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REPRODUCIBLE SHEETS



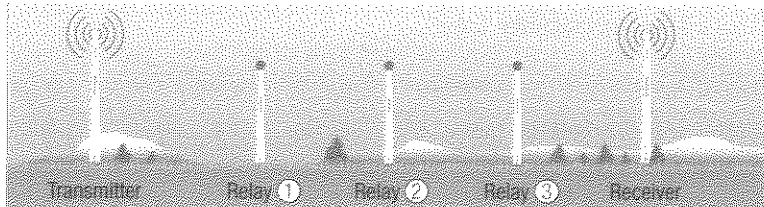
Probability and
voting procedures

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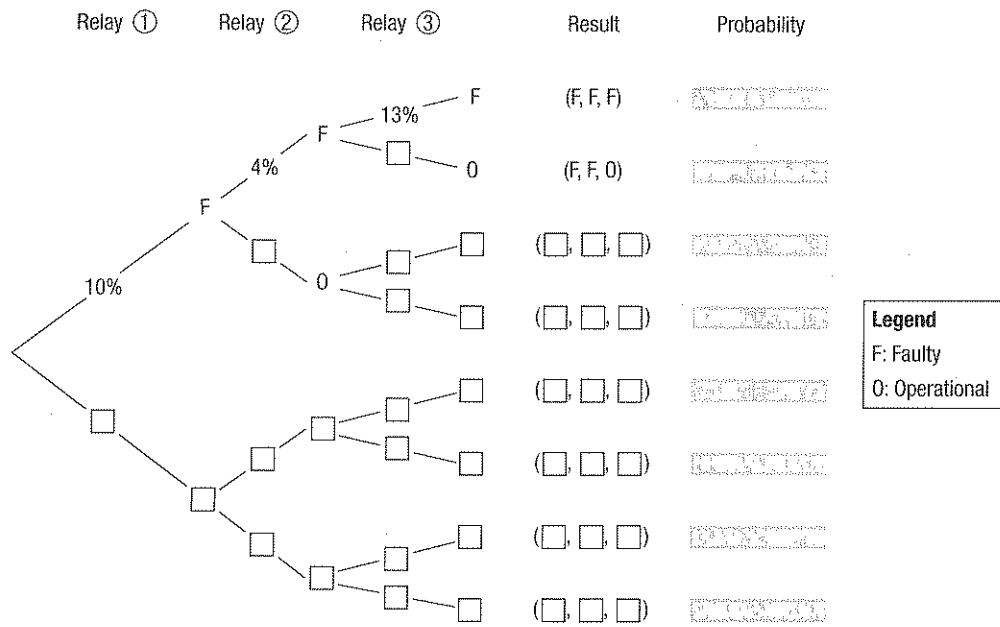
PRELIMINARY
VERSION

Transmitting a radio signal

The diagram below illustrates a transmitter, a receiver and three relays, which allow a radio signal to be transmitted. The probability of a power outage at Relay ① is 10%, that at Relay ② is 4% and that at Relay ③ is 13%.



a. Complete this probability tree.



b. What is the probability that:

- 1) Relays ① and ② are faulty?
- 2) the three relays are faulty?
- 3) at most two relays are faulty?
- 4) all relays are operational?

Name: _____

Group: _____ Date: _____

Types of events

1 A box contains 2 red marbles, 2 green marbles and 2 black marbles. Two consecutive selections are made.

a) If the marbles are replaced after each selection, do the following:

1) Construct the probability tree.

2) Will the result of the 1st selection affect the possibilities of the 2nd selection? Explain your answer.

3) Are the intermediate steps associated with this experiment dependent or independent? Explain your answer.

b) If the marbles are not replaced after each selection, do the following:

1) Construct the probability tree.

2) Will the result of the 1st selection affect the possibilities of the 2nd selection? Explain your answer.

3) Are the intermediate steps associated with this experiment dependent or independent? Explain your answer.

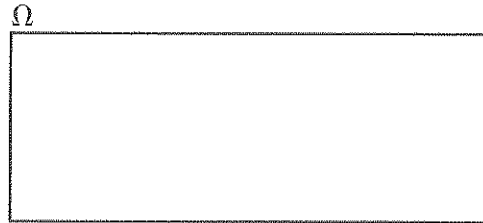
Name: _____

Group: _____ Date: _____

2 An experiment consists of rolling an eight-sided die numbered 1 to 8. The following are two possible events.

A: obtaining an even number B: obtaining a number less than 6

a) Represent these events in the adjacent Venn diagram.



b) Are events A and B mutually exclusive or non-mutually exclusive? Explain your answer.

c) Calculate:

- | | |
|------------------------|-------------------------|
| 1) $P(A)$ _____ | 2) $P(B)$ _____ |
| 3) $P(A \cup B)$ _____ | 4) $P(A \cap B)$ _____ |
| 5) $P(A')$ _____ | 6) $P(B' \cup A)$ _____ |

3 2 six-sided dice numbered 1 to 6 are rolled in succession. Consider the sum of the numbers on the top faces of these dice.

a) Are the intermediate steps associated with this experiment dependent or independent? Explain your answer.

The following two events are defined:

A: obtaining an even sum B: obtaining a sum greater than 7

b) Are events A and B mutually exclusive or non-mutually exclusive? Explain your answer.

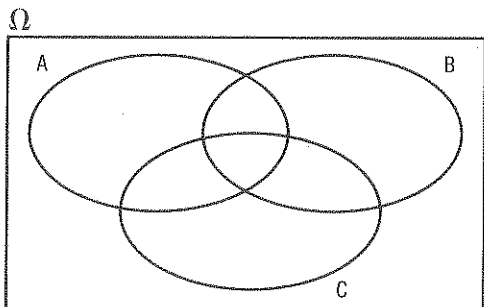
c) Calculate:

- | | |
|------------------------|-------------------------|
| 1) $P(A)$ _____ | 2) $P(B)$ _____ |
| 3) $P(A \cup B)$ _____ | 4) $P(A \cap B)$ _____ |
| 5) $P(A')$ _____ | 6) $P(A' \cup B)$ _____ |

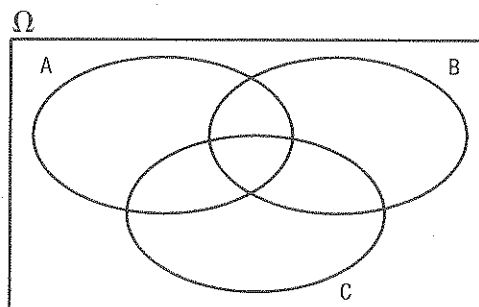
Types of events

1 For each case, shade the region described.

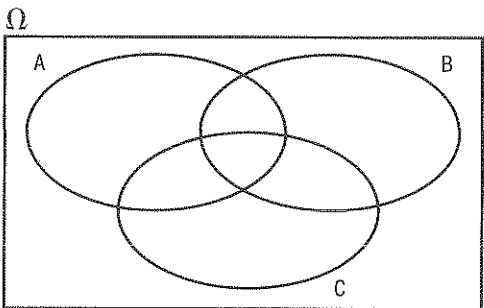
a) $A \cap B$



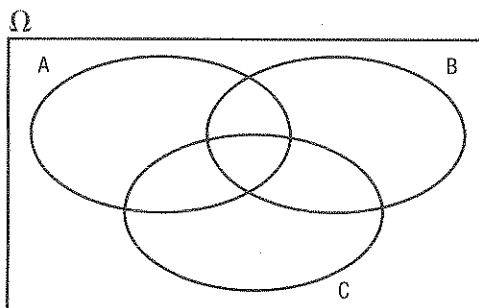
b) $A \cup C$



c) $A \cap (B \cup C)$



d) $A' \cup C$



2 The adjacent Venn diagram represents two events associated with rolling a six-sided die numbered 1 to 6.

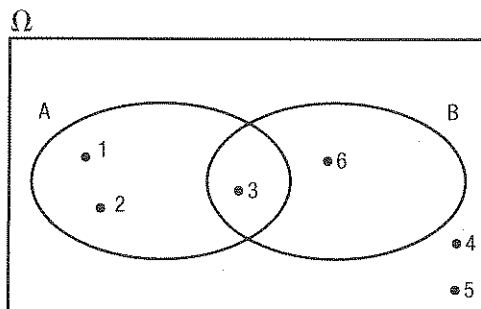
a) Define events A and B.

b) Are events A and B mutually exclusive or non-mutually exclusive? Explain your answer.

c) Calculate:

- | | | | |
|-------------------|-------|-------------------|-------|
| 1) $P(A)$ | _____ | 2) $P(B)$ | _____ |
| 3) $P(A \cup B)$ | _____ | 4) $P(A \cap B)$ | _____ |
| 5) $P(A' \cap B)$ | _____ | 6) $P(A' \cup B)$ | _____ |

d) Translate the following expression based on the context: $A' \cap B$.



Name: _____

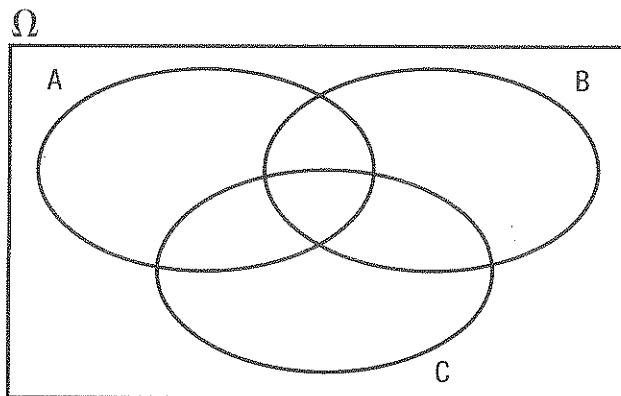
Group: _____ Date: _____

(cont'd)

3 A twelve-sided die numbered 1 to 12 is rolled. The following are three possible events:

A: obtaining an even number B: obtaining a multiple of 3 C: obtaining a divisor of 24

a) Complete the Venn diagram below.



b) Express the statements below using set-builder notation:

- 1) obtaining an even number that is a multiple of 3 _____
- 2) obtaining a multiple of 3 or a divisor of 24 _____
- 3) obtaining an odd number that is not a multiple of 3 _____
- 4) obtaining a divisor of 24 or an odd number _____

c) Translate the following expressions based on the context:

- 1) $A \cap C$ _____
- 2) $A \cup B$ _____
- 3) $A' \cap B$ _____
- 4) $B' \cup C'$ _____

d) Calculate:

