq 0$.

FALSE

$*\ (x-3)(x+2) = 9$

$x-3=9 \quad x+2=9$

Case 4: Solving quadratic equations of the form $\boxed{ax^2 + bx + c = 0}$ (Can Not Be Factored)

Discriminant Method

Pg. 33 #9-10

$$a = coefficient\ of\ x^2$$
$$b = coefficient\ of\ x$$
$$c = constant$$

To solve for the solutions to x,

➔ Use Quadratic Formula

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Greek (Delta) aka the Discriminant

Where the discriminant

$$\Delta = b^2 - 4ac$$

The formula tells us the number of solutions that an equation has.

Sign of Δ	# of solutions
$\Delta > 0$	2
$\Delta = 0$	1
$\Delta < 0$	0

The quadratic formula can be written as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex 1: Solve $x^2 - x - 20 = 0$ include negatives

a b c

By Factoring

$(x+4)(x-5)=0$

$x=-4$ $x=5$

By Quadratic Formula

$a=1$ $b=-1$ $c=-20$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-20)}}{2(1)}$$

$$x=\frac{1\pm\sqrt{81}}{2}=\frac{1\pm 9}{2}$$

5 -4

Ex 2: Solve $-3(x+1)^2 + 9 = 0$

By isolating x

$$\frac{-3(x+1)^2}{-3}=\frac{-9}{-3}$$

$$\sqrt{(x+1)^2}=\sqrt{3}$$

$x+1=\sqrt{3}$ $x+1=-\sqrt{3}$

$x=\sqrt{3}-1$ $x=-\sqrt{3}-1$

$x=0.73$ $x=-2.73$

By Quadratic Formula

$-3(x^2+2x+1)+9=0$

$-3x^2-6x-3+9=0$

$-3x^2-6x+6=0$

a b c

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(-3)(6)}}{2(-3)}$$

$$x=\frac{6\pm\sqrt{108}}{-6}$$

$$\frac{6+\sqrt{108}}{-6} \qquad \frac{6-\sqrt{108}}{-6}$$

-2.73 0.73

Ex 3: Solve $-5x - 8 + 3x^2 = 0$

$3x^2 - 5x - 8 = 0$

a b c

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-8)}}{2(3)}$$

$$\frac{5 \pm \sqrt{121}}{6}$$

Solving Rational Equations
Not in Workbook – (Handout Given)

$$\frac{2x}{3} + \frac{x}{5} = 13$$

The usual technique is to put all terms on the lowest common denominator (LCD) and then multiply the entire equation by the LCD, meaning both sides of the equation. The resulting equation will be cleared c fractions, and we can then proceed to solve for the variable.

Clearing the Fractions in an equation

Example 1:

Solve $15\left(\frac{2x}{3}+\frac{x}{5}=13\right)$

Verify

$\frac{2(15)}{3}+\frac{(15)}{5}=13$?

$13=13$ ✓

$10x+3x=195$

$13x=195$

$\frac{13x}{13}=\frac{195}{13}$

$x=15$

NOTE The LCM for 3 and 5 is 15.

The LCD for $\frac{2x}{3}$ and $\frac{x}{5}$ is 15.

Verify Answer: To check, substitute result in the original equation.

Be Careful! A common mistake is to confuse an *equation* such

$$\frac{2x}{3}+\frac{x}{5}=13$$

and an *expression* such as

$$\frac{2x}{3}+\frac{x}{5}$$

Let's compare.

Equation: $\frac{2x}{3}+\frac{x}{5}=13$

Here we want to *solve the equation for x*, as in Example 1. We multiply both sides by LCD to clear fractions and proceed as before.

