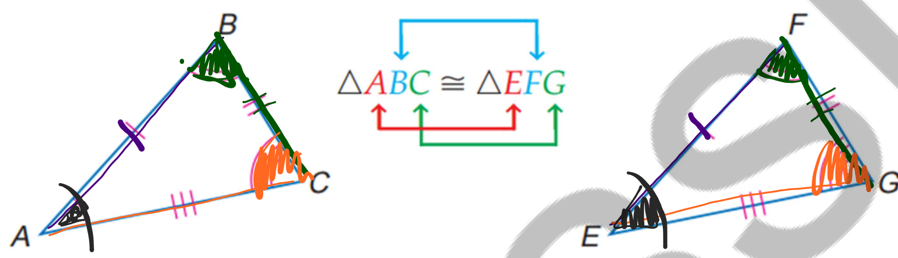
Chapter 7:

Isometric Triangles (Congruent)

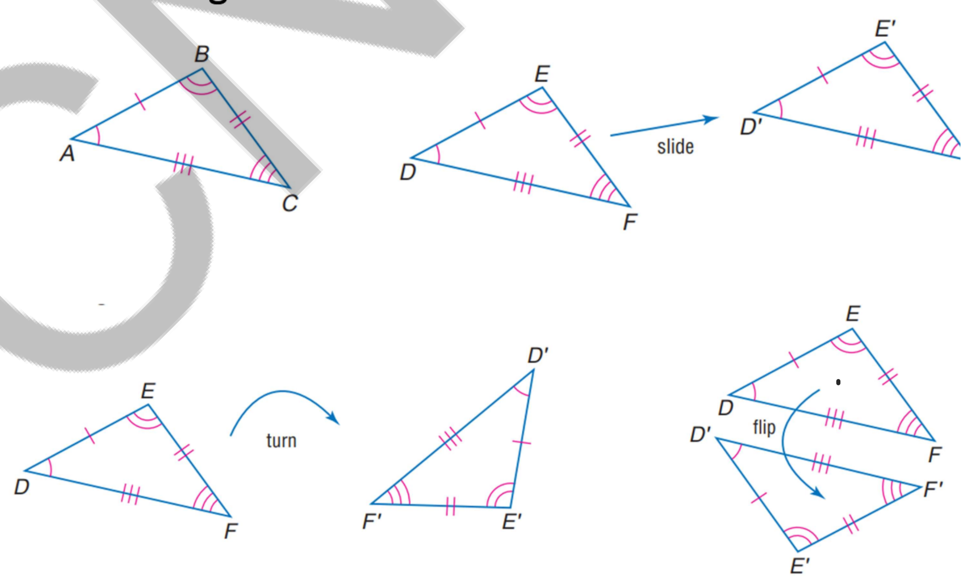
Triangles that are the same size and shape are called isometeic or congruent triangles.

Each triangle has three Sides and three angles.

If all six of the corresponding parts of two triangles are congruent, then the triangles are congruent.



If you slide, flip, or turn a triangle, the size and shape do not change.



Third Angle Theorem:

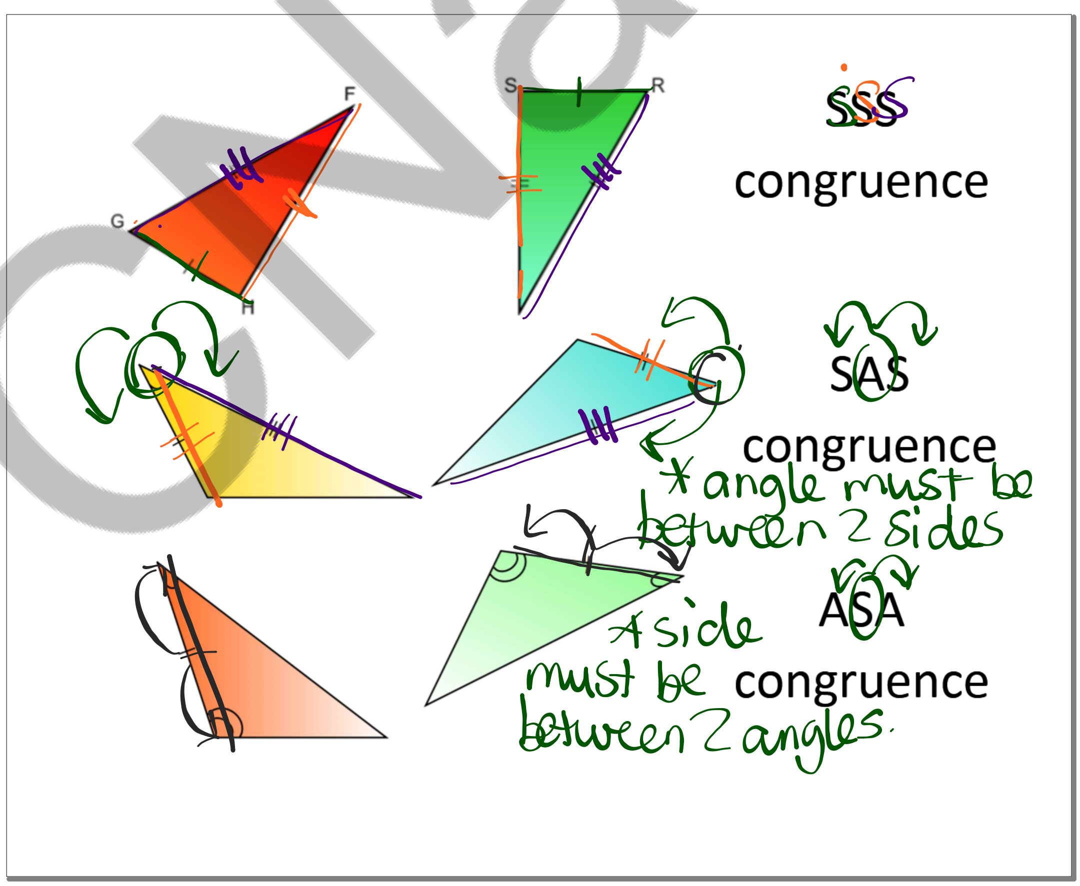
If you know two angles are equal then the third one is necessarily equal.

To prove that two triangles are congruent, it is not necessary to show all six conditions. There are minimum conditions for showing that triangles are congruent These are known as theorems of congruence.

Five Geometric Statements for Congruence

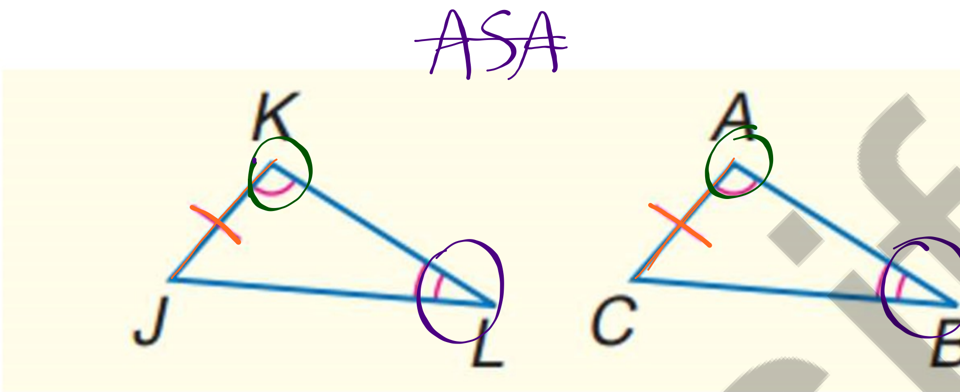
These 5 Rules state that 2 triangles will be congruent:

