Name_________________

Package 3

Which function has a range of $]-\infty, 4]$ and is positive for $x \in [-5, -1]$?

a) $f(x) = 4(x+5)^2 - 1$

b) $g(x) = -4(x+5)^2 - 1$

c) $h(x) = -x^2 - 6x - 5$

d) $k(x) = -x^2 + 6x + 5$

$\frac{-\Delta}{4a} = 4$

$-b^2 + 4ac = 16a$

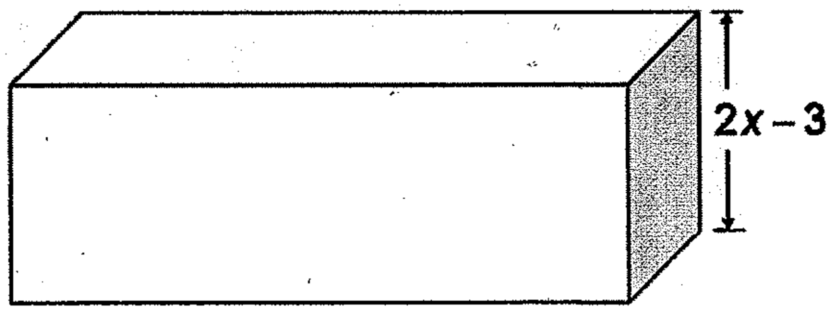
$-b^2 - 4c = -16$

$b^2 + 4c = 16$

$36 + 4(-5) = 16$

Name________________

The volume of the prism shown below is $2x^3 - 5x^2 + 5x - 3$. If its height is $2x - 3$ what algebraic expression represents the area of its base?



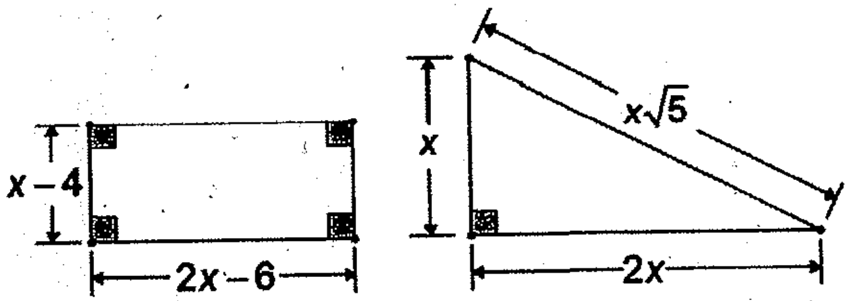
$$V = 2x^3 - 5x^2 + 5x - 3$$

$$A_b = \frac{V}{h} = \frac{2x^3 - 5x^2 + 5x - 3}{2x - 3}$$

$$= (x^2 - x + 1)$$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline 2x-3 & 2x^3 - 5x^2 + 5x - 3 \\ & 2x^3 - 3x^2 \\ \hline & -2x^2 + 5x - 3 \\ & -2x^2 + 3x \\ \hline & 2x - 3 \end{array}$$

The following rectangle and right triangle are equivalent. What are the numerical dimensions of both shapes?



$A_1 = A_2$

$l \times w = \frac{bh}{2}$

$(x-4)(2x-6) = \frac{x(2x)}{2}$

$2x^2 - 14x + 24 = x^2$

$x^2 - 14x + 24 = 0$

$(x-12)(x-2) = 0$

$x = 2$ or 12

$\therefore 2(2) - 6$

$= -2$

$\therefore x \neq 2$

$x = 12$

Rectangle

$(x-4) \quad (2x-6)$

$= (12-4) \quad [(12)2-6]$

$= 8 \times 18$

$\therefore$ The length & width of the rectangle is 8 and 18 units

Triangle

$(x)(2x)$

$= 12\,[2(12)]$

$= 12 \times 24$

$x\sqrt{5}$

$= 12\sqrt{5}$

The height and length of base is 12 and 24 units and the hypotenuse is $12\sqrt{5}$ units long.