- | | |
|-----------------------------------|-----------------------------------|
| 1) $P(A)$ _____ | 2) $P(B')$ _____ |
| 3) $P(A \cap B)$ _____ | 4) $P(B \cap C)$ _____ |
| 5) $P(A \cap B \cap C)$ _____ | 6) $P(A \cup B)$ _____ |
| 7) $P(B \cup C)$ _____ | 8) $P(A \cup B \cup C)$ _____ |
| 9) $P(A' \cap B)$ _____ | 10) $P(B' \cup C')$ _____ |
| 11) $P(A' \cap B' \cap C')$ _____ | 12) $P(A' \cup B' \cup C')$ _____ |

Name: _____

Group: _____ Date: _____

(cont'd)

4 For each of the following pairs of events A and B, indicate whether the events are mutually exclusive or non-mutually exclusive.

a) A: choosing a boy
B: choosing a girl

b) A: choosing a person with blue eyes
B: choosing a person with brown hair

c) A: choosing a person wearing running shoes
B: choosing a person who is at the gym

d) A: obtaining tails
B: obtaining heads

e) A: choosing an even number
B: choosing a multiple of 7

5 For each of the following pairs of events A and B, indicate whether the events are dependent or independent.

a) A: obtaining 3 when a die is rolled
B: obtaining 4 when this die is rolled a second time

b) A: choosing a chocolate doughnut from a box of doughnuts and eating it
B: choosing a second chocolate doughnut

c) A: drawing an ace from a deck of cards and leaving it on the game table
B: drawing a king from the remaining deck of cards

d) A: obtaining tails when a coin is tossed
B: obtaining heads when this coin is tossed a second time

e) A: choosing a red crayon from a box of crayons and placing it on the table
B: choosing a blue crayon from the same box of crayons

Name: _____

Group: _____ Date: _____

6 The table below shows the results of a survey on the favourite type of food of some high school students.

Students' favourite type of food

| Type of food \ Sex | Boy | Girl | Total |
|--------------------|-----|------|-------|
| Italian | | 52 | 127 |
| Greek | 68 | | 148 |
| Asian | | | |
| Total | | 197 | 375 |

- a) Complete the adjacent table.
b) If a person is chosen at random, calculate the probability of:

1) choosing a boy who prefers Italian food

2) choosing a boy

3) choosing a girl or someone who prefers Greek food

4) choosing a girl whose favourite cuisine is not Italian food

7 A random experiment consists of tossing a coin.

- If tails is obtained, a six-sided die numbered 1 to 6 is rolled.
- If heads is obtained, an eight-sided die numbered 1 to 8 is rolled.

a) Determine the universe of possible outcomes for this random experiment.

b) What is the probability of obtaining:

- 1) an even number? _____ 2) a number greater than 5? _____
3) tails or an even number? _____ 4) heads and an even number? _____

c) Are the compound events resulting from this experiment comprised of independent or dependent events? Explain your answer.

8 In a wildlife refuge, the probability of encountering a moose during a fall day is 75%.

a) What is the probability of encountering a moose on the first day and not encountering one the following day?

b) What is the probability of not encountering a moose during a 3-day stay at the wildlife refuge?

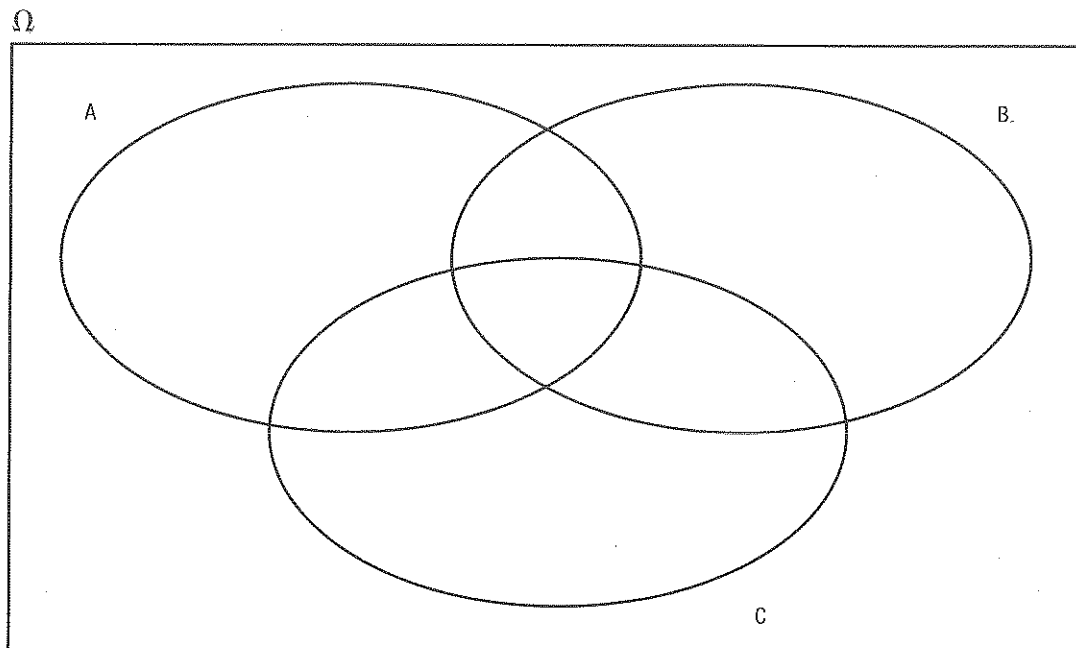
c) Do the events involved in this situation consist of unrelated events or related events? Explain your answer.

Types of events

1 A random experiment consists of rolling a tetrahedral die numbered 1 to 4 followed by an octahedral die numbered 1 to 8. The following events were defined:

- A: The result from the first die is a divisor of the result from the second die.
 B: The product of the result from the first die and that from the second is a multiple of 4.
 C: The sum of the results from the dice is an even number.

a) Complete the Venn diagram below.



b) Calculate:

- | | |
|-------------------------------|------------------------|
| 1) $P(A)$ _____ | 2) $P(B')$ _____ |
| 3) $P(A \cap B)$ _____ | 4) $P(B \cap C)$ _____ |
| 5) $P(A \cap B \cap C)$ _____ | 6) $P(A \cup B)$ _____ |

2 The information below is about two events A and B resulting from a random experiment.

$$P(A) = 50\%$$

$$P(B) = 0.4$$

$$A \cup B = \Omega$$

$$P(A' \cup B') = 0.9$$

a) Determine:

- | | |
|------------------------|------------------------|
| 1) $P(A)$ _____ | 2) $P(B)$ _____ |
| 3) $P(A \cap B)$ _____ | 4) $P(A \cup B)$ _____ |

Conditional probability

- 1** An eight-sided die numbered 1 to 8 is rolled and the number on the top face is observed.
- | | |
|---|--|
| a) How many possible results are there? _____ | b) Which results are greater than 3? _____ |
| c) How many even results are greater than 3? _____ | d) What is the probability of obtaining an even number given that it is greater than 3? _____ |
| e) Which results are less than 6? _____ | f) How many divisors of 8 are smaller than 6? _____ |
| g) What is the probability of obtaining a divisor of 8 given that it is less than 6? _____ | h) Which results are greater than or equal to 4? _____ |
| i) How many prime numbers are greater than or equal to 4? _____ | j) What is the probability of obtaining a prime number given that it is greater than or equal to 4? _____ |

2 A card is drawn from a regular 52-card deck and the following events are defined:

- | | |
|---------------------------|--------------------------|
| A: obtaining a black card | B: obtaining a club |
| C: obtaining a queen | D: obtaining a face card |

Calculate:

- | | |
|------------------------|------------------------|
| a) $P(A)$ _____ | b) $P(B)$ _____ |
| c) $P(C)$ _____ | d) $P(D)$ _____ |
| e) $P(A \cap B)$ _____ | f) $P(A \cap C)$ _____ |
| g) $P(B \cap D)$ _____ | h) $P(C \cap D)$ _____ |
| i) $P(A B)$ _____ | j) $P(A C)$ _____ |
| k) $P(A D)$ _____ | l) $P(B A)$ _____ |
| m) $P(C D)$ _____ | n) $P(D A)$ _____ |
| o) $P(D B)$ _____ | p) $P(D C)$ _____ |

3 The following are the probabilities of a few events associated with the same random experiment.

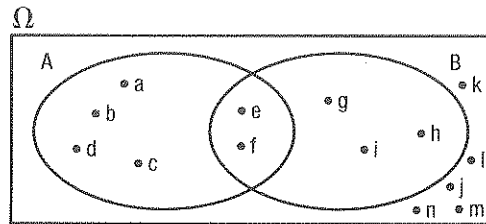
$P(A) = 0.4$ $P(B) = 0.5$ $P(A \cap B) = 0.15$ $P(A \cup B) = 0.75$

Calculate:

a) $P(A | B)$ _____ b) $P(B | A)$ _____

4 Given that all the results in the adjacent Venn diagram are equiprobable, calculate:

- a) $P(A)$ _____
- b) $P(B)$ _____
- c) $P(A \cap B)$ _____
- d) $P(A \cup B)$ _____
- e) $P(A | B)$ _____
- f) $P(B | A)$ _____



5 564 students from a school were asked about their lunch meal. The following results were obtained:

a) Complete the adjacent table.

Types of meals consumed

| Meal \ Sex | Boy | Girl | Total |
|-----------------|-----|------|-------|
| Cafeteria menu | 135 | | 243 |
| Lunch from home | | 137 | |
| Total | | | 564 |

One person was chosen at random among the 564 people surveyed. The following are four events associated with this situation.

- B: The person chosen is a boy.
- G: The person chosen is a girl.
- C: The person chosen orders from the cafeteria menu.
- H: The person chosen brings his or her lunch from home.

b) Calculate:

- 1) $P(B)$ _____
- 2) $P(G)$ _____
- 3) $P(C)$ _____
- 4) $P(H)$ _____
- 5) $P(B \cap H)$ _____
- 6) $P(B \cap C)$ _____
- 7) $P(G \cap H)$ _____
- 8) $P(G \cap C)$ _____
- 9) $P(B | C)$ _____
- 10) $P(B | H)$ _____
- 11) $P(G | C)$ _____
- 12) $P(G | H)$ _____
- 13) $P(C | B)$ _____
- 14) $P(H | B)$ _____

Conditional probability

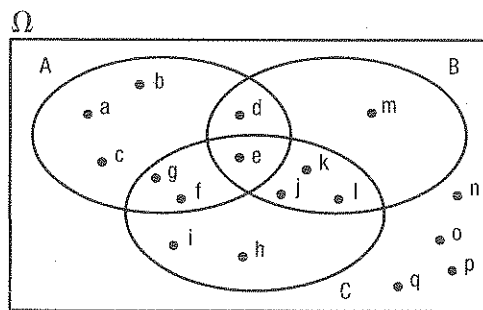
1 A random experiment consists of rolling a twelve-sided die numbered 1 to 12 and observing the top face.