SSS
SAS
ASA
AAS
HL



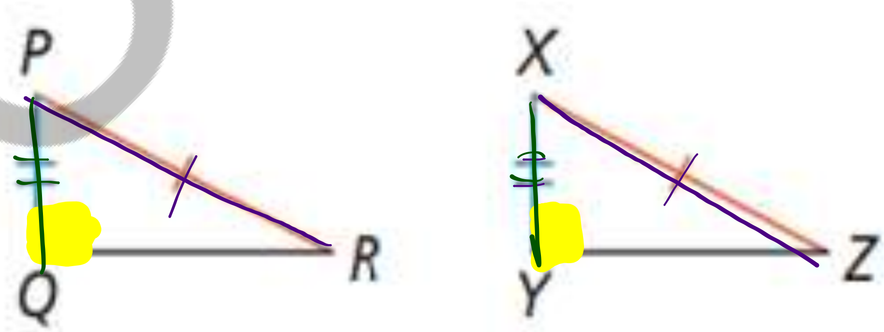
Angle-Angle-Side Congruence

AAS



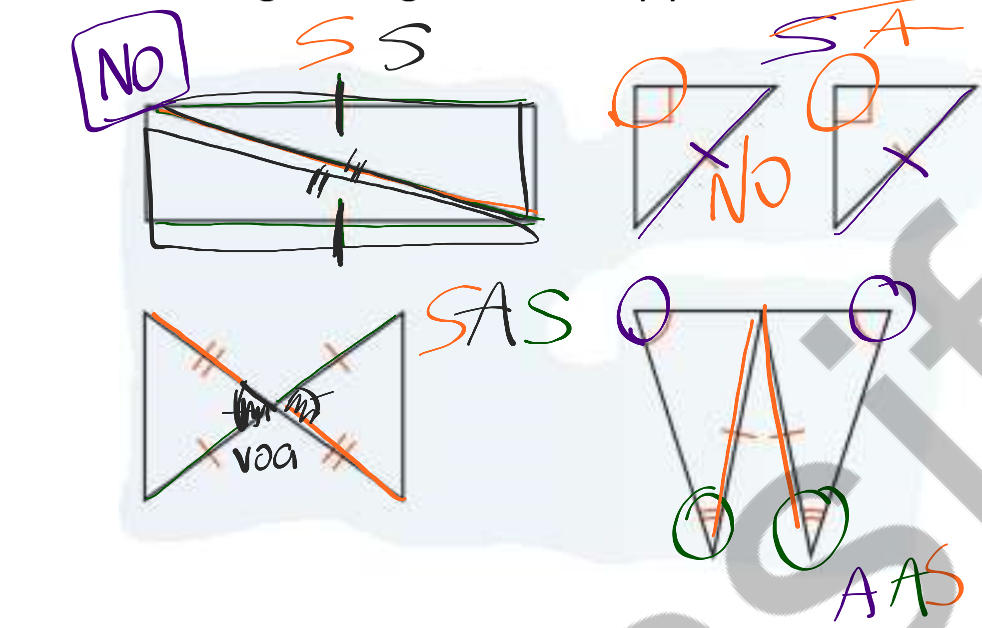
Hypotenuse-Leg Congruence

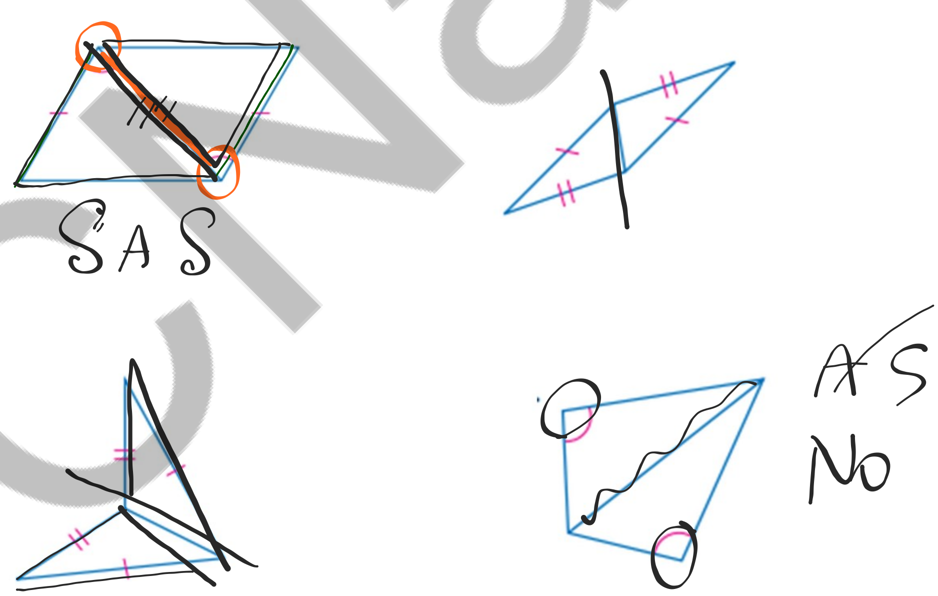
In a right triangle, if you know 2 sides are congruent then the 2 triangles are congruent. HL

