

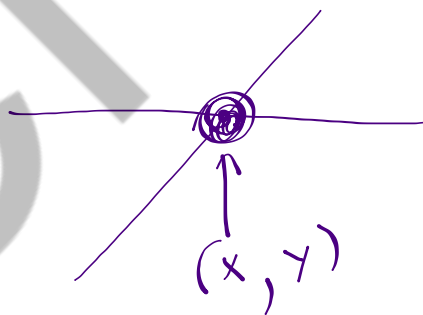
## Chapter 6: Systems of equations

### 6.1 System of two first degree equations in two variables pg169-171

In this section, we discuss how to solve systems of 2 linear equations in two variables.

We have studied how the graph of a linear equation in two variables is a line in the plane consisting of all points that satisfy the linear equation.

A solution to a system of two linear equations in two variables is a point that satisfies both linear equations and is therefore a point on both lines.



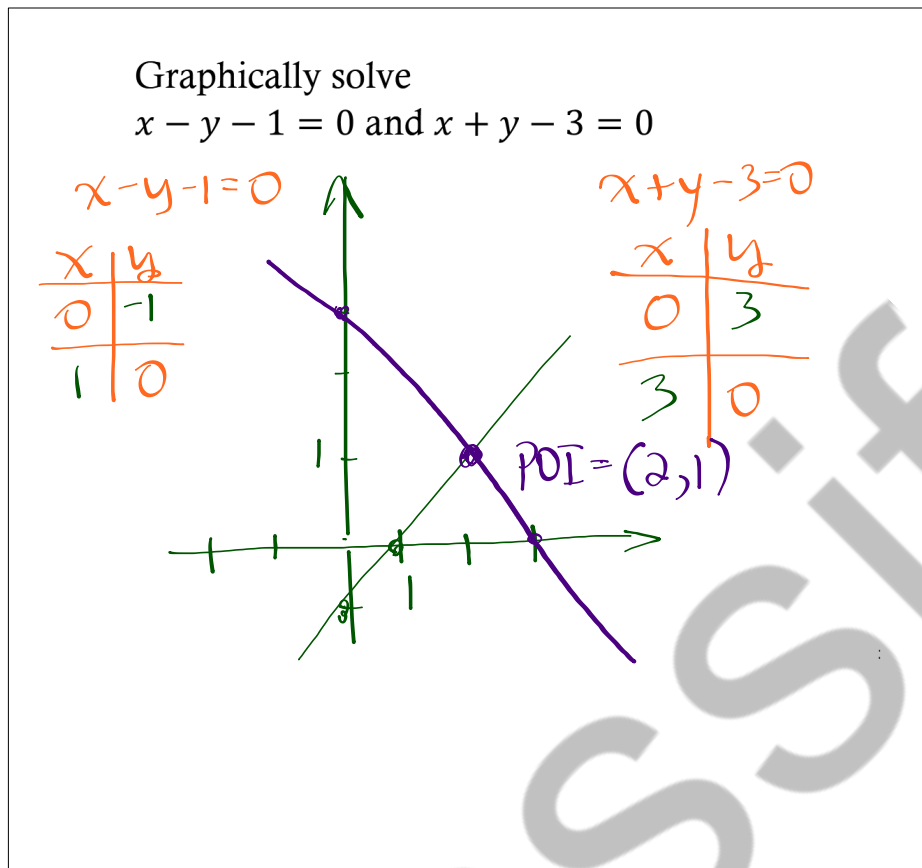
For example, consider the following system of equations:

$$\begin{array}{ll} x+4y=5 & (-3)+4(2)=5 \checkmark \text{ Equation of line 1} \\ 2x-y=-8 & 2(-3)-(2)=-8 \checkmark \text{ Equation of line 2} \end{array}$$

The point  $(-3, 2)$  satisfies the first equation since  $-3+4(2)=5$ , and so  $(-3, 2)$  is a point on line 1. The point  $(-3, 2)$  also satisfies the second equation since  $2(-3)-2=-8$ , and so  $(-3, 2)$  is a point on line 2. Therefore  $(-3, 2)$  is a *solution* to the system of equations because it satisfies *both* equations and is a *point* on both lines.