What is the probability of obtaining:

- | | |
|--|--|
| a) an even number given that the number obtained is greater than 6? _____ | b) a divisor of 24, given that the number obtained is less than 9? _____ |
| c) a multiple of 3 given that the number obtained is at least 7? _____ | d) a prime number given that the number obtained is greater than 5? _____ |

2 Given that all the results in the adjacent Venn diagram are equiprobable, calculate:

- | | |
|---------------------------------------|-------------------------------|
| a) $P(A B)$ _____ | |
| b) $P(A C)$ _____ | |
| c) $P(B A)$ _____ | |
| d) $P(B C)$ _____ | |
| e) $P(C A)$ _____ | |
| f) $P((A \cap B) B)$ _____ | g) $P(A (A \cap C))$ _____ |
| h) $P((A \cup B) (B \cap C))$ _____ | i) $P(A' (B \cup C))$ _____ |



3 The following are the probabilities of several events resulting from the same random experiment.

$P(A) = 0.35$ $P(B) = 0.46$ $P(C) = 0.42$

$P(A \cup B) = 0.65$ $P(A \cap C) = 0.12$ $P(B \cap C) = 0.1$

Calculate:

- | | |
|-------------------------------|-------------------------------|
| a) $P(A B)$ _____ | b) $P(B A)$ _____ |
| c) $P(A C)$ _____ | d) $P(B C)$ _____ |
| e) $P((A \cup B) B')$ _____ | f) $P((A \cup B) A')$ _____ |
| g) $P(B (B \cap C))$ _____ | h) $P(B' A')$ _____ |

Name: _____

Group: _____ Date: _____

4 The table below presents the results of a survey of men and women's favourite colours.

Favourite colour

| Colour \ Sex | Man | Woman | Total |
|--------------|------|-------|-------|
| Black | | 346 | |
| White | 571 | | 942 |
| Red | 361 | 842 | |
| Grey | 786 | | |
| Total | 2906 | | 5438 |

a) Complete the table above.

A person is chosen at random among the 5438 people surveyed. The following are six possible events.

B: The person chosen prefers black.

Wh: The person chosen prefers white.

R: The person chosen prefers red.

G: The person chosen prefers grey.

M: The person chosen is a man.

W: The person chosen is a woman.

b) Determine the following probabilities:

1) $P(M | B)$ _____

2) $P(G | W)$ _____

3) $P(Wh | W)$ _____

4) $P(M | G)$ _____

5) $P(G | M)$ _____

6) $P(W | Wh)$ _____

7) $P(M | R)$ _____

8) $P(R | M)$ _____

5 Two six-sided dice numbered 1 to 6, one green and the other red, are rolled and the number on the top face of each is then observed.

a) What is the probability that the following will be obtained:

1) 6 on the green die given that the number obtained on the red die is 4?

2) 6 on the green die?

3) 3 on the red die given that the number obtained on the green die is 2?

4) 3 on the red die?

b) Why are the probabilities in a) 1) and a) 2) and those in a) 3) and a) 4) identical?

Name: _____

Group: _____ Date: _____

8 A random experiment consists of tossing a coin and then rolling a die. If the result of the coin toss is tails, a six-sided die numbered 1 to 6 is rolled. If the result of the coin toss is heads, an eight-sided die numbered 1 to 8 is rolled. The following are four possible events:

T: obtaining tails H: obtaining heads F: obtaining 5

G: obtaining a number greater than 4

Determine the following probabilities:

- | | |
|------------------------|------------------------|
| a) $P(T \cap F)$ _____ | b) $P(H \cap F)$ _____ |
| c) $P(F H)$ _____ | d) $P(G \cap T)$ _____ |
| e) $P(F T)$ _____ | f) $P(F T)$ _____ |
| g) $P(G T)$ _____ | h) $P(G H)$ _____ |

9 The probability that a scheduled airline flight is on time is 85%.

a) What is the probability that the second flight will be on time given that the first is on time?

b) What is the probability that the second flight will be on time given that the first is late?

c) What explanation can be given for the fact that no matter what happens with the first flight, the probability that the second flight will be on time remains the same?

10 75% of the songs stored on an MP3 player are English songs, and approximately 47% of these are rock songs. Among the 932 songs stored on the player, there are 374 hip-hop songs and 106 French songs. There are 3 alternative French songs. If the random function of the player is activated, what is the probability that the song chosen will be:

a) a rock song given that it is a French song?

b) an English song given that it is an alternative song?

c) a song other than hip-hop given that it is an English song?

d) a French song, given that it is a rock song?

Conditional probability

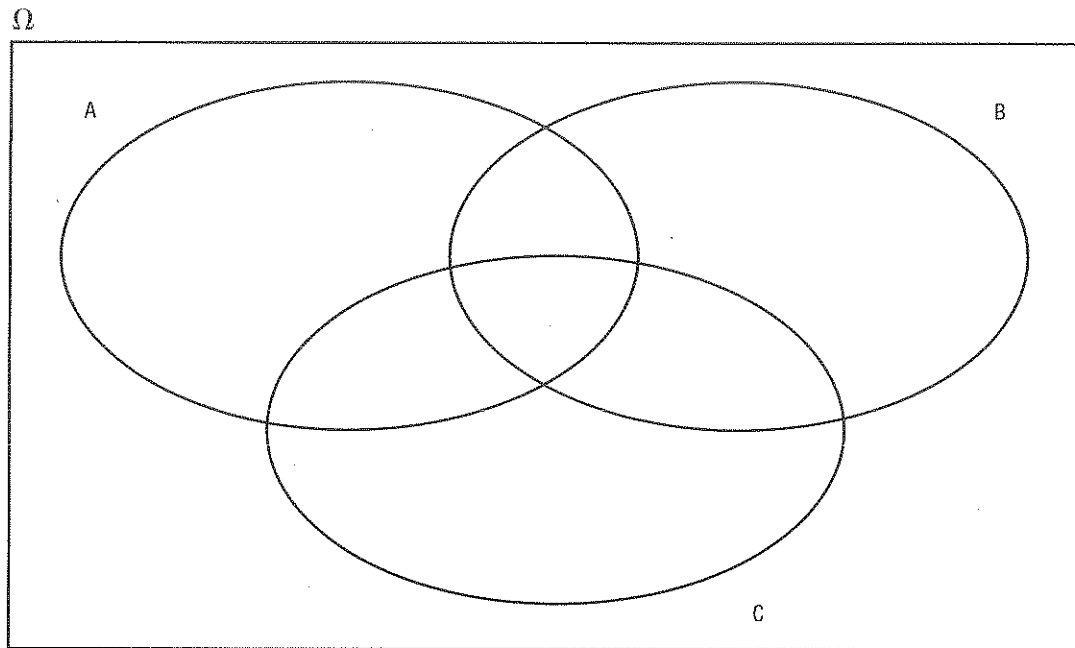
1 A random experiment consists of rolling a tetrahedral die numbered 1 to 4 followed by an octahedral die numbered 1 to 8. The following events were defined:

A: The result from the second die is less than or equal to the result from the first.

B: The product of the result from the first die and that from the second is a multiple of 4.

C: The sum of the results from the dice is an even number.

a) Complete the Venn diagram below.



b) Calculate:

1) $P(A \cap B)$ _____ 2) $P(B \cup A)$ _____ 3) $P(B \cap C)$ _____

4) $P(A | C)$ _____ 5) $P(A | B)$ _____ 6) $P(C | A)$ _____

7) $P(B | A)$ _____ 8) $P(B | C)$ _____ 9) $P(C | B)$ _____

2 A six-sided die is rolled twice and the numbers n_1 and n_2 are obtained. The following are 4 possible events:

A: the sum of n_1 and n_2 is even

B: the product of n_1 and n_2 is even

C: the product of n_1 and n_2 is odd

D: the sum of n_1 and n_2 is odd

a) What is the probability that the product of n_1 and n_2 is even given that the sum is even? _____

b) What is the probability that the sum of n_1 and n_2 is odd given that the product is odd? _____

Voting procedures

- 1** An election is held in a school to determine who will be its representative. This person is determined according to plurality voting. The following results are obtained:

Results of the election

| Candidate | Jeanne | Ruben | Patrick | Jasmine |
|-----------------|--------|-------|---------|---------|
| Number of votes | 123 | 145 | 120 | 131 |

- a) What is the percentage of votes obtained by:
 1) Jeanne? _____ 2) Ruben? _____ 3) Patrick? _____ 4) Jasmine? _____
- b) Who wins the election? _____
- c) What is the minimum number of votes that the elected candidate needs to win according to the majority rule? _____

- 2** The following are the results of an election:

Results

| Number of voters who ranked the candidates in this way | 46 | 34 | 31 | 29 |
|--|----|----|----|----|
| 1st choice | B | A | C | C |
| 2nd choice | A | B | A | B |
| 3rd choice | C | C | B | A |

According to the Borda count, each candidate is allocated:

- 3 points when he or she is a voter's 1st choice
 - 2 points when he or she is a voter's 2nd choice
 - 1 point when he or she is a voter's 3rd choice
- a) In how many of the ballots is:
 1) Candidate A the 1st choice? _____ 2) Candidate C the 3rd choice? _____
- b) How many points, in total, are allocated to:
 1) Candidate A? _____ 2) Candidate B? _____ 3) Candidate C? _____
- c) Based on the Borda count, who wins the election? _____
- d) 1) Complete the table below.

Table of duels

| Number of voters who prefer: | | Number of voters who prefer: | | Winner of the duel opposing: | |
|------------------------------|--|------------------------------|--|------------------------------|--|
| A to B | | B to A | | A and B | |
| A to C | | C to A | | A and C | |
| B to C | | C to B | | B and C | |

- 2) According to the Condorcet method, the winner is the candidate who wins all his or her duels with other candidates. Based on this criterion, who wins the election?

- 3** The following is the result of an election in which the winner is determined by the elimination method.

Results

| | | | | | | |
|--|----|----|----|----|----|----|
| Number of voters who ranked the candidates in this way | 46 | 38 | 35 | 34 | 32 | 22 |
| 1st choice | B | A | B | A | C | C |
| 2nd choice | A | C | C | B | A | B |
| 3rd choice | C | B | A | C | B | A |

- a) Is there a winner according to the majority rule? Explain your answer.