Name________________

The table below shows four algebraic operations. When they are performed and reduced to simplest form, they will show a pattern. Note that none of the denominators may be equal to zero.

#	Algebraic Operation	Reduced Result
1	$\frac{x^2+3x+2}{x+2}-\frac{(4x+1)^2}{16x^2+8x+1}$	
2	$\frac{(x^2+4x+3)(x+4)}{x^2+7x+12}$	
3	$\frac{2x^2-9x-5}{4x^2-1}\cdot\frac{2x^2+3x-2}{x-5}$	
4	$\frac{2x^2+11x+15}{?}$	

What is the missing denominator in the 4th algebraic operation if we want to continue the pattern in the reduced result column?

Operation #1

$$\frac{x^2+3x+2}{x+2}-\frac{(4x+1)^2}{16x^2+8x+1}$$

$$=\frac{\cancel{(x+2)}(x+1)}{\cancel{x+2}}-\frac{\cancel{(4x+1)^2}}{\cancel{(4x+1)^2}}$$

$$=x$$

Operation #2

$$\frac{(x^2+4x+3)(x+4)}{x^2+7x+12}$$

$$=\frac{\cancel{(x+3)}(x+1)\cancel{(x+4)}}{\cancel{(x+3)}\cancel{(x+4)}}$$

$$=x+1$$

Operation #3

$$\frac{2x^2-9x-5}{4x^2-1}\cdot\frac{2x^2+3x-2}{x-5}$$

$$=\frac{\cancel{(2x+1)}\cancel{(x-5)}}{\cancel{(2x+1)}\cancel{(2x-1)}}\cdot\frac{\cancel{(2x-1)}(x+2)}{\cancel{x-5}}$$

$$=x+2$$

∴ The forth operation must equal to $x+3$

Operation #4: Let y be the denominator

$$\frac{2x^2+11x+15}{y}=x+2$$

$(2x+5)(x+3)$

$$y=\frac{(2x^2+11x+15)}{x+2}$$

∴ The missing denominator of the 4th Algebraic expression is $\frac{2x^2+11x+15}{x+2}$

Name________________

Which of the following four systems of equations does not have a real solution when $m > 0$?

a) $y_1 = -m$
$y_2 = x^2 - m$

c) $y_1 = -x^2$
$y_2 = m$

b) $y_1 = -x^2$
$y_2 = -m$

d) $y_1 = -x^2 - n$
$y_2 = -n$

Name________________

Review
Math 4SN

Function g is defined by $g(x) = 2x^2 + 16x + 33$. Which of the following statements is false?

a) If $x \in \mathbb{R}$, then function g is positive.

b) If $x \in [-4, \infty[$, then function g is increasing.

c) If $x \in [1, \infty[$, then function g is decreasing.

d) The range is $y \in [1, \infty[$.

$\frac{-b}{2a} = \frac{-16}{2(2)} = -4$

$2(-4)^2 + 16(-4) + 33$

$= 1$

V $(-4, 1)$

Name________________

Review
Math 4SN

Perform the following operations and reduce your answers to their simplest forms with positive exponents.

1. $\dfrac{4x^2-4x+1}{4x^2-1}\cdot\dfrac{6x^3+3x^2-3x}{x^3+x^2}=?$ $2x^2+x-1$

$x \neq 0, \pm\frac{1}{2}, -1$

$= \dfrac{(2x-1)^2}{(2x+1)(\cancel{2x-1})}\cdot\dfrac{3\cancel{x}(\cancel{2x-1})(\cancel{x+1})}{x^{\cancel{2}}(\cancel{x+1})}$

