

Chapter 10: Statistics



10.1 Two variable distributions

Contingency Tables

A contingency table illustrates a two-variable distribution. It is another way to display statistical data, like a scatter plot.

Ex: Here is some data about the **number of hours of sleep** (x) that a student gets before an exam and their **grade on the exam** (y).

(2, 50) (3, 50) (4, 45) (4, 35) (5, 50) (5, 53) (6, 60) (6, 45) (7, 70) (7, 64) (8, 80) (8, 55) (8, 75) (8, 78) (8, 83) (9, 79) (9, 30) (9, 89) (9, 62) (9, 93) (9, 85) (10, 85) (10, 71) (10, 92) (10, 88)

Do you think there is a correlation between the two variables?
Fill in the table:

| Hours Slept (x) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----------------|---|---|---|---|---|---|---|---|----|
| Grade % (y) | | | | | | | | | |
| [30, 40[| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| [40, 50[| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| [50, 60[| 1 | 1 | 0 | 2 | 0 | 0 | 1 | 0 | 0 |
| [60, 70[| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| [70, 80[| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 |
| [80, 90[| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 |
| [90, 100[| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

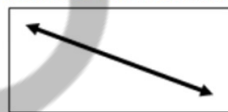
HHH
S

- Correlation exists if...

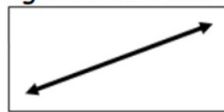
large numbers on the diagonal, small numbers (look for zeros) in the corners

- the more obvious the pattern = stronger correlation

Positive correlation:



Negative correlation:



when in a contingency table

| Hours Slept (X) | Total | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|-----------------|-------|---|---|---|---|---|---|---|---|----|-------|
| Grade % (Y) | | | | | | | | | | | |
| [30, 40[| 2 | 1 | 1 | | | | | | | | 2 |
| [40, 50[| 2 | | 1 | | | | | | | | 2 |
| [50, 60[| 5 | 1 | 2 | | | 1 | | | | | 5 |
| [60, 70[| 3 | | | 1 | | | | | | | 3 |
| [70, 80[| 5 | | | | 1 | 1 | 1 | 1 | 1 | | 5 |
| [80, 90[| 6 | | | | | 2 | 2 | 2 | | | 6 |
| [90, 100] | 2 | | | | | | | 1 | 1 | | 2 |
| Total | 25 | 1 | 2 | 2 | 2 | 2 | 2 | 5 | 6 | 4 | 25 |

Contingency Tables

1. What kind of correlation exists, given the table below?

| X \ Y | [0,1[| [1,2[| [2,3[| [3,4[| [4,5[|
|-------|-------|-------|-------|-------|-------|
| 2 | 0 | 0 | 0 | 0 | 4 |
| 4 | 0 | 0 | 0 | 3 | 0 |
| 6 | 0 | 0 | 3 | 0 | 0 |
| 8 | 0 | 2 | 0 | 0 | 0 |
| 10 | 3 | 0 | 0 | 0 | 0 |

neg
Strong

2. What kind of correlation exists, given the table below?

| X \ Y | [0,20[| [20,40[| [40,60[| [60,80[| [80,100[|
|-------|--------|---------|---------|---------|----------|
| 10 | 6 | 0 | 0 | 0 | 0 |
| 20 | 0 | 7 | 0 | 0 | 0 |
| 30 | 0 | 0 | 9 | 0 | 0 |
| 40 | 0 | 0 | 0 | 8 | 0 |
| 50 | 0 | 0 | 0 | 0 | 9 |

Strong

3. What kind of correlation exists, given the table below?

| X \ Y | [0,1[| [1,2[| [2,3[| [3,4[| [4,5[|
|-------|-------|-------|-------|-------|-------|
| 1 | 3 | 1 | 5 | 2 | 3 |
| 2 | 2 | 4 | 2 | 4 | 3 |
| 3 | 5 | 2 | 4 | 1 | 4 |
| 4 | 3 | 3 | 3 | 3 | 3 |
| 5 | 4 | 4 | 1 | 6 | 5 |