- b) Which candidate obtains the fewest 1st-choice votes? _____
- c) If this candidate is eliminated from the ballots, how many 1st-choice votes will be allocated to each of the other candidates? Explain your answer.

- d) Who wins the election based on this procedure? Explain your answer.

- 4** A country's parliament contains 114 seats which are allocated to parties according to proportional representation. The table below presents the results of the elections in this country.

Distribution of votes received by the parties

| | | | | | |
|-----------------|------|-----|------|------|-------|
| Party | A | B | C | D | Total |
| Number of votes | 1525 | 675 | 2836 | 1243 | 6279 |

- a) Complete the adjacent table.
- b) Of the 114 seats, how many seats were not allocated? _____
- c) At the most, one of the remaining seats is distributed to each party according to the decreasing order of their remainder.

| Party | Minimum number of seats won | Remainder |
|-------|--|----------------|
| A | $\frac{1525}{6279} \times 114 \approx 27.69$, or at least 27 seats | ≈ 0.69 |
| B | | |
| C | | |
| D | | |

How many additional seats will be allocated to:

- 1) Party A? _____ 2) Party B? _____ 3) Party C? _____ 4) Party D? _____

- d) In total, how many seats were won by:

- 1) Party A? _____ 2) Party B? _____ 3) Party C? _____ 4) Party D? _____

- e) Does the party in power have a majority? Explain your answer.

Voting procedures

- 1** The following are the results of an election during which the voters had to rank the candidates.

Results

| Number of voters who ranked the candidates in this way | 132 | 140 | 156 | 231 | 91 | 104 |
|--|-----|-----|-----|-----|----|-----|
| 1st choice | A | B | C | D | C | B |
| 2nd choice | B | A | D | C | A | D |
| 3rd choice | D | C | A | B | D | A |
| 4th choice | C | D | B | A | B | C |

Which candidate wins this election if the winner is determined according to:

- a) the Condorcet method? _____
- b) plurality voting? _____
- c) majority rule? _____
- d) the Borda count? _____
- e) the elimination method? _____

- 2** The person in charge of intramural activities in a school must decide which sport will be played next month. He asks students to rank which of the four available sports they prefer. The following is the result of this survey:

Students' favourite sport

| Number of voters who ranked the candidates in this way | 161 | 152 | 143 | 138 |
|--|------------|------------|------------|------------|
| 1st choice | Volleyball | Basketball | Badminton | Soccer |
| 2nd choice | Basketball | Volleyball | Volleyball | Badminton |
| 3rd choice | Badminton | Soccer | Soccer | Volleyball |
| 4th choice | Soccer | Badminton | Basketball | Basketball |

Which sport will be chosen if it is determined according to:

- a) the Condorcet method? _____
- b) plurality voting? _____
- c) majority rule? _____
- d) the Borda count? _____
- e) the elimination method? _____

Name: _____

Group: _____ Date: _____

- 3** A country's parliament is made up of 32 seats. This country is divided into 8 districts that have the same number of voters. The following are the results of the last elections:

Election results

| Party candidate \ District | District | | | | | | | |
|----------------------------|----------|-----|-----|-----|-----|-----|-----|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| A | 42% | 31% | 23% | 13% | 6% | 62% | 21% | 48% |
| B | 36% | 35% | 44% | 31% | 28% | 13% | 34% | 20% |
| C | 22% | 34% | 33% | 56% | 66% | 25% | 45% | 32% |

- a) 4 seats are allocated to each district. Determine the composition of the parliament of this country if, in a district, the 4 seats are given to the winning party according to plurality voting.

- b) 1) Determine the composition of the parliament of this country if the seats are allocated according to proportional representation.

- 2) Does the party in power have a majority? Explain your answer.

- 4** The parliament of a country is divided into 8 districts comprised of 41 seats. The seats are allocated to each district proportionally based on the voting population and the seats in each district are allocated to the political parties proportionally based on the number of votes obtained. The following are the results of the last elections in this country.

Distribution of voters in the districts

| District | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------------|----|----|---|----|---|----|----|---|
| Percentage of voters (%) | 17 | 19 | 7 | 10 | 5 | 18 | 21 | 3 |

Number of votes for each party in each district

| Party \ District | District | | | | | | | |
|------------------|----------|-----|-----|-----|-----|-----|-----|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| A | 25% | 43% | 17% | 25% | 34% | 52% | 25% | 35% |
| B | 48% | 34% | 63% | 28% | 40% | 33% | 31% | 44% |
| C | 27% | 23% | 20% | 47% | 26% | 15% | 44% | 21% |

Determine the composition of the parliament in this country.

Name: _____

Group: _____ Date: _____

(cont'd)

- 5** A group of students must choose an activity as a reward they have earned. Each student in the group writes down the available activities on a sheet of paper in order of preference. The result is as follows:

Favourite activities

| | | | | | |
|--|----------|----------|----------|----------|----------|
| Number of students who ranked the activities in this way | 16 | 20 | 11 | 16 | 10 |
| 1st choice | Gym | Movies | Computer | Movies | Computer |
| 2nd choice | Movies | Gym | Gym | Computer | Movies |
| 3rd choice | Computer | Computer | Movies | Gym | Gym |

Which activity was chosen according to:

- a) the Borda count?

- b) the Condorcet method?

- c) the elimination method?

- 6** The parliament of a country is divided into 10 districts and has 29 seats. A seat is allocated to each district according to plurality voting, and the other 19 seats are allocated to the political parties in proportion to the number of votes obtained. The following are the results of the elections that took place in this country.

Distribution of votes according to the parties in each district

| Party \ District | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------------------|------|------|------|------|-----|------|------|-----|-----|------|
| A | 1568 | 2567 | 298 | 658 | 821 | 2167 | 2121 | 222 | 568 | 649 |
| B | 2985 | 2129 | 1267 | 185 | 598 | 1179 | 1777 | 908 | 221 | 1347 |
| C | 1925 | 1221 | 967 | 1901 | 911 | 182 | 3261 | 218 | 875 | 298 |

Determine the composition of this country's parliament.

Name: _____

Group: _____ Date: _____

(cont'd)

7 An auditorium will be built in one of the four cities in a region. The citizens of these cities want the auditorium to be located as close as possible to their city. The following is some information about this:

Distance (in km) between the cities

| | | | | |
|--------------------|----------|----------|----------|----------|
| From To | A | B | C | D |
| A | 0 | 13 | 23 | 20 |
| B | 13 | 0 | 18 | 15 |
| C | 23 | 18 | 0 | 16 |
| D | 20 | 15 | 16 | 0 |

Population of city

| | |
|-------------|-------------------|
| City | Population |
| A | 26 900 |
| B | 28 500 |
| C | 29 000 |
| D | 27 500 |

a) Complete the table below.

Choice of location of the auditorium

| | | | | |
|---|---------------|---------------|---------------|---------------|
| Number of citizens that ordered the cities in this way | 26 900 | 28 500 | 29 000 | 27 500 |
| 1st choice | | | | |
| 2nd choice | | | | |
| 3rd choice | | | | |
| 4th choice | | | | |

b) Which city will be selected based on:

1) the Condorcet method?

2) the Borda count?

3) plurality voting?

4) the elimination method?

Voting procedures

1 The following are the results of the last municipal elections, in which three parties competed.

Municipal elections

| | | | |
|---|---------------|---------------|---------------|
| Number of people who ranked the parties in this way | 32 500 | 32 500 | 32 500 |
| 1st choice | Renouveau | Enviroplus | Vision Avenir |
| 2nd choice | Enviroplus | Vision Avenir | Renouveau |
| 3rd choice | Vision Avenir | Renouveau | Enviroplus |

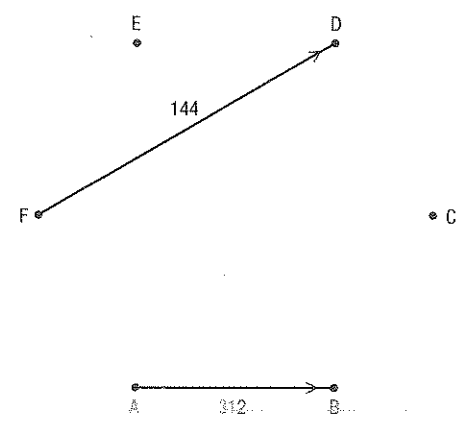
Based on your current knowledge, verify that it is impossible to determine a winning party.

2 The following are the results of an election in which the winner is determined using the Condorcet method.

Election results

| | | | | | | |
|--|-----|-----|-----|-----|-----|----|
| Number of people who ranked the candidates in this way | 301 | 267 | 254 | 156 | 132 | 72 |
| 1st choice | C | B | A | A | E | D |
| 2nd choice | B | C | B | F | A | E |
| 3rd choice | D | A | C | B | C | A |
| 4th choice | A | F | D | D | B | F |
| 5th choice | E | D | E | E | F | C |
| 6th choice | F | E | F | C | D | B |

a) Complete the adjacent duel graph, on which each directed edge gives the number of votes that allowed a candidate to win a duel against another candidate. (For example, an edge that has a value of 312 directed from A towards B means that Candidate A has won his or her duel against Candidate B by 312 votes).



b) Who won this election? Explain your answer.

Decision-making in the context of social choices

1 The following are the results of an election in which the voters had to rank 3 candidates:

Election results

| | | | | | |
|--|----|----|----|----|----|
| Number of voters who ranked the candidates in this way | 16 | 14 | 17 | 23 | 12 |
| 1st choice | C | A | B | C | B |
| 2nd choice | B | B | A | A | C |
| 3rd choice | A | C | C | B | A |

- a) Who wins this election if the winner is determined using:
- 1) the Condorcet method? _____
 - 2) plurality voting? _____
 - 3) the Borda count? _____
 - 4) the elimination method? _____

b) Using the results obtained, complete the following table given that the following is true:

- When the winner is a voter's 1st choice, the elector is very satisfied.
- When the winner is a voter's 2nd choice, the elector is satisfied.
- When the winner is a voter's 3rd choice, the elector is dissatisfied.

Number of voters according to the level of satisfaction and procedure used

| Level of satisfaction \ Procedure used | Condorcet method | Plurality voting | Borda count | Elimination method |
|--|------------------|------------------|-------------|--------------------|
| Very satisfied | | | | |
| Satisfied | | | | |
| Dissatisfied | | | | |

- c) Which voting procedure would you recommend using in this situation? Explain your answer.
- _____
- _____
- _____

Name: _____

Group: _____ Date: _____

- 2** The following are the results of the last elections in a country that is divided into 10 districts that have the same number of voters. A seat is allocated to each district in this country.