$= \dfrac{3(2x-1)^2}{x(2x+1)}$

2. $\dfrac{5x+4y}{2ax+by+ay+2bx}-\dfrac{3a+3b}{a^2+2ab+b^2}=?$

$= \dfrac{5x+4y}{a(2x+y)+b(2x+y)} - \dfrac{3\cancel{(a+b)}}{(a+b)^{\cancel{2}}}$

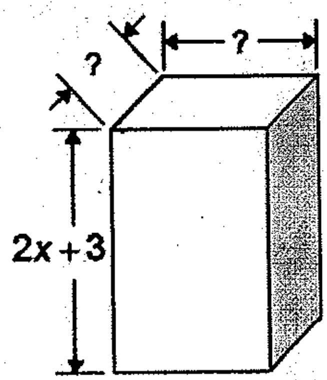
$= \dfrac{5x+4y}{(a+b)(2x+y)} - \dfrac{3}{a+b}$

$= \dfrac{5x+4y-3(2x+y)}{(a+b)(2x+y)}$

$= \dfrac{-x+y}{(a+b)(2x+y)}$

Name________________

The volume of the prism shown below is $2x^3+11x^2+18x+9$. If its height is $2x+3$ what two binomials represent the dimensions of the base?



$V = 2x^3+11x^2+18x+9$

$$\begin{array}{r} x^2+4x+3 \\ 2x+3 \,\big)\, \overline{2x^3+11x^2+18x+9} \\ \underline{2x^3+3x^2 \qquad\qquad\quad} \\ 8x^2+18x+9 \\ \underline{4x^2+12x \quad\;} \\ 6x+9 \\ \underline{6x+9} \end{array}$$

$Ab = \frac{V}{h} = \frac{2x^3+11x^2+18x+9}{2x+3}$

$= x^2+4x+3$

$= (x+3)(x+1)$

$\therefore$ $(x+3)$ and $(x+1)$ represents the dimension of the base

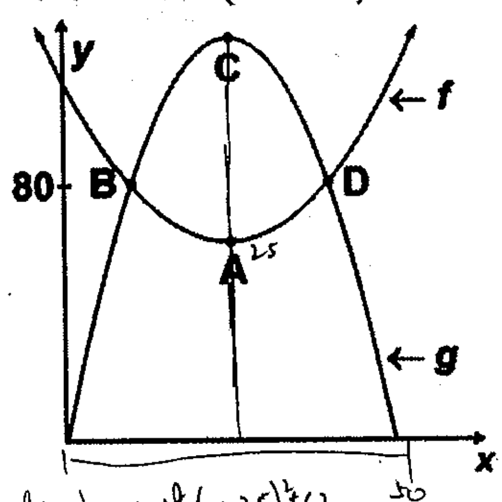
Name________________

Review
Math 4SN

. The Sunshine Circus Company has some amazing acrobats. In the Cartesian plane below, function f shows the path of a person on a zip-line, while function g displays the path of another performer bouncing from a trampoline. At a height of 80 feet the two performer's paths meet at points B and D.

$f(x) = 0.08(x-25)^2 + 62$

The maximum height of the trampoline specialist and the minimum height of the zip-line performer have the same x-value on the Cartesian plane.



What is the rule of the function g that maps the path of the trampoline specialist?

$f(x) = 0.08(x-25)^2 + 62$

$C = (25, 62)$

$25 - 0 = 25$

$25 + 25 = 50$

$g(x) = a(x - x_1)(x - x_2)$

$80 = a(10-0)(10-50)$

$80 = -40a$

$a = -2$

$g(x) = -2x(x-50)$

$= -2x^2 + 100x$

$f(x) = 0.08(x-25)^2 + 62$

$80 = 0.08(x-25)^2 + 62$

$(x-25)^2 = 225$

$x = 10$ and 40

$B(10, 80)$ $D(40, 80)$

Name________________

Perform the following subtraction, reduce your result and state all restrictions on the variable.