Expression: $\frac{2x}{3}+\frac{x}{5}$

Here we want to find *a third fraction* that is equivalent to the given expression. We w each fraction as an equivalent fraction with the LCD as a common denominator.

$$\frac{2x}{3} + \frac{x}{5} = \frac{2x \cdot 5}{3 \cdot 5} + \frac{x \cdot 3}{5 \cdot 3}$$
$$= \frac{10x}{15} + \frac{3x}{15} = \frac{10x + 3x}{15}$$
$$= \frac{13x}{15}$$

Steps to Solve a Rational Equation

1. Factor the denominators of all rational expressions. Identify any restrictions on the variable.
2. Identify the LCD of all expressions in the equation.
3. Multiply both sides of the equation by the LCD.
4. Solve the resulting equation.
5. Check each potential solution.

Solving an equation Involving Rational Expressions

Example 2

LCD: $4x^2$

Take highest exponent of x.

NOTE We assume that x cannot have the value 0. Do you see why?

Solve.

$$(4x^2)\frac{7}{4x} - (4x^2)\frac{3}{x^2} = (4x^2)\frac{1}{2x^2}$$

The LCM of $4x$, x^2, and $2x^2$ is $4x^2$. So, the LCD for the equation is $4x^2$.

$$\frac{7(4x^2)}{4x} - \frac{3(4x^2)}{x^2} = \frac{1(4x^2)}{2x^2}$$

$$7x - 12 = 2$$

$$7x = 14$$

$$x = 2$$

Be sure to return to the original equation and substitute the result for x.

Example 3: Solve

(Always mention restrictions.)

LCD: $(10+m)(10-m)$

In this case its the product of both denom.

$$\frac{24}{10+m} + 1 = \frac{24}{10-m}$$

$m \neq -10$ $\quad$ $m \neq 10$

$$\frac{24(10+m)(10-m)}{(10+m)} + 1(10+m)(10-m) = \frac{24(10+m)(10-m)}{(10-m)}$$

$$24(10-m) + (100-m^2) = 24(10+m)$$

$$240 - 24m + 100 - m^2 = 240 + 24m$$

$$0 = m^2 + 48m - 100$$

$$0 = (m+50)(m-2)$$

$m = -50$ $\quad$ $m = 2$

Always Checks.

$S = \{2, -50\}$

Example 4: Solve

LCD: $(x+2)(x-2)$

$$\frac{11}{\cancel{x^2-4}} - \frac{(x+3)}{2-x} = \frac{2x-3}{x+2}$$

$(x-2)(x+2)$ $\quad$ $(x-2)$

$x \neq 2, -2$

$11 - (x+3)(x+2) = (2x-3)(x-2)$

$11 - (x^2+5x+6) = 2x^2 - 7x + 6$

$11 - x^2 - 5x - 6 = 2x^2 - 7x + 6$

$0 = 3x^2 - 2x + 1$

~~$0 = (3x+1)(x-1)$~~

$\Delta = b^2 - 4ac$

$= (-2)^2 - 4(3)(1)$

$\Delta = -8$

No Solution

Example 5: Solve and check

$$\frac{x}{x-2} - 7 = \frac{2}{x-2}$$

Bell Work Question

Solve

$$\frac{(x+2)}{x+3} - \frac{x^2}{\cancel{x^2-9}} = 1 + \frac{x-1}{\cancel{3-x}}$$

$(x-3)(x+3)$ $(x-3)$

$(x+2)(x-3) - x^2 = (x-3)(x+3) + (x-1)(x+3)$

$\cancel{x^2} - 1x - 6 - \cancel{x^2} = x^2 - 9 + x^2 + 2x - 3$

$0 = 2x^2 + 3x - 6$

$\Delta = 3^2 - 4(2)(-6)$

$\Delta = 57$ Quadratic Formula

$x = \frac{-3 \pm \sqrt{57}}{2(2)}$

$\frac{-3+\sqrt{57}}{4} \approx 1.14$

$\frac{-3-\sqrt{57}}{4} \approx -2.6$

always check

Bell work Question

Solve.

$$\frac{x}{x-4} = \frac{15}{x-3} - \frac{2x}{x^2-7x+12}$$

Bell Work Question

$$\frac{x+3}{x^2-x} - \frac{8}{x^2-1} = 0$$

Step by Step: Solving Equations Containing Rational Expressions

Step 1 Clear the equation of fractions by multiplying both sides of the equation by the LCD of all the fractions that appear.