Election results

| Party candidate \ District | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A | 34% | 35% | 5% | 43% | 36% | 34% | 37% | 40% | 10% | 4% |
| B | 33% | 34% | 40% | 3% | 34% | 33% | 29% | 35% | 80% | 50% |
| C | 33% | 31% | 55% | 54% | 30% | 33% | 34% | 25% | 10% | 46% |

- a) If the seat in each district is allocated according to plurality voting, how many seats are gained by:

1) Party A? _____ 2) Party B? _____ 3) Party C? _____

- b) Describe the government in this case.

- c) If the seats are allocated to the parties according to proportional representation, how many seats are gained by:

1) Party A? _____ 2) Party B? _____ 3) Party C? _____

- d) Describe the government in this case.

- e) An analyst asserts that this type of voting procedure has a minimal impact on the outcome of the elections. In light of this situation, how would you respond?

Decision-making in the context of social choices

- 1** A company organizing a trip to the South for 137 travellers had these people take part in a survey to determine which destination should be chosen. The following results were obtained:

Favourite destinations

| | | | | | | |
|---|------------|------------|------------|------------|------------|------------|
| Number of travellers who ranked the destinations in this way | 26 | 25 | 23 | 23 | 21 | 19 |
| 1st choice | Martinique | Cuba | Florida | Mexico | Florida | Martinique |
| 2nd choice | Mexico | Florida | Cuba | Martinique | Mexico | Cuba |
| 3rd choice | Cuba | Mexico | Martinique | Florida | Cuba | Mexico |
| 4th choice | Florida | Martinique | Mexico | Cuba | Martinique | Florida |

- a) When the chosen destination is:
- a voter's 1st choice, he or she is very satisfied.
 - a voter's 2nd choice, he or she is satisfied.
 - a voter's 3rd choice, he or she is dissatisfied.
 - a voter's 4th choice, he or she is very dissatisfied.

Complete the table below.

Number of travellers based on the level of satisfaction and the procedure used

| Level of satisfaction \ Procedure used | Plurality voting | Borda count | Elimination method |
|--|------------------|-------------|--------------------|
| Very satisfied | | | |
| Satisfied | | | |
| Dissatisfied | | | |
| Very dissatisfied | | | |

- b) Among the proposed procedures, which one can be used to determine the destination that generates:

- 1) the highest level of satisfaction among the group of travellers?
Explain your answer.

- 2) the lowest level of satisfaction among the group of travellers?
Explain your answer.

- c) Can the Condorcet method be used to determine a destination that generates a higher level of satisfaction from the group of travellers than the one determined by the procedure given in b) 1)? Explain your answer.

Name: _____

Group: _____ Date: _____

2 A country's parliament is made up of 24 seats. This country is divided into 8 districts that have the same number of voters. The following are the results of the last elections:

Election results

| Party candidate \ District | District | | | | | | | |
|----------------------------|----------|-----|-----|-----|-----|-----|-----|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| A | 33% | 35% | 55% | 34% | 6% | 41% | 21% | 48% |
| B | 32% | 26% | 40% | 25% | 28% | 47% | 34% | 20% |
| C | 35% | 39% | 5% | 41% | 66% | 12% | 45% | 32% |

a) Determine the composition of this country's parliament if:

1) 3 seats are allocated to the winning party for each district according to plurality voting in each district.

2) The 24 seats are allocated according to proportional representation.

b) Which of the voting procedures leads to a parliament whose composition is more representative of the wishes of the electorate. Explain your answer.

c) List two advantages of the procedure described in a) 1).

d) Explain why, in this situation, the following is true:

1) The procedure described in a) 1) leads to a parliament in which the decision-making process is generally quick.

2) The procedure described in a) 2) leads to a parliament in which the decision-making process can be slower.

Name: _____

Group: _____ Date: _____

3 The following are the results of a country's presidential election.

Presidential election

| Number of voters who ranked the candidates in this way | 30 | 22 | 21 | 17 | 10 |
|--|----|----|----|----|----|
| 1st choice | A | B | A | D | C |
| 2nd choice | C | C | C | C | D |
| 3rd choice | D | D | B | B | B |
| 4th choice | B | A | D | A | A |

Once the winner is announced, approximately half of the electorate expresses their dissatisfaction with the result.

a) Which procedure was used to determine the winner? Explain your answer.

b) Which candidate appears to be the most appropriate choice to rally the electorate? Explain your answer.

c) Which voting procedure would allow this candidate to come to power?

4 A country's parliament revises its electoral system. The current system allocates a seat to the party candidate who receives the most votes in each district.

a) 1) Describe the current voting procedure in this country.

2) In your opinion, what is the main advantage of this procedure?

3) In your opinion, what is the main disadvantage of this procedure?

b) 1) As a result of the population's dissatisfaction, the government recommends that the parties be represented more equitably. Which voting procedure can it adopt?

2) In your opinion, what is the main disadvantage of this procedure?

Decision-making in the context of social choices

1 The following is a description of three voting procedures that can be used to allocate the 40 seats of a parliament.

Procedure 1

For each district, four seats are allocated to the party that wins the elections based on plurality voting in this district.

Procedure 2

Two seats for each district are allocated to the party that wins the elections based on plurality voting in this district. The remaining seats are allocated to the parties according to proportional representation.

Procedure 3

Each district is required to allocate a number of seats in proportion to its population. In each district, the seats are allocated according to proportional representation.

The following information concerns the last elections in this country:

Distribution of the electorate

| District | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------------------------|----|---|----|----|---|---|---|---|----|----|
| Percentage of voters (%) | 10 | 9 | 11 | 15 | 7 | 8 | 7 | 9 | 11 | 13 |

Election results

| Party candidate \ District | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | A | 34% | 35% | 33% | 41% | 34% | 29% | 50% | 33% | 81% |
| B | 31% | 25% | 33% | 55% | 30% | 34% | 46% | 33% | 9% | 54% |
| C | 35% | 40% | 34% | 4% | 36% | 37% | 4% | 34% | 10% | 43% |

The following formula can be used to calculate the index I of disproportionality of a voting system.

$$I = \sqrt{\frac{1}{2}((V_A - S_A)^2 + (V_B - S_B)^2 + (V_C - S_C)^2)}$$

The higher the index I , the more the voting system is disproportional, in other words the allocation of seats is less and less proportional to the number of votes. In this formula, the following can be noted:

- V_A , V_B and V_C correspond to the percentage of votes received by Parties A, B and C, respectively.
- S_A , S_B and S_C correspond to the percentage of seats received by Parties A, B and C, respectively.

Which procedure leads to the best index of disproportionality?

Number 13

13 The Condorcet method can be illustrated using a directed graph in which the following is true:

- Each vertex corresponds to a candidate.
- Each edge corresponds to a preference.

For example, an edge with a value of 5 directed from A to B means that 5 voters prefer A to B.

a) Complete the adjacent Graph ① which represents the results of the election.

Based on this graph, a duel graph can be created in which only the highest-value edges between two vertices are retained.

- b) Complete the adjacent duel Graph ②.
- c) Graphically, how can the winner of this election be identified?

The table and Graph ③ below present a summary of the results of another election.

Election results

| Number of voters who ordered the candidates in this way | 15 | 13 | 10 | 9 | 6 |
|---|----|----|----|---|---|
| 1st choice | B | D | C | A | B |
| 2nd choice | A | B | D | C | C |
| 3rd choice | D | C | A | B | A |
| 4th choice | C | A | B | D | D |

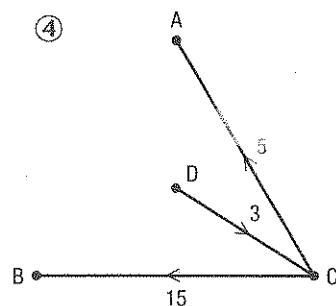
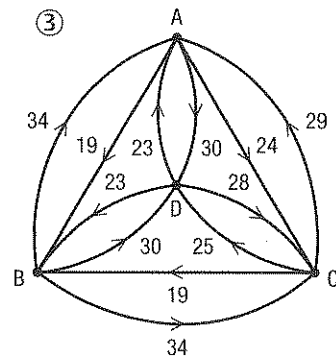
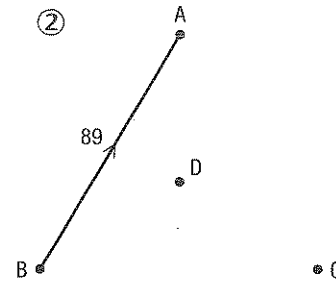
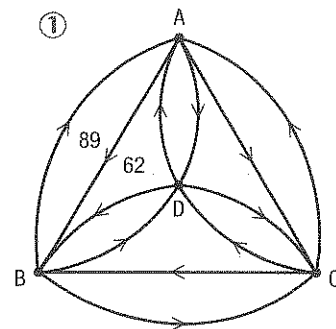
d) Using the results above, complete the adjacent duel Graph ④ in which each edge represents the number of votes by which a candidate won the duel with another candidate. For example, an edge directed from A to B of value 5 means that A won the duel over B by 5 votes.

e) Explain why it is impossible to determine the winner using the Condorcet method.

f) One way to determine the winner consists of eliminating the edges of lowest value, one by one, until a candidate who loses no duels has been identified. Determine the winner by applying this method.

Election results

| Number of voters who ordered the candidates in this way | 62 | 45 | 23 | 21 |
|---|----|----|----|----|
| 1st choice | A | B | D | C |
| 2nd choice | B | C | C | D |
| 3rd choice | C | D | B | B |
| 4th choice | D | A | A | A |



- 1** **SO, ARE YOU SATISFIED?** To choose a school's Monday meal, 248 people were asked to rank four meals according to their preferences. The following results were obtained:

Favourite meal

| Number of people who ranked the meals in this way | 64 | 62 | 59 | 35 | 28 |
|---|----|----|----|----|----|
| 1st choice | A | B | A | D | C |
| 2nd choice | B | D | C | C | D |
| 3rd choice | C | A | B | B | A |
| 4th choice | D | C | D | A | B |

If the chosen meal is:

- a person's 1st choice, he or she is very satisfied.
- a person's 2nd choice, he or she is satisfied.
- a person's 3rd choice, he or she is dissatisfied.
- a person's 4th choice, he or she is very dissatisfied.

A person is chosen at random among those who have stated their preferences. What is the probability that the person chosen is very satisfied or satisfied, given that the procedure used is the Borda count or the elimination method?

Name: _____

Group: _____ Date: _____

2 **QUITE AN EVENT** A, B and C are three events such that the following is true:

- A and B are non-mutually exclusive.
- B and C are non-mutually exclusive.
- A and C are non-mutually exclusive.

