OPTIMIZATION 566

2- Correction key

1	C

2 Work : (example)

Inequalities representing the constraints

Given x : measure of each of the congruent sides

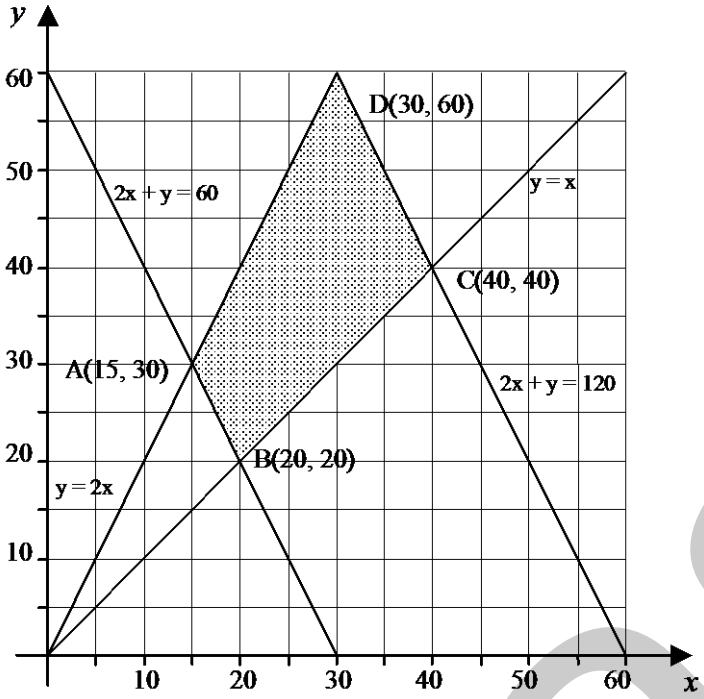
y : measure of the third side

$x \geq 0$ $y \geq 0$

$2x + y \leq 120$ $2x + y \geq 60$

$y \leq 2x$ $y \geq x$

Polygon of constraints and co-ordinates of the vertices



Function to be optimized

$$h = \sqrt{x^2 - 0.25y^2}$$

Value of the function to be optimized for each of the vertices

At A, $h = \sqrt{15^2 - 0.25 \times 30^2} = 0$

At B, $h = \sqrt{20^2 - 0.25 \times 20^2} = \sqrt{300} \approx 17.3$

At C, $h = \sqrt{40^2 - 0.25 \times 40^2} = \sqrt{1200} \approx 34.6$

At D, $h = \sqrt{30^2 - 0.25 \times 60^2} = 0$

Maximum value of the function

The maximum height is 34.6.

Result : The maximum height of the triangle is 34.6 cm.