Step 2 Solve the equation resulting from step 1.

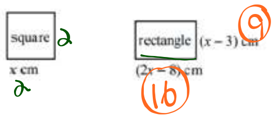
Step 3 Check all solutions by substitution in the original equation.

Quadratic Word Problems

pg 34 #14-22

1. Find the perimeter of a rectangle if the two shapes are equal in area.

$A_{sq} = x^2$



$A_{rect} = (2x-8)(x-3)$

$2x^2-14x+24$

$-x^2$ Make Equal

$x^2 = 2x^2-14x+24$

$0 = x^2-14x+24$

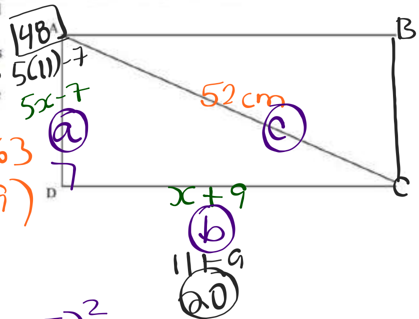
$0 = (x-2)(x-12)$

$\cancel{x=2}$ $\boxed{x=12}$

cannot be because it gives a neg. dimension.

$P = 16(2)+9(2) = 50$ cm

2. The length of the sides of ~~rectangle~~ ABCD below can be represented by binomials. The area of this rectangle is then represented by the trinomial $\boxed{5x^2+38x-63}$. In addition, the length of diagonal AC of this rectangle is 52 cm. → no variables What is the numerical perimeter of rectangle ABCD in centimetres?



$A_{rect} = 5x^2+38x-63$

$A_{rect} = (5x-7)(x+9)$

$a^2+b^2=c^2$

$(5x-7)^2+(x+9)^2 = 52^2$

$(25x^2-70x+49)+(x^2+18x+81) = 2704$

$26x^2-52x-2574=0$

$26(x^2-2x-99)=0$

$26(x-11)(x+9)=0$

$x=11$ $\cancel{x=9}$

$P = 2(20)+2(48) = 136$ cm

3. Sam is 10 years older than Frank. Ten years from now, $\boxed{\text{the product of their ages will be 1200}}$ What's Sam's present age?

	present	future
Frank	x	$x+10$
Sam	$x+10$	$x+20$

$(x+10)(x+20) = 1200$

$x^2 + 30x + 200 = 1200$

$x^2 + 30x - 1000 = 0$

$(x-20)(x+50) = 0$

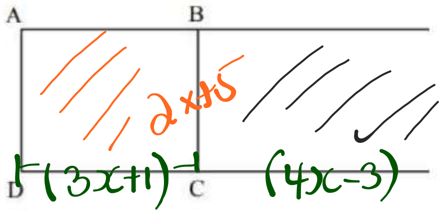
$x - 20 = 0$ $\qquad$ $x + 50 = 0$

$x = 20$ Frank $\qquad$ $\cancel{x = -50}$

$30 =$ Sam

4.

Rectangles ABCD and BEFC below have side BC in common.
The lengths of their bases and their heights can be represented by binomials. This therefore means that the area of each rectangle is represented by a trinomial as follows:



- The area of rectangle ABCD is represented by the trinomial $6x^2 + 17x + 5$. $A =$
- The area of rectangle BEFC is represented by the trinomial $8x^2 + 14x - 15$. What binomial represents the length of side BC?

$A_{ABCD} = 6x^2 + 17x + 5$
$= (3x+1)(2x+5)$

$A_{BEFC} = 8x^2 + 14x - 15$
$= (4x-3)(2x+5)$

$\overline{BC} = (2x+5)$