Show that:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

Name: _____

Group: _____ Date: _____

3 A GOOD BALLOT The following information concerns elections:

During the same election, the voters must vote for:

- a municipal representative
- a regional representative
- a national representative

For the same election, the following is noted:

- 3 people run as candidates in each municipality.
- 4 people run as candidates in each region.
- 6 people run as candidates nationally.

For the same election, the following is noted:

- Each municipal representative is elected according to plurality voting.
- Each regional representative is elected according to the Condorcet method.
- Each national representative is elected by approval voting.

Create an effective ballot that can be used for this election.

Name: _____

Group: _____ Date: _____

- 4** **THE MUSIC LIST** The following information concerns two MP3 players that look exactly the same.

| Player 1 | | Player 2 | |
|----------|-----------------|----------|-----------------|
| Artist | Number of songs | Artist | Number of songs |
| Jonas | 5 | Jonas | 13 |
| Michaël | 5 | Michaël | 1 |
| Denise | 5 | Denise | 1 |

A person chooses one of these players at random and activates the “random list” function. This function creates a list of three songs that allows for repetitions. If this list contains two songs from the artist Jonas, what is the probability that the person chose Player 2?

Name: _____

Group: _____ Date: _____

- 5 PROCEDURES** A group of students must choose its president from among 4 candidates. Since the students do not agree on which voting procedure should be followed, they are asked to rank three voting procedures as well as the four candidates. The following results were obtained.

Preferences regarding the voting procedure to be followed

| Number of people who ranked the procedures in this way | 9 | 8 | 7 | 6 | 5 |
|--|-------------|-------------|-------------|-------------|-------------|
| 1st choice | Borda | Condorcet | Borda | Elimination | Elimination |
| 2nd choice | Condorcet | Borda | Elimination | Condorcet | Borda |
| 3rd choice | Elimination | Elimination | Condorcet | Borda | Condorcet |

Preferences regarding the candidates

| Number of people who ranked the candidates in this way | 12 | 8 | 6 | 5 | 4 |
|--|----|---|---|---|---|
| 2nd choice | B | A | C | C | D |
| 4th choice | D | D | D | A | B |
| 1st choice | A | B | A | D | C |
| 3rd choice | C | C | B | B | A |

The voting procedure to be followed can be determined with the Borda count, the Condorcet method or by the elimination method. The result obtained indicates which procedure should be used to determine the winning candidate.

A student makes the following conjecture for this situation:

“Whether we use the Borda count, the Condorcet method or the elimination method to determine which voting procedure should be followed, the winner of the election will be the same.”

Confirm or refute this conjecture.

Name: _____

Group: _____ Date: _____

(cont'd)

6 PARTICIPATION In a region divided into 6 districts, the seat in each district is allocated according to plurality voting. A political journal published the results of a survey concerning the voting intentions in this region. The results are the following:

Results of the election

| Party candidate \ District | District | | | | | |
|----------------------------|----------|-----|-----|-----|-----|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 40% | 42% | 5% | 43% | 28% | 50% |
| B | 33% | 24% | 60% | 54% | 30% | 25% |
| C | 27% | 34% | 35% | 3% | 42% | 25% |

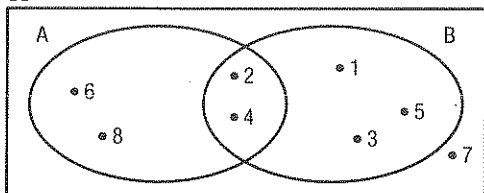
Write a brief article for this journal explaining the ways in which proportional representation further encourages the population to vote in this situation.

| b) 1) | 1st selection | 2nd selection | Result | Probability |
|-------|---------------|---------------|--------|----------------|
| | R | R | (R, R) | $\frac{1}{15}$ |
| | R | G | (R, G) | $\frac{2}{15}$ |
| | R | B | (R, B) | $\frac{2}{15}$ |
| | G | R | (G, R) | $\frac{2}{15}$ |
| | G | G | (G, G) | $\frac{1}{15}$ |
| | G | B | (G, B) | $\frac{2}{15}$ |
| | B | R | (B, R) | $\frac{2}{15}$ |
| | B | G | (B, G) | $\frac{2}{15}$ |
| | B | B | (B, B) | $\frac{1}{15}$ |

- 2) Yes, since the 2nd selection is not always done using the same marbles.
 3) They are dependent since the marbles available for the 2nd selection change depending on the marble that was drawn during the 1st selection.

Support 4.1 (cont'd)

2. a) Ω



b) Several answers possible. Example:

The events are non-mutually exclusive since they have elements in common.

- c) 1) $\frac{1}{2} = 0.5$ 2) $\frac{5}{8} = 0.625$ 3) $\frac{7}{8} = 0.875$
 4) $\frac{1}{4} = 0.25$ 5) $\frac{1}{2} = 0.5$ 6) $\frac{5}{8} = 0.625$

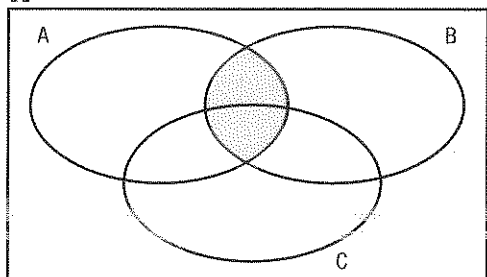
3. a) The intermediate steps are independent since the first does not have an effect on the possible probabilities of the second.

b) The events are non-mutually exclusive since they have elements in common.

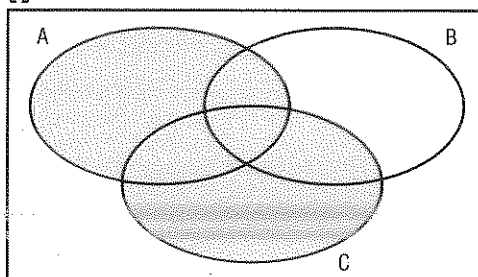
- c) 1) $\frac{1}{2} = 0.5$ 2) $\frac{5}{12} \approx 0.42$ 3) $\frac{2}{3} \approx 0.67$
 4) $\frac{1}{4} = 0.25$ 5) $\frac{1}{2} = 0.5$ 6) $\frac{3}{4} = 0.75$

Consolidation 4.1

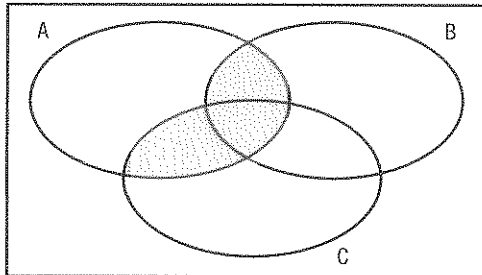
1. a) Ω



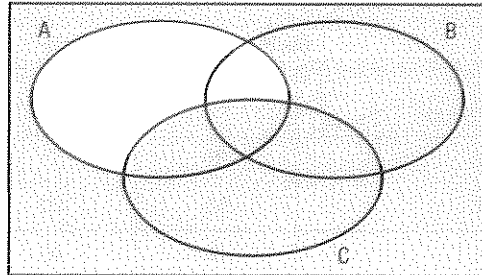
b) Ω



c) Ω



d) Ω



2. a) Several answers possible. Example:

A: obtaining a number less than 4, B: obtaining a multiple of 3.

b) The events are non-mutually exclusive since they have an element in common.

c) 1) $\frac{1}{2} = 0.5$ 2) $\frac{1}{3} \approx 0.33$ 3) $\frac{2}{3} \approx 0.67$

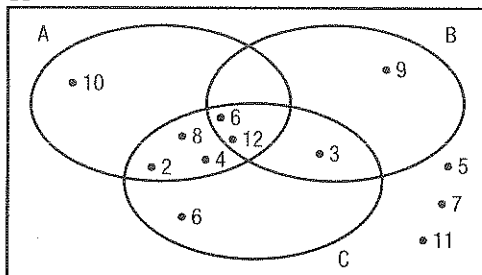
4) $\frac{1}{6} \approx 0.17$ 5) $\frac{1}{6} \approx 0.17$ 6) $\frac{2}{3} \approx 0.67$

d) Several answers possible. Example:

Obtaining a number that is both greater than 3 and a multiple of 3.

Consolidation 4.1 (cont'd)

3. a) Ω



- b) 1) $A \cap B$ 2) $B \cup C$
 3) $A' \cap B'$ 4) $C \cup A'$

- c) 1) Obtaining an even divisor of 24.
 2) Obtaining an even number or a multiple of 3.
 3) Obtaining an odd multiple of 3.
 4) Obtaining a number that is not a multiple of 3 or that is not a divisor of 24.

- d) 1) $\frac{1}{2} = 0.5$ 2) $\frac{2}{3} \approx 0.67$ 3) $\frac{1}{6} \approx 0.17$ 4) $\frac{1}{4} = 0.25$
 5) $\frac{1}{6} \approx 0.17$ 6) $\frac{2}{3} \approx 0.67$ 7) $\frac{2}{3} \approx 0.67$ 8) $\frac{3}{4} = 0.75$
 9) $\frac{1}{6} \approx 0.17$ 10) $\frac{3}{4} = 0.75$ 11) $\frac{1}{4} = 0.25$ 12) $\frac{3}{4} = 0.75$

Consolidation 4.1 (cont'd)

4. a) Mutually exclusive events. b) Non-mutually exclusive events.
 c) Non-mutually exclusive events. d) Mutually exclusive events.
 e) Non-mutually exclusive events.
5. a) Independent events. b) Dependent events.
 c) Dependent events d) Independent events.
 e) Dependent events.