$$\frac{x+1}{x^2-5x+6}-\frac{x+5}{x^2-4x+3}=?$$

$$\frac{x+1}{x^2-5x+6}-\frac{x+5}{x^2-4x+3}$$

$x \neq 1, 2, 3$

$$=\frac{x+1}{(x-2)(x-3)}-\frac{x+5}{(x-3)(x-1)}$$

$$=\frac{(x+1)(x-1)-(x+5)(x-2)}{(x-2)(x-3)(x-1)}$$

$$=\frac{(x^2-1)-(x^2+3x-10)}{(x-2)(x-3)(x-1)}$$

$$=\frac{-3x+9}{(x-2)(x-3)(x+1)}$$

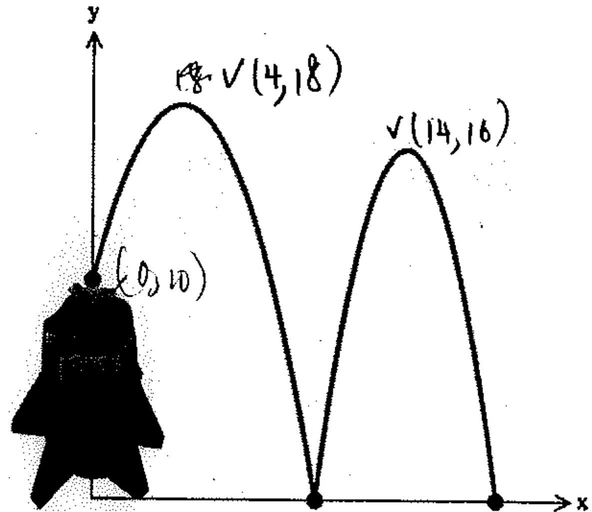
$$=\frac{-3(x-3)}{(x-2)(x-3)(x+1)}$$

$$=\frac{-3}{(x-2)(x+1)}$$

Name________________

A grasshopper is standing on a stump that is 10 cm above the ground. It jumps from the stump and lands on the ground and then makes a second jump and stops.

- On its first jump, the grasshopper reached a maximum height of 18 cm off the ground when it had traveled a horizontal distance of 4 cm.
- During its second jump, the grasshopper reached a maximum height of 16 cm off the ground, a horizontal distance of 14 cm from the stump.



At what height was the grasshopper during its second jump when it was a horizontal distance of 17 cm from the stump?

First Jump : $f(x)$
Second jump : $g(x)$
x : horizontal distance
$f(x)$, $g(x)$: Vertical distance

$f(x) = a(x-h)^2+k$
$10 = a(0-4)^2+18$
$-8 = 16a$
$a = \frac{-1}{2}$
$f(x) = \frac{-1}{2}(x-4)^2+18$
$0 = \frac{-1}{2}(x-4)^2+18$
$(x-4)^2 = 36$
$x = -2$ and 10

$g(x) = a(x-h)^2+k$
$0 = a(10-14)^2+16$
$-16 = 16a$
$a = -1$
$g(x) = -(x-14)^2+16$
$g(17) = -(17-14)^2+16$
$= 7$cm

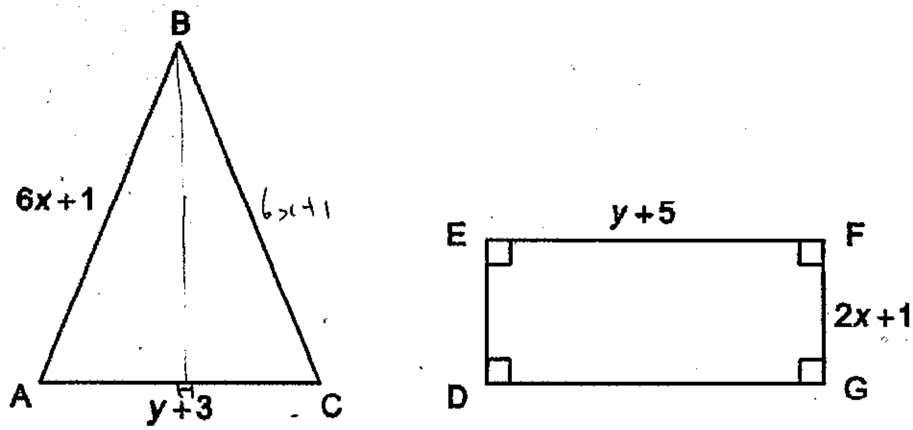
∴ The grasshopper is 7cm above the ground

Name________________

Review
Math 4SN

The polygons shown below have the following properties:

- ΔABC is an isosceles triangle and DEFG is a rectangle
- The perimeter of ΔABC is 36 units
- The perimeter of DEFG is 34 units



Given the above information, determine if the two figures are equivalent in area. Justify your decision.