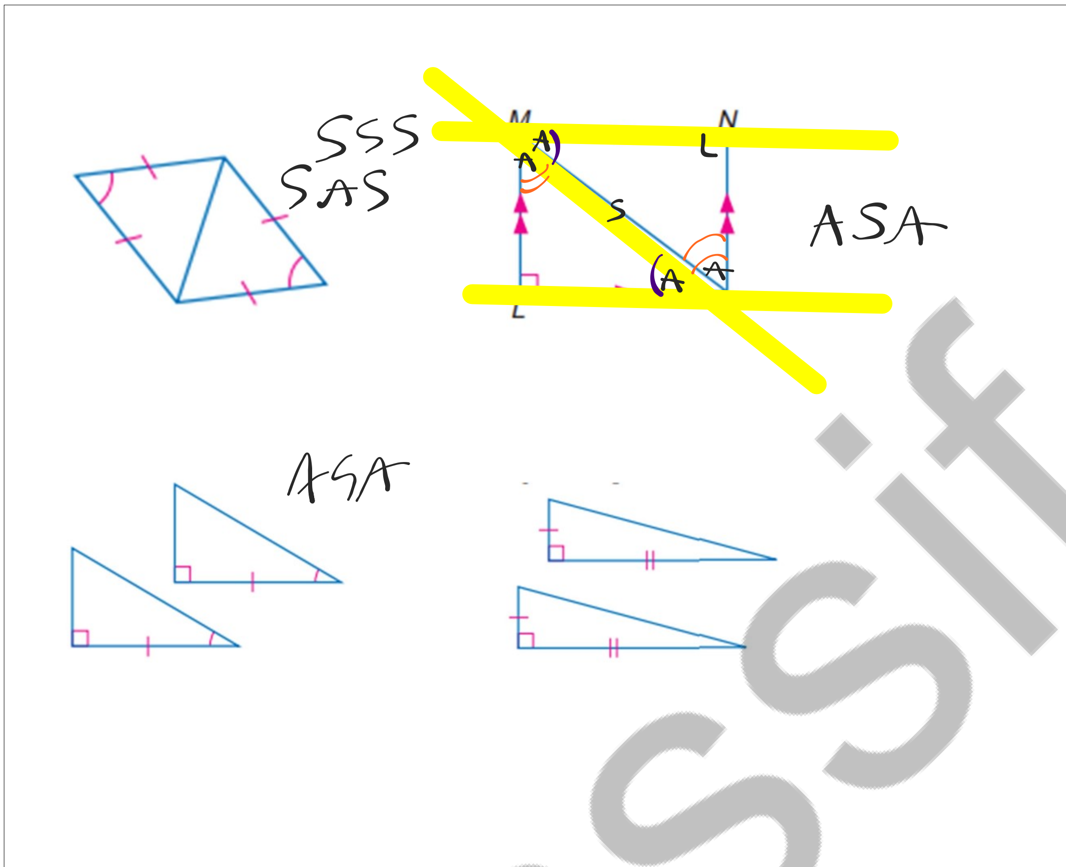


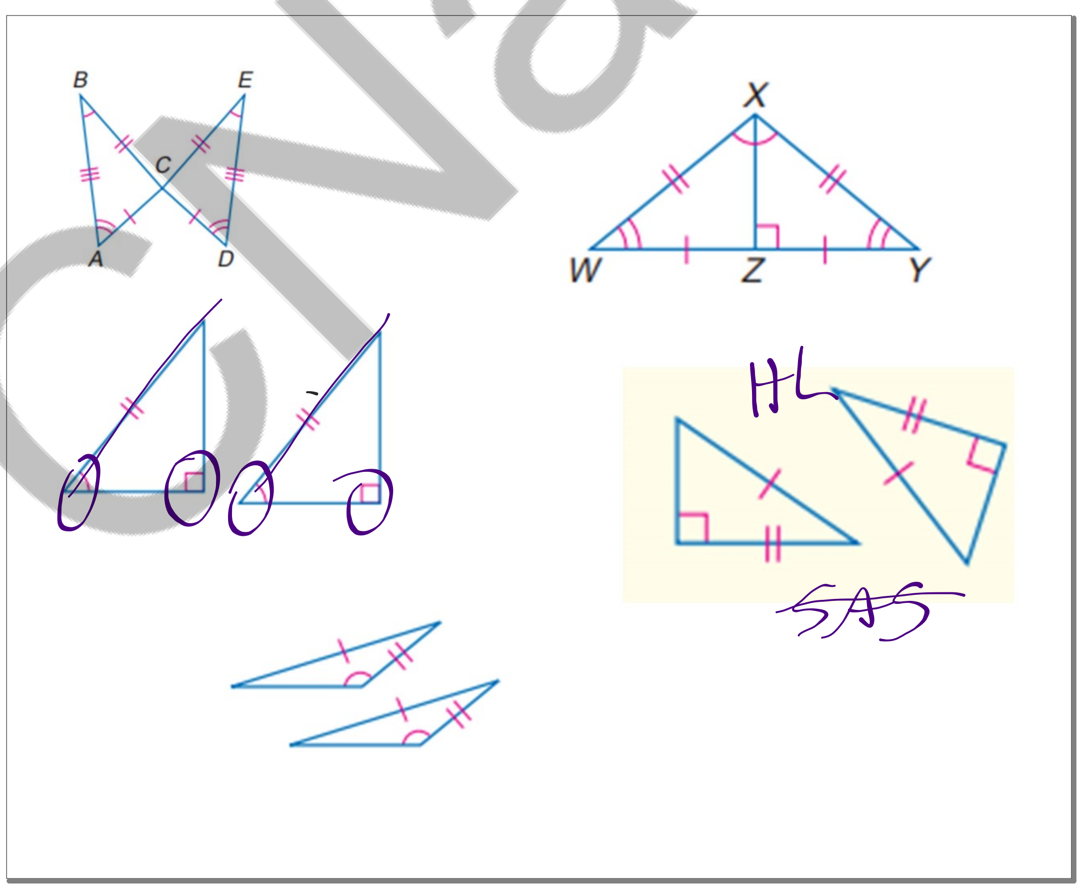
SAS SSS ASA AAS HL

Are the triangles congruent? Justify your answer.