Consolidation 4.1 (cont'd)

6. a) Students' favourite type of food

| Type of food \ Sex | Boy | Girl | Total |
|--------------------|-----|------|-------|
| Italian | 75 | 52 | 127 |
| Greek | 68 | 80 | 148 |
| Asian | 35 | 65 | 100 |
| Total | 178 | 197 | 375 |

- b) 1) $\frac{1}{5} = 0.2$
 2) $\frac{178}{375} \approx 0.47$
 3) $\frac{53}{75} \approx 0.71$
 4) $\frac{29}{75} \approx 0.39$

7. a) $\Omega = \{(P, 1), (P, 2), (P, 3), (P, 4), (P, 5), (P, 6), (F, 1), (F, 2), (F, 3), (F, 4), (F, 5), (F, 6), (F, 7), (F, 8)\}$

- b) 1) $\frac{1}{2} = 0.5$ 2) $\frac{2}{7} \approx 0.29$ 3) $\frac{5}{7} \approx 0.71$ 4) $\frac{2}{7} \approx 0.29$

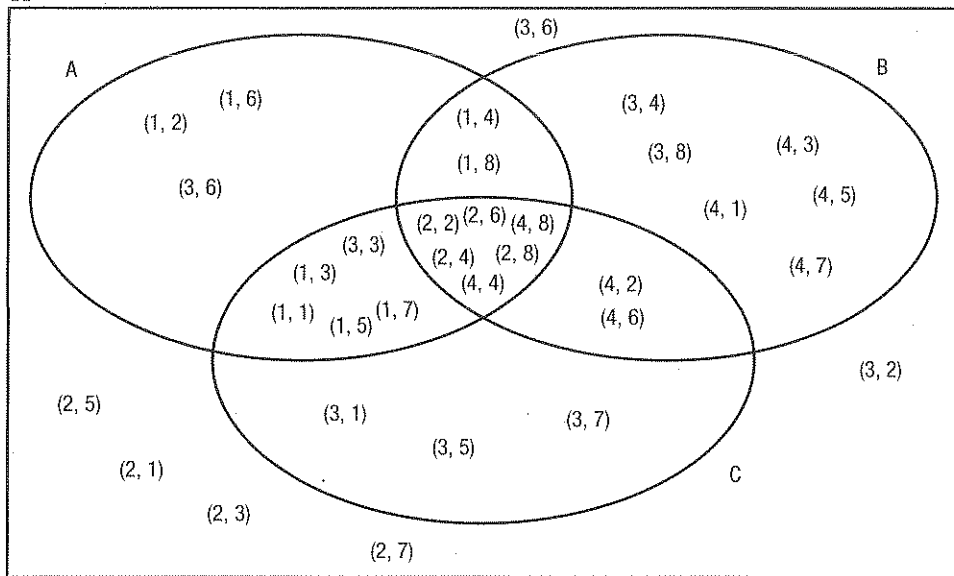
c) The compound events are independent, since the events resulting from the first experiment do not affect the probabilities of the events resulting from the second.

8. a) 18.75% b) 1.5625%

c) The compound events are independent since seeing or not seeing a moose one day does not affect the probability of seeing a moose on another day.

Enrichment 4.1

1. a) Ω



- b) 1) $\frac{1}{2} = 0.5$ 2) $\frac{1}{2} = 0.5$
 3) $\frac{1}{4} = 0.25$ 4) $\frac{1}{4} = 0.25$
 5) $\frac{3}{16} = 0.1875$ 6) $\frac{3}{4} = 0.75$

2. a) 1) 0.5 2) 0.6 3) 0.1 4) 1

Support 4.2

1. a) 8 possible results. b) {4, 5, 6, 7, 8} c) 3 results. d) $\frac{3}{5} = 0.6$
 e) {1, 2, 3, 4, 5} f) 3 divisors. g) $\frac{3}{5} = 0.6$ h) {4, 5, 6, 7, 8}
 i) 2 prime numbers. j) $\frac{2}{5} = 0.4$
2. a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{13}$ d) $\frac{3}{13}$
 e) $\frac{1}{4}$ f) $\frac{1}{26}$ g) $\frac{3}{52}$ h) $\frac{1}{13}$
 i) 1 j) $\frac{1}{2}$ k) $\frac{1}{2}$ l) $\frac{1}{2}$
 m) $\frac{1}{3}$ n) $\frac{3}{13}$ o) $\frac{3}{13}$ p) 1

Support 4.2 (cont'd)

3. a) 0.375 b) 0.3
4. a) $\frac{3}{7}$ b) $\frac{5}{14}$ c) $\frac{1}{7}$
 d) $\frac{9}{14}$ e) $\frac{2}{5}$ f) $\frac{1}{3}$

5. a) **Types of meals consumed**

| Meal \ Sex | Boy | Girl | Total |
|-----------------|-----|------|-------|
| Cafeteria menu | 135 | 108 | 243 |
| Lunch from home | 184 | 137 | 321 |
| Total | 319 | 245 | 564 |

- b) 1) ≈ 0.57 2) ≈ 0.43 3) ≈ 0.43 4) ≈ 0.57
 5) ≈ 0.33 6) ≈ 0.24 7) ≈ 0.24 8) ≈ 0.19
 9) ≈ 0.56 10) ≈ 0.57 11) ≈ 0.44 12) ≈ 0.43
 13) ≈ 0.42 14) ≈ 0.58

Consolidation 4.2

1. a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{1}{3}$ d) $\frac{2}{7}$
2. a) $\frac{1}{3}$ b) $\frac{3}{8}$ c) $\frac{2}{7}$
 d) $\frac{1}{2}$ e) $\frac{3}{7}$ f) $\frac{1}{3}$
 g) 1 h) 1 i) $\frac{3}{5}$
3. a) ≈ 0.35 b) ≈ 0.46 c) ≈ 0.29 d) ≈ 0.24
 e) ≈ 0.35 f) ≈ 0.46 g) 1 h) ≈ 0.54

Consolidation 4.2 (cont'd)

4. a) **Favourite colour**

| Colour \ Sex | Man | Woman | Total |
|--------------|------|-------|-------|
| Black | 1188 | 346 | 1534 |
| White | 571 | 371 | 942 |
| Red | 361 | 842 | 1203 |
| Grey | 786 | 973 | 1759 |
| Total | 2906 | 2532 | 5438 |

- b) 1) ≈ 0.77
 2) ≈ 0.38
 3) ≈ 0.15
 4) ≈ 0.45
 5) ≈ 0.27
 6) ≈ 0.39
 7) ≈ 0.3
 8) ≈ 0.12

5. a) 1) $\frac{1}{6}$ 2) $\frac{1}{6}$ 3) $\frac{1}{6}$ 4) $\frac{1}{6}$
 b) The events are independent. Therefore, the first event will not affect the probability of the second event.

Consolidation 4.2 (cont'd)

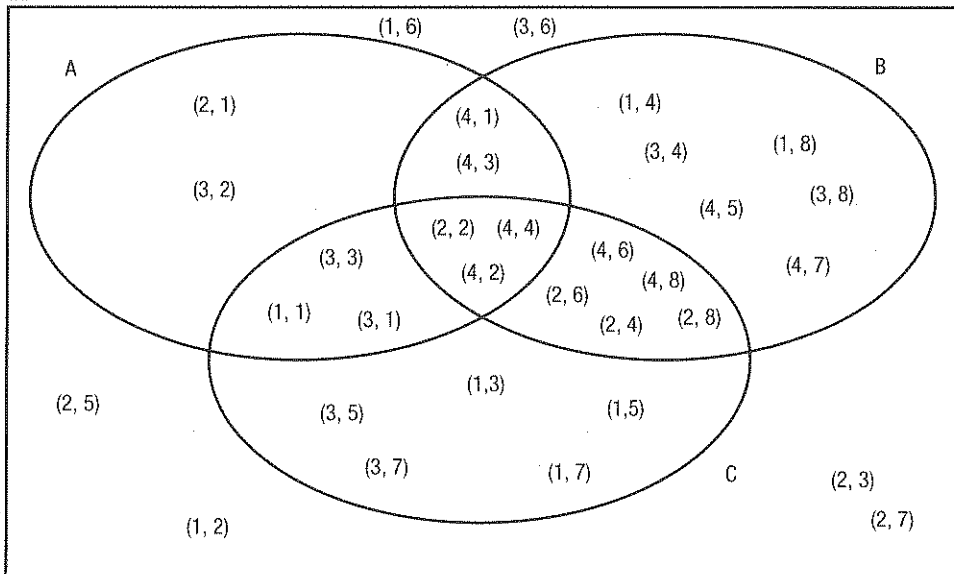
6. a) 1) $\frac{1}{6}$ 2) $\frac{1}{9}$ 3) $\frac{3}{8}$ 4) $\frac{1}{12}$
 5) $\frac{1}{4}$ 6) $\frac{3}{8}$ 7) $\frac{1}{2}$ 8) $\frac{3}{8}$
 b) 1) $\frac{4}{27}$ 2) $\frac{8}{81}$ 3) $\frac{1}{3}$ 4) $\frac{2}{27}$
 5) $\frac{2}{9}$ 6) $\frac{1}{3}$ 7) $\frac{4}{9}$ 8) $\frac{1}{3}$
 7. a) 1) $\frac{4}{13}$ 2) $\frac{2}{13}$ 3) $\frac{1}{2}$ 4) $\frac{1}{26}$
 5) $\frac{8}{13}$ 6) $\frac{1}{13}$ 7) $\frac{1}{8}$ 8) $\frac{5}{13}$
 b) 1) $P | N$ 2) $C' | P$ 3) $P | N'$ 4) $C | P$

Consolidation 4.2 (cont'd)

8. a) $\frac{1}{12}$ b) $\frac{1}{16}$ c) $\frac{1}{8}$ d) $\frac{1}{6}$
 e) $\frac{1}{6}$ f) $\frac{5}{6}$ g) $\frac{1}{3}$ h) $\frac{1}{2}$
 9. a) 85% b) 85%
 c) Since the events are independent, each flight is not affected by what happened during previous flights.
 10. a) $\frac{31}{58}$ b) $\frac{34}{35}$ c) $\frac{431}{699}$ d) $\frac{124}{453}$

Enrichment 4.2

1. a) Ω



- b) 1) $\frac{5}{32}$ 2) $\frac{21}{32}$ 3) $\frac{1}{4}$
 4) $\frac{3}{8}$ 5) $\frac{5}{16}$ 6) $\frac{3}{5}$
 7) $\frac{1}{2}$ 8) $\frac{1}{2}$ 9) $\frac{1}{2}$
 2. a) $\frac{1}{2}$ b) 0

Support 4.3

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1. a) 1) $\approx 23.7\%$ 2) $\approx 27.94\%$ 3) $\approx 23.12\%$ 4) $\approx 25.24\%$
 b) Ruben wins the election.
 c) He or she needs 115 votes.
2. a) 1) 34 ballots. 2) 80 ballots.
 b) 1) 285 points. 2) 295 points. 3) 260 points.
 c) Candidate B wins the election.
 d) 1)

Table of duels

| Number of voters who prefer: | | Number of voters who prefer: | | Winner of the duel opposing: | |
|------------------------------|----|------------------------------|----|------------------------------|---|
| A to B | 65 | B to A | 75 | A and B | B |
| A to C | 80 | C to A | 60 | A and C | A |
| B to C | 80 | C to B | 60 | B and C | B |

- 2) Candidate B wins the election.