$P_\triangle = (6x+1)2 + (y+3) = 36$

$12x + y + 5 = 36 \ldots (1)$

$P_\square = 2(l+w) = 34$

$2[(y+5)+(2x+1)] = 34$

$2x + y + 6 = 17 \ldots (2)$

Add a point so that AH=HC

$BC^2 - HC^2 = BH^2$

$(6x+1)^2 - \left(\frac{y+3}{2}\right)^2 = BH^2$

$[6(2)+1]^2 - \left(\frac{7+3}{2}\right)^2 = BH^2$

$BH^2 = 144$

$BH = 12u$

$A_\triangle = \frac{bh}{2}$

$= \frac{12(y+3)}{2}$

$= \frac{12(7+3)}{2} = 60u^2$

(1) - (2)

$12x + y + 5 = 36$

$2x + y + 6 = 17$

$10x - 1 = 19$

$10x = 20$

$x = 2$

Put $x=2$ into (2)

$2x + y + 6 = 17$

$2(2) + y + 6 = 17$

$y = 7$

$A_\square = lw$

$= (y+5)(2x+1)$

$= (7+5)[2(2)+1]$

$= 12 \times 5$

$= 60u^2$

$60u^2 = 60u^2$

$A_\triangle = A_\square$

$\therefore$ Yes, they are equivalent figures

Name________________

Review
Math 4SN

Consider the following systems of equations where $a > 0$, $b > 0$ & $c > 0$ and $a \neq b \neq c$.

1) $y = \frac{a}{b}x - c$ & $y = \frac{a}{b}x + c$ $\frac{a}{b}x - c = \frac{a}{b}x + c$

2) $y = x^2 + a^2$ & $y = -2ax$

3) $y = \frac{a}{b}x - c$ & $y = -\frac{a}{b}x + c$

Which of the above systems of equations have only one solution?

a) Systems 1 and 2 only

b) System 3 only

c) Systems 2 and 3 only

d) System 2 only

Name________________

In the chain of operations below, the numerators and denominators of the rational expressions are never equal to zero.

$$\frac{7x-3}{x^2+3x-4}+\frac{x^2+x-12}{x^2-9}\times\frac{x^2+2x-3}{x^2+5x+4}=?$$

Perform the operations and reduce your final answer.

$$\frac{7x-3}{x^2+3x-4}+\frac{x^2+x-12}{x^2-9}\times\frac{x^2+2x-3}{x^2+5x+4}$$

$$=\frac{7x-3}{(x+4)(x-1)}+\frac{\cancel{(x+4)}\cancel{(x-3)}}{\cancel{(x+3)}\cancel{(x-3)}}\times\frac{\cancel{(x+3)}(x-1)}{\cancel{(x+4)}(x+1)}$$