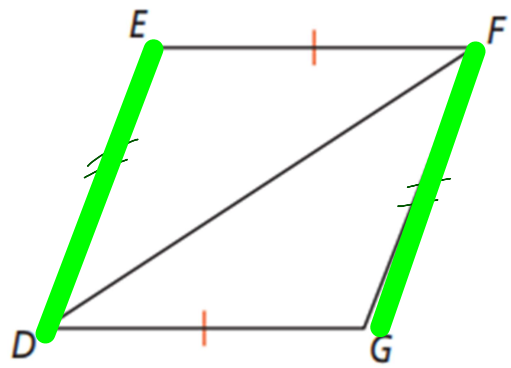



SSS
SAS
M
N
L
ASA
ASA

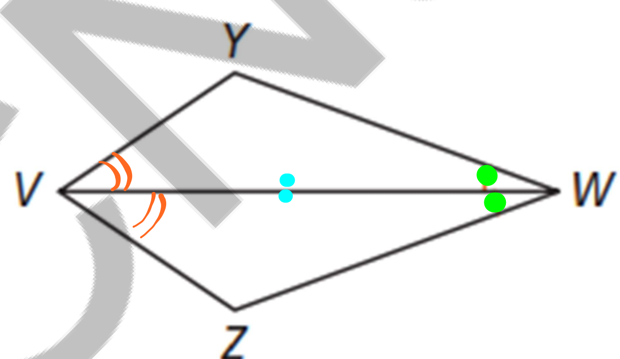

B
E
C
A
D
X
W
Z
Y
HL
SAS

What other information do you need to prove that the triangles are congruent?

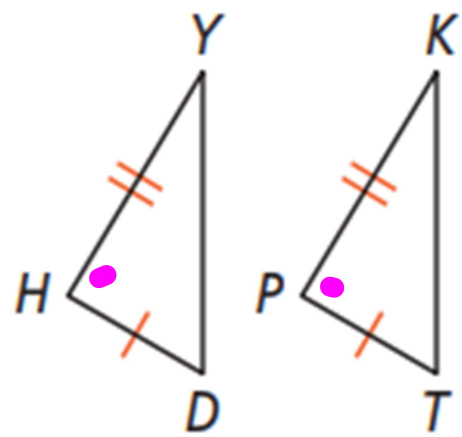
By SSS



By ASA



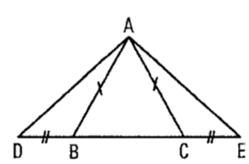
By SAS



Proofs:

Ex:1

Triangle ABC is isosceles with principal vertex A. If segments BD and CE are congruent, justify the statements proving that triangles ABD and ACE are congruent.



Hypothesis: – $\triangle ABC$ isosceles
– $\overline{BD} \cong \overline{CE}$

Statement	Justification
1. $\angle ABC \cong \angle ACB$	The angles at the base of an isosceles triangle are congruent.
2. $\angle ABD \cong \angle ACE$	The supplementary angles to two congruent angles are congruent.
3. $\overline{AB} \cong \overline{AC}$	The sides meeting at the main vertex of an isosceles triangle are congruent.
4. $\overline{BD} \cong \overline{CE}$	Hypothesis
5. $\triangle ABD \cong \triangle ACE$	SAS

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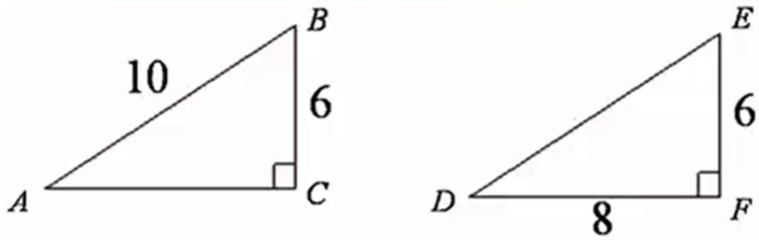
Ex 2:

2. Determine whether the given triangles are isometric. If they are, state the theorem that proves congruency.



By the pythagorean theorem, ED =10

HL

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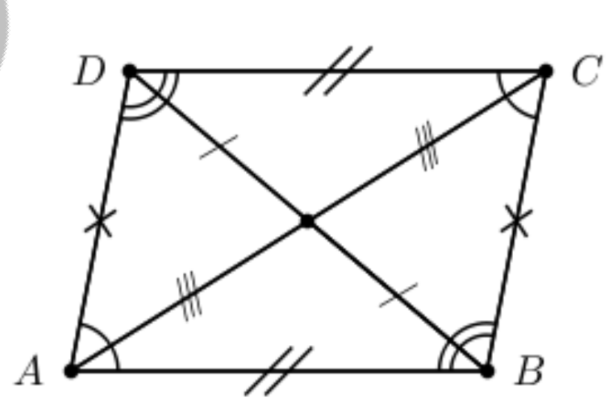
Parallelogram properties

Definition: A parallelogram is any quadrilateral with parallel opposite sides.

State the characteristics…



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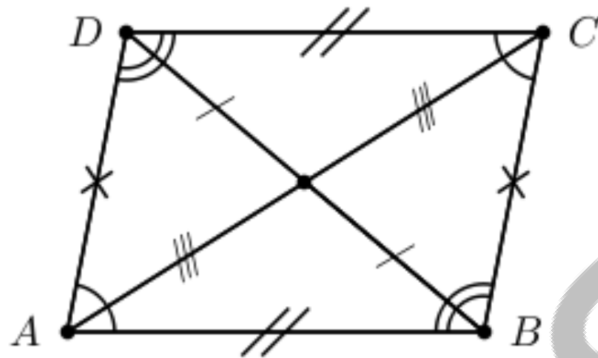
- Opposite sides are parallel by definition.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.



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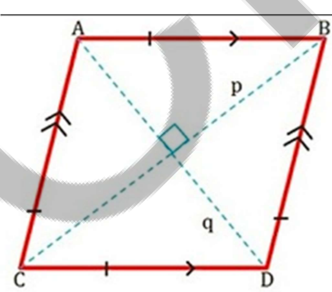
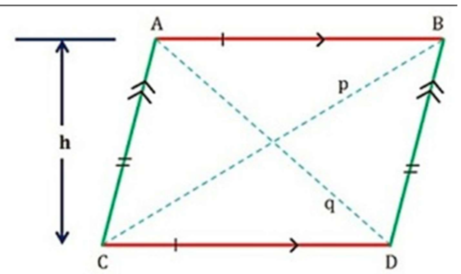
True or False?

A rhombus is a parallelogram. false

A parallelogram is a rhombus. true



Rhombus



Parallelogram

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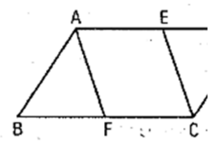
Ex 3:

In the parallelogram on the right, E and F are the respective mid-points of sides AD and BC. Prove that triangles ABF and DCE are congruent.



Statement	Justification
1. $\overline{AB} \cong \overline{CD}$	The opposite sides of a parallelogram are congruent
2. $\angle ABC \cong \angle ADC$	The opposite angles of a parallelogram are congruent.
3. $\overline{BF} \cong \overline{DE}$	BF and DE are each mid-points of congruent segments, AD and BC being congruent because they are opposite sides of a parallelogram.
4. $\Delta ABF \cong \Delta DCE$	SAS

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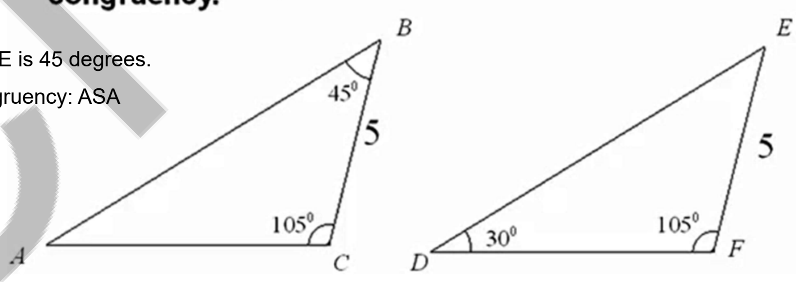
Ex 4:

4. Determine whether the given triangles are isometric. If they are, state the theorem that proves their congruency.

By Calculation, <E is 45 degrees.