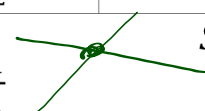
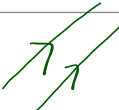
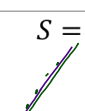
## GRAPHICAL METHOD

The graphical method of solving a system of two equations involves graphing the line corresponding to each equation and finding a point that is on both these lines.



### NUMBER OF SOLUTIONS

An easy way to determine the number of solutions is to analyze the coefficients.

Case	$a_1x + b_1y = c_1$ $a_2x + b_2y = c_2$	Solution is:
1	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ 	$S = \{x, y\}$
2	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ 	$S = \emptyset$ NONE <sup>0</sup>
3	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ 	$S = \{(x, y)   a_1x + b_1y = c_1\}$ every point on the line infinitely.

Ex: Determine the number of solutions

1.  $\begin{cases} 2x + 3y = 6 \\ 4x + 6y = 0 \end{cases}$   $\frac{2}{4} = \frac{3}{6} \neq \frac{6}{0}$   
NONE (Scenario 2)

2.  $\begin{cases} x + 2y = 4 \\ 3x + 6y = 12 \end{cases}$   $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$  (scenario 3)  
 $\rightarrow$  subtrah  $S = \{(x, y) | x + 2y = 4\}$

SOLVING WORD PROBLEMS GRAPHICALLY  
PG 172

Ex: A charity sells t-shirts and long sleeve shirts.

# of T-shirts	# of long sleeve shirts	Profit
150	200	1000\$
100	400	1200\$

Find the profit when 120 t-shirts and 300 long sleeve shirts are sold.

**Step 1**

# of T-shirts	# of long sleeve shirts	Profit
150	200	1000\$
100	400	1200\$

Find the profit when 120 t-shirts and 300 long sleeve shirts are so

Step

- 1) Define variab
- 2) Write the 2 equatic
- 3) Solve graphica
- 4) Answer the questi

**Unknowns**

x: Profit / t shirt  
y: Profit / long sleeve

**Step 2**

$$150x_1 + 200y_1 = 1000$$

$$100x_2 + 400y_2 = 1200$$

**Step 3**

$x_1$	$y_1$	$x_2$	$y_2$
0	5	0	3
6.6	0	12	0

**Step 4**

$$120(4) + 300(2)$$

$$1080\$$$

x: 4\$ = profit / t shirt  
y: 2\$ = profit / long sleeve

## 6.2 Algebraic solving of a two-variable first degree system

### COMPARISON PG176 #6-8

Format

$$y = ax + b$$

$$y = cx + d$$

Example

$$y = -\frac{3}{5}x + 5$$

$$y = \frac{1}{2}x - 3$$

Solve the example

$$y = -\frac{3}{5}x + 5$$

$$y = \frac{1}{2}x - 3$$

LCM of 5 & 2 is 10

$$-3x + 5 = \frac{1}{2}x - 3$$

$$-6x + 10 = 5x - 6$$

$$-6x - 5x = -6 - 10$$

$$-11x = -16$$

$$x = \frac{16}{11}$$

Find y by plugging it in

$$y = \frac{1}{2} \left( \frac{16}{11} \right) - 3$$

$$y = \frac{8}{11} - 3$$

$$y = \frac{8}{11} - \frac{33}{11}$$

$$y = -\frac{25}{11}$$

PoI =  $\left( \frac{16}{11}, -\frac{25}{11} \right)$

In Word Problems:

$$y = ax + b$$

Hint: 'a' is the amount per  $\left\{ \begin{array}{l} \text{hour} \\ \text{day} \\ \text{student} \end{array} \right.$

'b' is the amount paid once

ex:  $\left\{ \begin{array}{l} \text{a membership to a gym} \\ \text{bonus at a job} \end{array} \right.$

9.80\$/hr Tip  
 $ax + b$  200

Ex: Two video clubs compete for clients. Club A has no membership fee but charges 4\$ per movie. Club B charges 20\$ for membership but only 3\$ per movie. What is the cost when the number of movies is the same?

x: # of movies  
 y: Total Cost \$

Comparison

$$4x = 3x + 20$$

$$x = 20 \text{ movies}$$

$$\text{Club A: } y = 4x$$

$$\text{Club B: } y = 3x + 20$$

$$y = 4(20)$$

$$y = 80\$$$