$$=\frac{7x-3}{(x+4)(x-1)}+\frac{x-1}{x+1}$$

$$=\frac{(7x-3)(x+1)+(x-1)(x-1)(x+4)}{(x+4)(x-1)(x+1)}$$

$$=\frac{(7x^2+4x-3)+(x^2-2x+1)(x+4)}{(x+4)(x-1)(x+3)}$$

$$=\frac{(7x^2+4x-3)+(x^3+2x^2-7x+4)}{(x+4)(x-1)(x+3)}$$

$$=\frac{x^3+9x^2-3x+1}{(x+4)(x-1)(x+3)}$$

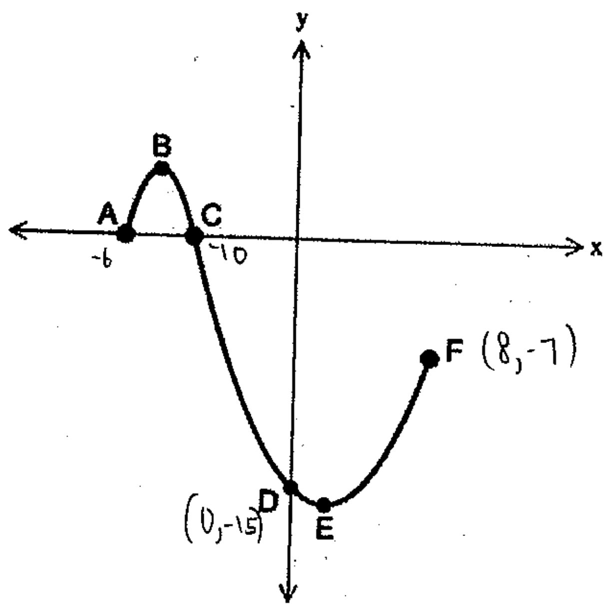
//

Name________________

Review
Math 4SN

A bug crawled along a graph projected on the screen of a Smart Board as shown in the diagram below.

- **It started along the parabola defined by $f(x) = -x^2 - 16x - 60$ through points A, B and C.**
- **At point C, it followed a new parabola defined by $g(x)$ and passed through the points C, D, E and F.**
- **When it reached point F, it flew off the Smart Board and out an open window.**
- **Point B is the maximum height of $f(x)$ and point E is the minimum height of $g(x)$.**
- **Point D is located at (0,-15) and point F is located at (8,-7).**



Calculate coordinates B and E

$f(x) = -x^2 - 16x - 60$

$0 = -x^2 - 16x - 60$

$x^2 + 16x + 60 = 0$

$(x+6)(x+10) = 0$

$x = -6$ and -10

$A(-6, 0)$

$B(-10, 0)$

$V(x, y)$

$x = \frac{-b}{2a}$

$= \frac{-(-16)}{2(-2)}$

$= -8$

$f(x) = -x^2 - 16x - 60$

$f(-8) = -1(-8)^2 - 16(-8) - 60$

$= 4$

$B(-8, 4)$

$y = ax^2 + bx + c$

$g(x) = ax^2 + bx - 15$

$0 = a(-10)^2 + b(-10) - 15$

$0 = 100a - 10b - 15 \quad \ldots(1)$

$f(7) = a(8)^2 + b(8) - 15$

$-7 = 64a + 8b - 15$

$8b = -64a + 8$

$b = -8a + 1 \quad \ldots(2)$

Put (2) into (1)

$0 = 100a - 10b - 15$

$0 = 100a - 10(-8a + 1) - 15$

$15 = 180a - 10$

$25 = 180a$

$a = \frac{5}{36}$

Put $a = \frac{5}{36}$ into (2)

$b = -8a + 1$

$b = -8\left(\frac{5}{36}\right) + 1$

$b = -\frac{1}{9}$

$g(x) = \frac{5}{36}x^2 + \frac{1}{9}x - 15$

$E\left(\frac{-b}{2a}, y\right)$

$x = \frac{-b}{2a} = \frac{-(-\frac{1}{9})x}{2(\frac{5}{36})} = 0.4$

$g(x) = \frac{5}{36}x^2 - \frac{1}{9}x - 15$

$g(0.4) = \frac{5}{36}(0.4)^2 - \frac{1}{9}(0.4) - 15$

$= -30.0\overline{2} = \frac{-676}{45}$

$E(0.4, -30.0\overline{2})$

Name________________

Review
Math 4SN

Perform the following operations and present your final answer as a simplified rational expression where $x \neq -3$ and $x \neq 3$.