Support 4.3 (cont'd)

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3. a) No, since none of them win more than half the votes.
 b) Candidate C.
 c) Since Candidate A is the 2nd choice of 32 voters who ranked Candidate C as their 1st choice and Candidate B is the 2nd choice of 22 voters who ranked Candidate C as their 1st choice, 32 1st-choice votes will be allocated to Candidate A, and 22 1st-choice votes will be allocated to Candidate B.
 d) Candidate A wins the election since he or she receives more than half of the 1st-choice votes once Candidate C is eliminated.

4. a)

| Party | Minimum number of seats won | Remainder |
|-------|---|----------------|
| A | $\frac{1525}{6279} \times 114 \approx 27.69$ or at least 27 seats | ≈ 0.69 |
| B | $\frac{675}{6279} \times 114 \approx 12.26$ or at least 12 seats | ≈ 0.26 |
| C | $\frac{2836}{6279} \times 114 \approx 51.49$ or at least 51 seats | ≈ 0.49 |
| D | $\frac{1243}{6279} \times 114 \approx 22.57$ or at least 22 seats | ≈ 0.57 |

- b) 2 seats.
 c) 1) 1 seat. 2) 0 seats. 3) 0 seats. 4) 1 seat.
 d) 1) 28 seats. 2) 12 seats. 3) 51 seats. 4) 23 seats.
 e) No, since the party in power (Party C) wins less than half of the seats.

Consolidation 4.3

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1. a) Candidate D. b) Candidate C.
 c) None of the candidates. d) Candidate D.
 e) Candidate C.
2. a) Volleyball. b) Volleyball. c) None.
 d) Volleyball. e) Volleyball.

Consolidation 4.3 (cont'd)

3. a) 12 seats are allocated to Party A, 8 seats to Party B and 12 seats to Party C.
 - b) 1) 10 seats are allocated to Party A, 10 seats to Party B and 12 seats to Party C.
 - 2) No, since the winning party (Party C) wins less than half of the seats.
4. 13 seats are allocated to Party A, 16 seats are allocated to Party B and 12 seats are allocated to Party C.

Consolidation 4.3 (cont'd)

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5. a) The movies. b) The movies. c) The movies.
6. 8 seats are allocated to Party A, 11 seats to Party B and 10 seats to Party C.

Consolidation 4.3 (cont'd)

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7. a) Choice of location of the auditorium

| Number of citizens that ordered the cities in this way | 26 900 | 28 500 | 29 000 | 27 500 |
|--|--------|--------|--------|--------|
| 1st choice | A | B | C | D |
| 2nd choice | B | A | D | B |
| 3rd choice | D | D | B | C |
| 4th choice | C | C | A | A |

- b) 1) City D. 2) City B. 3) City C. 4) City B.

Enrichment 4.3

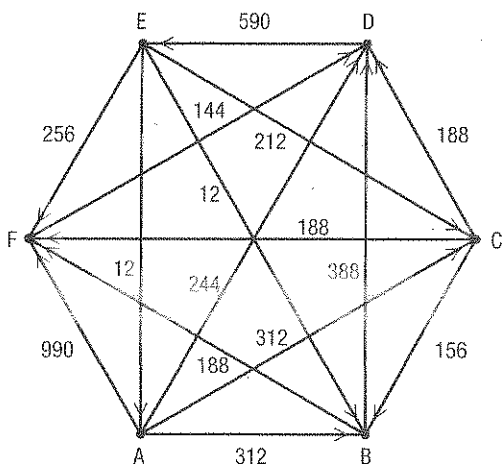
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1. The following is determined:

- The number of points received by each party according to the Borda count is identical.
- The Condorcet method cannot be used to determine the winner since the Renouveau party is preferred to the Enviroplus party, which is preferred to the Vision Avenir party, which is preferred to the Renouveau party (each party wins and loses a duel).
- The elimination method does not apply since none of the parties receive fewer 1st-choice votes than the others.
- Plurality voting and majority rule do not yield a winner since none of the parties receive more 1st-choice votes than the others.

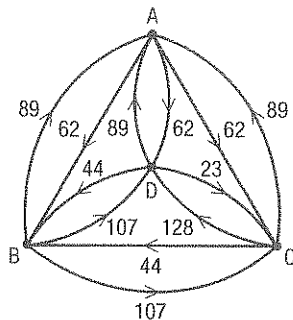
Therefore, a winner cannot be determined.

2. a)

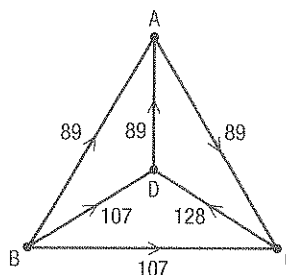


Overview

13. a)

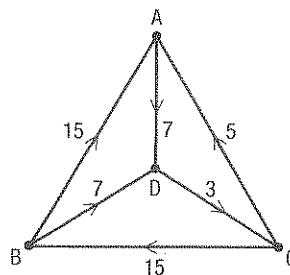


b)



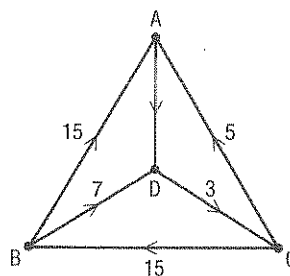
c) Since each arrow that meets a vertex indicates that a candidate has won the duel over the candidate associated with this vertex, the winner is associated with the vertex that does not meet an arrow.

d)



e) Every vertex is met by an arrow. This means that all the candidates lose at least one duel.

f) The winner in Candidate C.

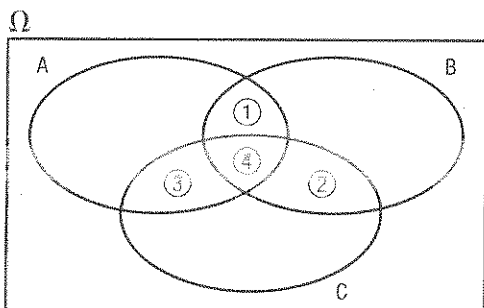


Snapshot 4

1. Meal A is chosen according to the Borda count, and Meal D is chosen according to the elimination method. Therefore, the probability that the person chosen will rank Candidates A or D as his or her 1st or 2nd choice must be calculated. Since all the voters have Candidate A or Candidate B as their two first choices, the probability is 100%.

Snapshot 4 (cont'd)

2. The following is a representation of events A, B and C.



- ①: $(A \cap B) - (A \cap B \cap C)$
- ②: $(B \cap C) - (A \cap B \cap C)$
- ③: $(A \cap C) - (A \cap B \cap C)$
- ④: $(A \cap B \cap C)$

By uniting events A, B and C:

- Region ① is counted an extra time
- Region ② is counted an extra time
- Region ③ is counted an extra time
- Region ④ is counted two extra times

$$A \cup B \cup C = A + B + C - \text{Region ①} - \text{Region ②} - \text{Region ③} - \text{Region ④} - \text{Region ④}$$

$$A \cup B \cup C = A + B + C - [(A \cap B) - (A \cap B \cap C)] - [(B \cap C) - (A \cap B \cap C)] - [(A \cap C) - (A \cap B \cap C)] - (A \cap B \cap C) - (A \cap B \cap C) - (A \cap B \cap C)$$

$$A \cup B \cup C = A + B + C - (A \cap B) + (A \cap B \cap C) - (B \cap C) + (A \cap B \cap C) - (A \cap C) + (A \cap B \cap C) - (A \cap B \cap C) - (A \cap B \cap C)$$

It may be deduced that:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

Snapshot 4 (cont'd)

3. Several answers possible. Example:

Marbleot

| Municipal representative | Regional representative | National representative |
|---|--|---|
| From the three following candidates, select the person you wish to elect as a municipal representative. | Indicate your preference for each of the following candidates by using numbers 1 to 4 (1 indicates that this is the candidate you prefer). | Select the candidate(s) whom you would support as national representative(s). |
| Candidate A <input type="checkbox"/> | Candidate D <input type="checkbox"/> | Candidate H <input type="checkbox"/> |
| Candidate B <input type="checkbox"/> | Candidate E <input type="checkbox"/> | Candidate I <input type="checkbox"/> |
| Candidate C <input type="checkbox"/> | Candidate F <input type="checkbox"/> | Candidate J <input type="checkbox"/> |
| | Candidate G <input type="checkbox"/> | Candidate K <input type="checkbox"/> |
| | | Candidate L <input type="checkbox"/> |
| | | Candidate M <input type="checkbox"/> |

Snapshot 4 (cont'd)

4. The following two events should be considered in this situation:

A: The songs come from Player 2.

B: Two out of three songs are by the artist Jonas.

The question amounts to calculating $P(A | B)$. Therefore, $P(A | B) = \frac{P(A \cap B)}{P(B)}$.

• $P(A) = \frac{1}{2}$

For $P(B)$, it is necessary to include the case in which the two songs by Jonas come from Player 1 and the case in which these songs come from Player 2. For each case, there are three equiprobable chances that two out of three songs are by the artist Jonas. Therefore:

• $P(B) = \frac{1}{2} \times \frac{5}{15} \times \frac{5}{15} \times \frac{10}{15} \times 3 + \frac{1}{2} \times \frac{13}{15} \times \frac{13}{15} \times \frac{2}{15} \times 3$

$$P(B) = \frac{1}{9} + \frac{507}{3375} = \frac{1257}{6750}$$

• $P(A \cap B) = P(A) \times P(B | A)$ since B depends on A.

$$= \frac{1}{2} \times \frac{13}{15} \times \frac{13}{15} \times \frac{2}{15} \times 3$$

$$= \frac{507}{3375}$$

Therefore, $P(A | B) = \frac{\frac{507}{3375}}{\frac{1257}{6750}}$, or approximately 80.67%.

Snapshot 4 (cont'd)

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5.

| Method used to choose the procedure | Procedure obtained | Winning candidate based on the procedure obtained |
|-------------------------------------|--------------------|---|
| Borda count | Borda count | Candidate D |
| Condorcet method | Borda count | Candidate D |
| Elimination method | Borda count | Candidate D |

The conjecture is true.

Snapshot 4 (cont'd)

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6. In each district, the leading party generally has a fairly large advantage. Therefore, it is highly likely that this party will win on the day of the election if the seat of the district is allocated according to plurality voting. In this case, the votes cast by the voters for the other parties are lost and do not count. These voters might conclude that, given that their vote will not affect the election in any way, it is not necessary for them to vote. Based on plurality voting, 3 seats would be allocated to Party A, 2 to Party B and 1 to Party C.

In proportional representation, each vote cast for each party counts and can allow the party to win some seats even if in the same district, a party receives a lower percentage of votes than another. The voters realize that although their preferred party might not come into power, their vote may allow it to obtain a certain degree of political clout. Based on proportional representation, each party would win 2 seats.

