Factoring by **Completing the square**
Stencil #1-8 on completing the square

→not in workbook but in the program, as well as in sec 5SN

$$ax^2 + bx + c \rightarrow \underbrace{a(x+d)^2 - e}_{\text{DOS}} \rightarrow \underbrace{a(x+f)(x+g)}_{\text{End}}$$

1 2 3

Case 1: $ax^2 + bx + c$ where a =1

Procedure:

1) Write the first two terms in a bracket and the constant further away
2) Calculate $(\frac{b}{2})^2$ and add that term in the bracket. Remember to immediately subtract the same term outside the bracket so that the total is 0.
3) Factor by Perfect Square Trinomial with the terms inside the bracket. Combine the constants outside the bracket.

4) Factor by Difference of Squares.

Example: Factor $x^2 + 6x - 16$

$$(\underbrace{x^2 + 6x + 9}_{\text{Perf ST}}) \underbrace{-16 - 9}_{\text{combine}}$$

$b = 6$

$\left(\frac{6}{2}\right)^2$

$(3)^2$

9

$$\underbrace{(x+3)^2 - 25}_{\text{DOS}}$$

$$(x+3-5)(x+3+5)$$

$$(x-2)(x+8)$$

Case 2: $ax^2 + bx + c$ where $a \neq 1$

Procedure:

1) Factor out the leading coefficient a using [square brackets]. This may produce fractions, but the important thing is to get a leading coefficient of 1 inside the brackets.
2) Write the first two terms in (round brackets). Add and subtract $\left(\frac{b}{2}\right)^2$.
3) Write the terms inside the round brackets a a perfect square trinomial in factored form. Combine the last two terms.
4) Factor using difference of squares.

Example: Factor $2x^2 + 12x - 14$

$2[x^2 + 6x - 7]$

Always positive

Always Neg.

$2[\underbrace{(x^2 + 6x + 9)}_{PST} - 7 - 9]$

$b = 6$

$\left(\frac{6}{2}\right)^2$

$(3)^2$

9

$2\underbrace{[(x+3)^2 - 16]}_{DOS}$

$2[x+3-4][x+3+4]$

$2(x-1)(x+7)$

1.5 Rational Expressions pg.25-27

Def: An expression in the form $\frac{P(x)}{Q(x)}$, where P(x) and Q(x) are polynomials, and Q(x)≠0, is a rational expression.

For what values is the denominator zero?

Restrictions

1) $\frac{5x^2+60x}{\boxed{x-3}}$ $3-3=0$

$x-3 \neq 0$

$x \neq 3$

2) $\frac{3x+4}{\boxed{2x}}$ $2(0)=0$

$\frac{2x}{2} \neq \frac{0}{2}$

$x \neq 0$

3) $\frac{7}{\boxed{3x+10}}$

$3x+10 \neq 0$

$\frac{3x}{3} \neq \frac{-10}{3}$

$x \neq \frac{-10}{3}$

A rational expression is undefined for the values where its denominator is equal to 0. When simplifying rational expressions, you must specify the restrictions.

* Remember it all your life.

Zero Product Principle:

If $a \cdot b = 0$ then either $a=0$ ~~or~~ and $b=0$

Examples: Determine the restrictions.

1) $\frac{x^2-3x}{\boxed{9x^2+24x+16}}$

$9x^2+24x+16 \neq 0$

$(3x+4)(3x+4)=0$

using zero product principle

$3x+4=0$ $3x+4=0$

$x=-4/3$. $x=-4/3$

Restriction is $x \neq -4/3$.

Reminder: You can not cancel terms; you can only cancel factors.

2) $\frac{3x^2+15x-9}{x^2+2x-15}$

$x^2+2x-15=0$

$(x-3)(x+5)=0$

P/S method

zero product principle

$x-3=0$ $x+5=0$

$x \neq 3$ $x \neq -5$

Restrictions are $x \neq 3, -5$

Testing

$x^2+2x-15$

$(3)^2+2(3)-15$

0

$x^2+2x-15$

$(-5)^2+2(-5)-15$

0

3) $\frac{8xy+9y}{\boxed{y^4-81}}$

$(y^2-9)(y^2+9)=0$

$(y-3)(y+3)(y^2+9)=0$

$y-3=0$ $\quad$ $y+3=0$ $\quad$ $y^2+9=0$

$y\neq 3$ $\quad$ $y\neq -3$ $\quad$ $y^2=-9$

cannot do

cannot square root negative

Simplifying Rational Expressions

$\frac{20}{70}=\frac{\cancel{2}\times 2\times\cancel{5}}{\cancel{2}\times\cancel{5}\times 7}=\frac{2}{7}$ ← Factor top and factor bottom & cancel

Examples: Simplify and state restrictions.

1) $\frac{12x^2-4x}{5x^2+15x}=\frac{(4\cancel{x})(3x-1)}{(5)\cancel{(x)}(x+3)}$ → The factors here are 4 and x and (3x−1).

The factors are 5 and x and (x+3).

$=\frac{4(3x-1)}{5(x+3)}$

Restrictions: Take the denominator from the original problem.

Restrictions

$5x^2+15x=0$

$5x(x+3)=0$

$5x=0$ $\quad$ $x+3=0$

$x\neq 0$ $\quad$ $x\neq -3$

Restrictions are 0, −3.