$$\frac{1}{x^2-6x+9} - \frac{1}{x^2-9} \div \frac{-3x^2+27}{x^2+6x+9} = ?$$

a) $\frac{-2}{(x-3)^3}$ b) $\frac{4}{3(x-3)^2}$ c) $-\frac{1}{3}$ d) $-\frac{3(x+3)^2}{(x-3)^2}$

$$\frac{1}{x^2-6x+9} - \frac{1}{x^2-9} \div \frac{-3x^2+27}{x^2+6x+9}$$

$$= \frac{1}{(x-3)^2} - \frac{1}{\cancel{(x+3)}(x-3)} \times \frac{\cancel{(x+3)}^2}{-3\cancel{(x+3)}(x-3)}$$

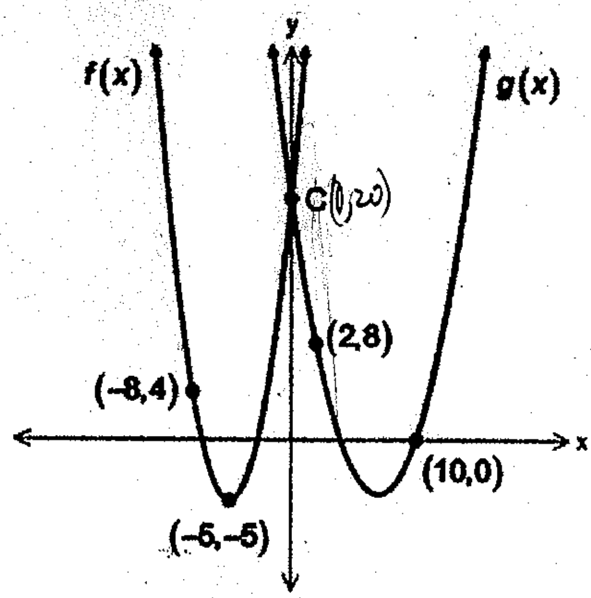
$$= \frac{1}{(x-3)^2} - \frac{1}{-3(x-3)^2}$$

$$= \frac{-4}{(x-3)^2}$$

Name________________

Consider the quadratic functions shown in the Cartesian plane below. The following additional information is also true:

- **$f(x)$ and $g(x)$ share the same y-intercept at point C**
- **The vertex of $f(x)$ is located at $(-5,-5)$**



State the rule for $g(x)$ in factored form.

$f(x) = a(x-h)^2+k$
$4 = a(-8+5)^2-5$
$9 = 9a$
$a = 1$
$f(x) = 1(x+5)^2-5$
$f(0) = (0+5)^2-5$
$= 20$
$C(0,20)$

$g(x) = ax^2 + bx + 20$
$8 = a(2)^2 + 2b + 20$
$-12 = 4a + 2b$
$-6 = 2a + b \ldots (1)$
$0 = (10^2)a + b(10) + 20$
$-20 = 100a + 10b$
$-2 = 10a + b \ldots (2)$
$(1) - (2)$
$-6 = 2a + b$
$-2 = 10a + b$
$-4 = -8a$
$a = \frac{1}{2}$
Put $a = \frac{1}{2}$ into (2)
$-2 = 10a + b$
$-2 = (\frac{1}{2})(10) + b$
$b = -7$

$g(x) = \frac{1}{2}x^2 - 7x + 20$
$= \frac{1}{2}(x^2 - 14x + 40)$
$= \frac{1}{2}(x-4)(x-10)$

SOLUTION #2

$y = a(x-$

$$y = a(x - x_1)(x - 10)$$
$$8 = a(2 - x_1)(2 - 10) \qquad a = \frac{8}{(2-x_1)(2-10)} \ldots (1)$$
$$20 = a(0 - x_1)(0 - 10) \quad (2)$$

Put (1) into (2)

$$20 = \frac{8}{(2-x_1)(2-10)}(0 - x_1)(0 - 10)$$
$$20(2 - x_1)(2 - 10) = 80x_1$$
$$40 - 20x_1 = -10x_1$$
$$10x_1 = 40$$
$$x_1 = 4$$

Put x_1 into (1)

$$a = \frac{8}{(2-x_1)(2-10)}$$
$$= \frac{8}{(2-4)(2-10)}$$
$$= \frac{8}{16}$$
$$= \frac{1}{2}$$