Strength

Direction

Zero

Weak

Strong

Positive

Negative

4. What kind of correlation exists, given the table below?

| X \ Y | [0,2[| [2,4[| [4,6[| [6,8[| [8,10[|
|-------|-------|-------|-------|-------|--------|
| 10 | 0 | 0 | 0 | 2 | 2 |
| 20 | 0 | 0 | 3 | 4 | 2 |
| 30 | 1 | 1 | 4 | 1 | 0 |
| 40 | 2 | 3 | 3 | 2 | 0 |
| 50 | 4 | 2 | 0 | 0 | 0 |

Strength

Direction

Zero

Weak

Strong

Positive

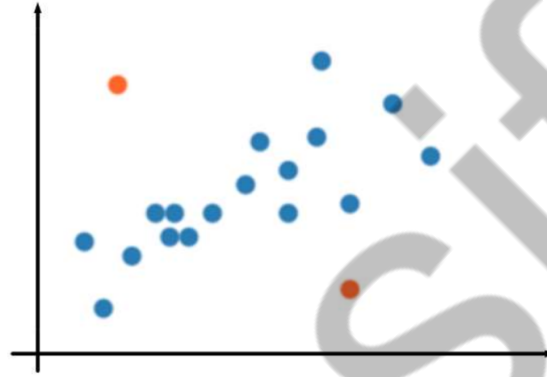
Negative

Handout

10.2 Linear correlation

Scatterplots

A scatterplot is a graph with a bunch of dots on it. We do not connect the dots. The dot that is far away from the rest is called the **OUTLIER**.



Linear Correlation

A linear correlation is a relationship between the x and y variables.

Ex: smoking and cancer

Ex: studying for a test and passing

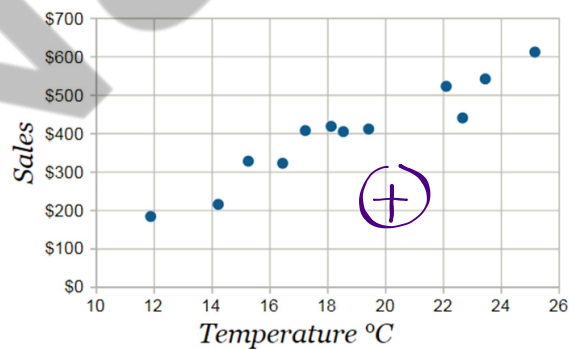
There is a correlation when the points on the graph tend to make a line.

Example: Ice Cream Sales

The local ice cream shop keeps track of how much ice cream they sell versus the temperature on that day, here are their figures for the last 12 days:

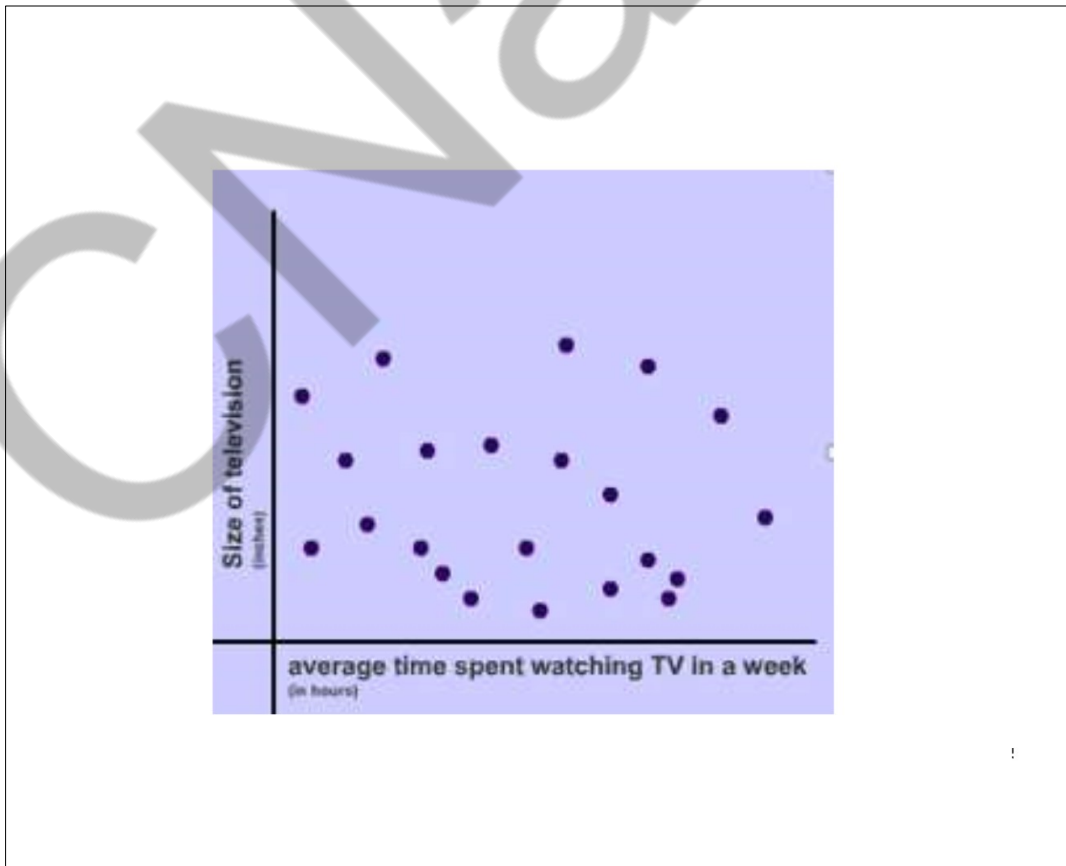
| <i>Ice Cream Sales vs Temperature</i> | |
|---------------------------------------|-----------------|
| Temperature °C | Ice Cream Sales |
| 14.2° | \$215 |
| 16.4° | \$325 |
| 11.9° | \$185 |
| 15.2° | \$332 |
| 18.5° | \$406 |
| 22.1° | \$522 |
| 19.4° | \$412 |
| 25.1° | \$614 |
| 23.4° | \$544 |
| 18.1° | \$421 |
| 22.6° | \$445 |
| 17.2° | \$408 |

And here is the same data as a [Scatter Plot](#):



We can easily see that warmer weather and higher sales go together. The relationship is good but not perfect. It is **0.9575**.

| POSITIVE CORRELATION | NEGATIVE CORRELATION |
|----------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| Both values increase together | As one value increases, the other decreases |
| <p>Hours spent studying</p> <p>Grade Point Average</p> <p>$r \approx +$</p> <p>$1 - 0.36 = 0.64$</p> | <p>Soup Sales</p> <p>Temperature</p> <p>$r \approx - \left(1 - \frac{1}{4}\right) = -(1 - 0.25) = -0.75$</p> |



Correlation coefficient (r)

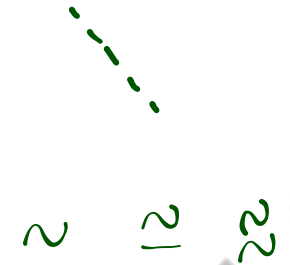
$$r \approx \begin{matrix} \text{approx} \\ \uparrow \\ \oplus \end{matrix} \left(1 - \frac{\text{short side}}{\text{long side}} \right)$$

It ranges from -1 to 1.

Use \oplus if the line is going up

Use \ominus if the line is going down

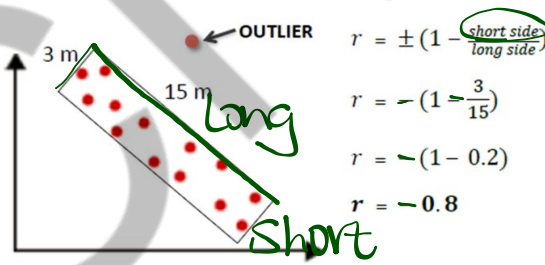
Pick the sign



INTERPRETING CORRELATION COEFFICIENT



CORRELATION (ESTIMATING COEFFICIENT)