2) $\frac{x^2+17x+70}{x^2+5x-14}$ P/S P/S

Restrictions

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$x+7=0$ $x-2=0$

$x\neq -7$ $x\neq 2$

Simplify

$\frac{(x+10)\cancel{(x+7)}}{\cancel{(x+7)}(x-2)}$

$\frac{x+10}{x-2}$

3) $\frac{x^2+8x+16}{x^2-16}$

Restrictions

$x^2-16=0$

$(x-4)(x+4)=0$

$x\neq 4$ $x\neq -4$

Simplify

$\frac{(x+4)\boxed{\cancel{(x+4)}}}{(x-4)\boxed{\cancel{(x+4)}}}=\boxed{\frac{x+4}{x-4}}$

4) $\frac{(x+2)^2-16}{x^2-36}$

$$= \frac{(x+2-4)(x+2+4)}{(x-6)(x+6)}$$

$$= \frac{(x-2)\cancel{(x+6)}}{(x-6)\cancel{(x+6)}}$$

$$= \frac{x-2}{x-6}$$

Rest.

$x \neq 6, -6$

5) $\frac{3x^2+7x+2}{9x^2-1}$

$$= \frac{\cancel{(3x+1)}(x+2)}{(3x-1)\cancel{(3x+1)}}$$

$$= \frac{x+2}{3x-1}$$

Restr.

$x \neq 1/3, -1/3$

Multiplication of Rational Expressions

p25#1-2

More required

$$\frac{1}{4} \times \frac{24}{30} \times \frac{2}{8} \times \frac{15}{20}$$

FACTOR THE NUMERATORS AND DENOMINATORS.

$$\frac{1}{\cancel{2}\cdot\cancel{2}} \cdot \frac{3\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}}{\cancel{5}\cdot\cancel{3}\cdot\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{2}\cdot 2\cdot 2} \cdot \frac{\cancel{5}\cdot 3}{2\cdot 2\cdot 5}$$

$$= \frac{3}{80}$$

Reminder: You can cancel downwards and sideways
Multiply on the same line.

Simplify and state restrictions

DETERMINE IF THERE ARE ANY EXCLUDED VALUES. TO DO THIS, SET THE DENOMINATORS EQUAL TO 0 AND SOLVE FOR X.

1. $\frac{x^2+4x+4}{2x^2-x-10} \times \frac{x+5}{x^2+2x}$

$$\frac{(x+2)\cancel{(x+2)}}{(2x-5)\cancel{(x+2)}} \cdot \frac{x+5}{x\cancel{(x+2)}}$$

$$= \frac{x+5}{x(2x-5)}$$

Restric

$x \neq 5/2, -2, 0$

Note: Final Answers are left in factored form.

2. $\frac{y^2-4}{y^2-2y} \times \frac{y}{y^2+10y+16}$ See next page

3. $\frac{c^2-10c+25}{10c-100} \times \frac{c-10}{45-9c}$

$$\frac{(c-5)\cancel{(c-5)}}{10\cancel{(c-10)}} \cdot \frac{\cancel{(c-10)}}{9(5-c)}$$

$9(-1)\cancel{(-5+c)}$ ← Negative Trick

$$= \frac{c-5}{-90}$$

Rest: $c \neq 10, 5$

4. $\frac{9r^3-54r^2}{9r^2+45r} \times \frac{9r^2+9r}{9r^3-54r^2}$

$$\frac{\cancel{9r^2}\cancel{(r-6)}}{\cancel{9r}(r+5)} \cdot \frac{\cancel{9r}(r+1)}{\cancel{9r^2}\cancel{(r-6)}}$$

$$= \frac{r+1}{r+5}$$

Rest: $r \neq 0, -5, 6$

Division of Rational Expressions pg26 #3-4

$$\frac{9}{16} \div \frac{24}{18}$$

Reminder: Rewrite division as multiplication by the reciprocal.

$$= \frac{9}{16} \cdot \frac{18}{24}$$

$$= \frac{\cancel{3}\cdot 3}{2\cdot 2\cdot 2\cdot 2} \cdot \frac{3\cdot\cancel{2}\cdot 3}{2\cdot\cancel{2}\cdot\cancel{3}\cdot 2}$$

$$= \frac{27}{64}$$

Simplify and state restrictions (always)

1) $\frac{5x}{x-3} \div \frac{3x}{x-3}$ ← Restriction in yellow

$$\frac{5\cancel{x}}{\cancel{x-3}} \cdot \frac{\cancel{x-3}}{3\cancel{x}}$$

$$= \frac{5}{3}$$

Rest: $x \neq 3, 0$

Because 3x becomes the denominator in the reciprocal of $\frac{3x}{x-3}$, we must find the values of x that would make that factor equal to 0.

$$\frac{16x-56}{8} \div \frac{8x-28}{4}$$

Determine the excluded values that make the denominators & the numerator of the divisor equal to 0.

$$\frac{\cancel{8}(\cancel{2x-7})}{\cancel{8}} \cdot \frac{\cancel{4}}{\cancel{4}(\cancel{2x-7})}$$

$$= 1$$

Rest:

$x \neq 7/2$

2) $$\frac{c^2+3x-40}{c^2+2c-35} \div \frac{c^2+2c-48}{c^2+3c-18}$$

Rest in yellow

$$= \frac{\cancel{(c-5)}\cancel{(c+8)}}{\cancel{(c-5)}(c+7)} \cdot \frac{(c-3)(c+6)}{(c-6)\cancel{(c+8)}}$$

$$= \frac{(c-3)(c+6)}{(c+7)(c-6)}$$

Keep in factored form!

Rest: $c \neq 5, -7, 3, -6, 6, -8$

3) $\frac{x^2+2x-15}{x^2-4x-45} \div \frac{x^2+x-12}{x^2-5x-36}$

$$\frac{\cancel{(x-3)}\cancel{(x+5)}}{\cancel{(x+5)}\cancel{(x-9)}} \cdot \frac{\cancel{(x-9)}\cancel{(x+4)}}{\cancel{(x+4)}\cancel{(x-3)}}$$

$= 1$

Rest: $x = -5, 9, -4, 3$

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