Theorem of Congruency: ASA



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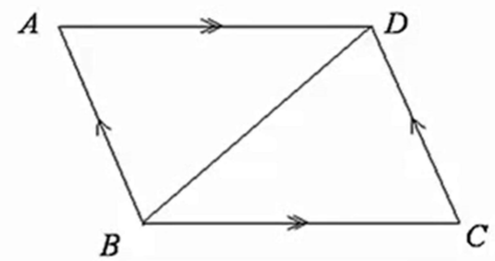
5. Using a statement/justification table. Prove that $\Delta ABD \cong \Delta CDB$ in the parallelogram below.



Statement	Justification
Tri ABD = Tri CDB	Alternate-Interior angles formed by the transversal line BD through parallel lines AD and BC
<ABD = <BDC	Alternate-Interior angles formed by the transversal line BD through parallel lines AB and DC
BD = BD	BD is a common side to triangles ABD and CDB
Tri ABD = Tri CDB	ASA
AB = DC and AD=BC	Corresponding elements in isometric triangles are congruent

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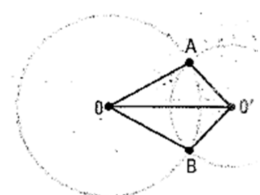
Ex 6:

Two circles centered at 0 and 0′ intersect each other at two points A and B. Complete the steps proving that angles 0A0′ and 0B0′ are congruent.



Hypothesis: – 0 and 0′ are the centres of two distinct circles.
– A and B are the intersection points of the two circles.

Consider the triangles *0A0′ and 0B0′.*

Statement	Justification
1. $\overline{0A} \cong \overline{0B}$	The radii of the circle centered at O are congruent.
2. $\overline{0'A} \cong \overline{0'B}$	The radii of the circle centered at O' are congruent.
3. $\overline{00'} \cong \overline{00'}$	Common side to triangles OAO' and OBO'
4. $\Delta 0A0' \cong \Delta 0B0'$	SSS
5. $\angle 0A0' \cong \angle 0B0'$	Corresponding elements in isometric triangles are congruent.

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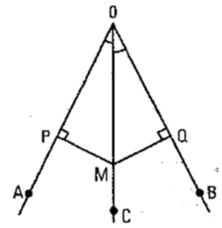
Ex 7:

- Consider the angle A0B and its bisector 0C. From a point M chosen at random on the bisector, segments MP and MQ are drawn perpendicularly to the sides 0A and 0B respectively. Justify the statements proving the following theorem: **Any point on an angle's bisector is equidistant to the sides of the angle.**



Hypothesis: – 0C is the bisector of $\angle A0B$
– $m\angle 0PM = m\angle 0QM = 90°$

Consider the right triangles 0PM and 0QM.

Statement	Justification
1. $\angle PM0$ is complementary to $\angle P0M$.	The acute angles of the right tri OPM are complementary
2. $\angle QM0$ is complementary to $\angle Q0M$.	The acute angles of the right triQOM are completementary
3. $\angle P0M \cong \angle Q0M$	OM is the bisector of <POQ
4. $\angle PM0 \cong \angle QM0$	The complementary angles of congruent angles are congruent
5. $\overline{0M} \cong \overline{0M}$	OM is a common side to both triangles
6. $\Delta 0PM \cong \Delta 0QM$	ASA
7. $m\overline{MP} \cong m\overline{MQ}$	Corresponding elements in congruent triangles are congruent.

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