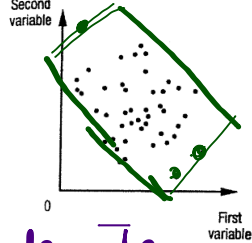


STEPS TO CALCULATE "r"

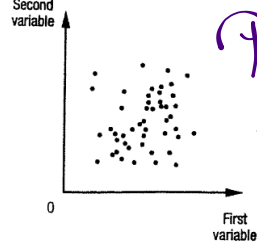
1. DRAW A RECTANGLE (MEASURE SIDE TO BE SURE)
2. LEAVE OUT OUTLIERS
3. CHECK IF THE TREND IS GOING "UP" OR "DOWN" (POSITIVE OR NEGATIVE SLOPE)
4. PUT MEASUREMENTS INTO THE FORMULA

Linear Correlation Coefficient

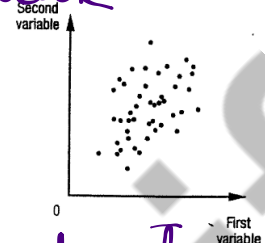
Negative weak Correlation



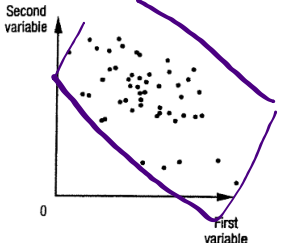
zero Correlation



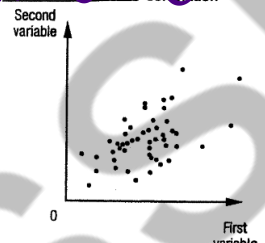
positive weak Correlation



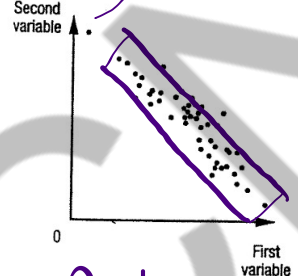
moderate Correlation



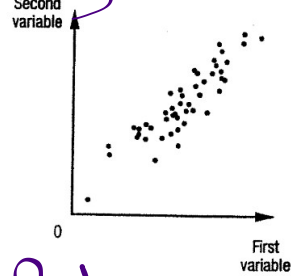
moderate Correlation



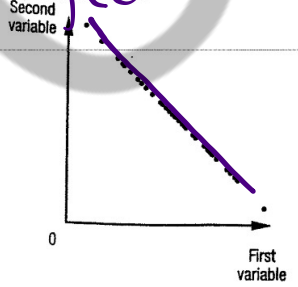
Strong Correlation



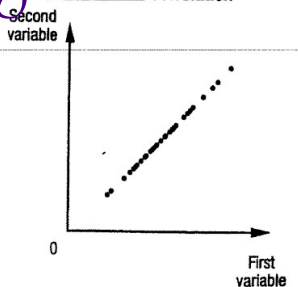
Strong Correlation



perfect Correlation



perfect Correlation

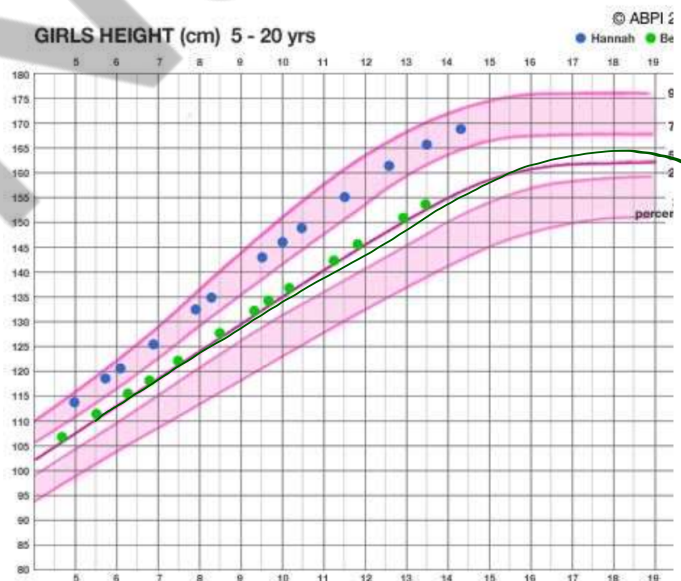


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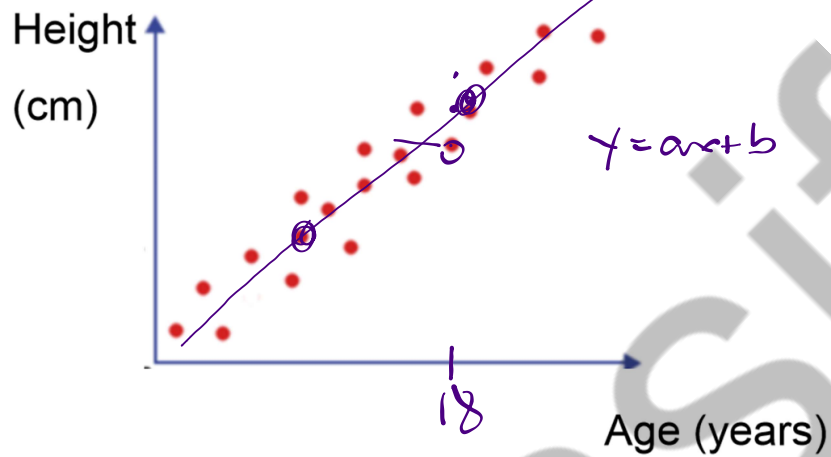
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10.3 Regression Line

When a correlation is strong, we can predict values that occur in the future. Ex: doctor predicts how tall you will get as an adult.



How can we figure out your height at 18 years old given the following scatterplot which represents the age and height of 20 kids?

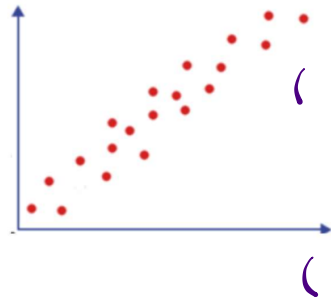


REGRESSION LINE

In a scatterplot relating two variables, the line that best represents all of the points is called a **regression line**.

To find the regression line, we use the **Mayer Line Method**.

Mayer Line Method



| | X values | Y values | |
|-------|----------|----------|---|
| AVG | ... | ... | A |
| | ... | ... | |
| | ... | ... | |
| | ... | ... | |
| | ... | ... | |
| ----- | | | |
| AVG | ... | ... | A |
| | ... | ... | |
| | ... | ... | |
| | ... | ... | |
| | ... | ... | |

Steps

- 1) Put the x values in order.
- 2) Cut the table in half.
- 3) Find the average of each section.
- 4) Find the equation $y = ax + b$

Ex: The following points compares x: the age of a tree (years) in relation to y: circumference of a tree (cm):

(10, 39), (15, 55), (23, 68), (18, 50), (7, 26),
 (25, 72), (13, 44), (19, 65), (6, 23), (14, 48)

1. Find the regression line.
2. Find the circumference of a 15 year old tree
3. Find the age for a tree with a 100 cm circumference.

① Mayer

(6,23)
 (7,26)
 (10,39)
 (13,44)
 (14,48)
 (15,55)
 (18,50)
 (19,65)
 (23,68)
 (25,72)

(10,36) x_1, y_1

(20,62) x_2, y_2

$a = \frac{62 - 36}{20 - 10} = 2.6$

$y = 2.6x + b$
 $62 = 2.6(20) + b$
 $b = 10$
 $y = 2.6x + 10$

1.) Find the regression line
 2.) Find the circumference of a 15 year old tree
 3.) Find the age for a tree with a 100 cm circumference

② $y = 2.6x + 10$
 $y = 2.6(15) + 10$
 $y = 49 \text{ cm}$

③ $y = 2.6x + 10$
 $100 = 2.6x + 10$
 $x = 34.5$
 Year

No Solution Key Available

Write on looseleaf and hand in

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