3 Work : (example)

Given x : Number of all-terrain vehicles sold

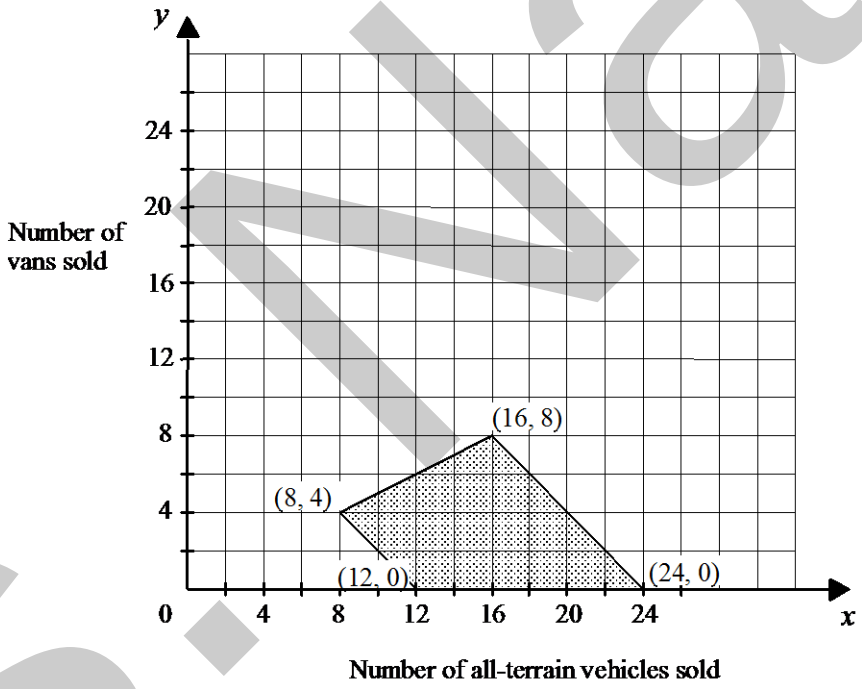
y : Number of vans sold

Mathematizing the constraints

$x \geq 0$ $\quad x + y \geq 12$ $\quad x \geq 2y$

$y \geq 0$ $\quad x + y \leq 24$

Polygon of constraints and co-ordinates of the vertices



Function to be optimized : $P = 700x + 1000y$

Value of the function to be optimized for each of the vertices

Vertex	$P = 700x + 1000y$
(12, 0)	8400
(8, 4)	9600
(16, 8)	19 200
(24, 0)	16 800

Maximum value of the function

Profit is maximal for 16 all-terrain vehicles and 8 vans.

Result : The maximum profit the dealer can expect to make on his sales is $19 200.

4 Work : (example)

Mathematizing the constraints

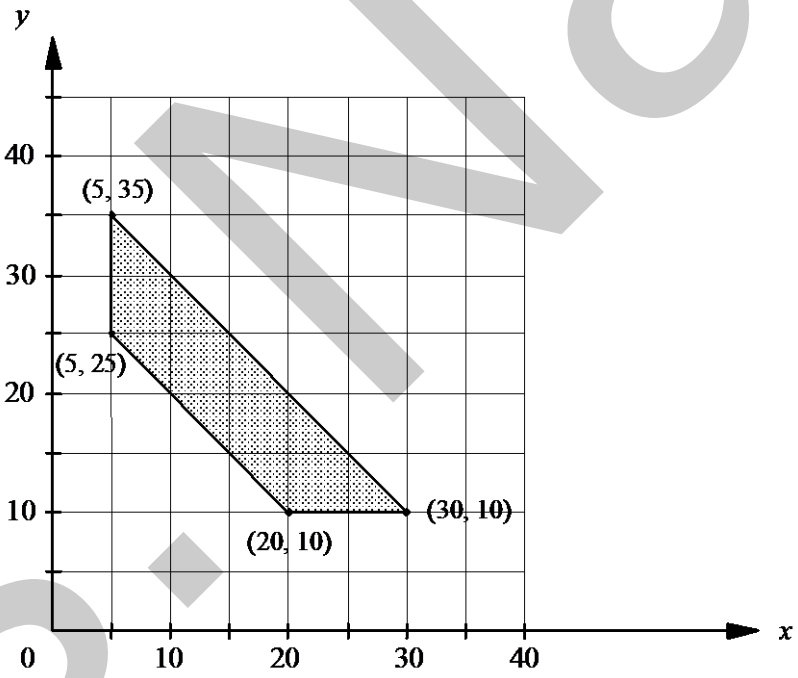
Given x : number of oak trees transplanted

y : number of fir trees transplanted

$x \geq 5$ $\quad$ $x + y \geq 30$

$y \geq 10$ $\quad$ $x + y \leq 40$

Polygon of constraints and coordinates of vertices



Mathematizing the function to be optimized

$P = 35x + 25y$

The value of the function to be optimized for each of the vertices

Vertex	$P = 35x + 25y$
(5, 25)	800
(5, 35)	1050
(20, 10)	950
(30, 10)	1300

Result : To maximize his profits, the landscaper must transplant 30 oak trees and 10 fir trees.

5

Work : (example)

Given x : number of work hours for electrician

y : number of work hours for apprentice

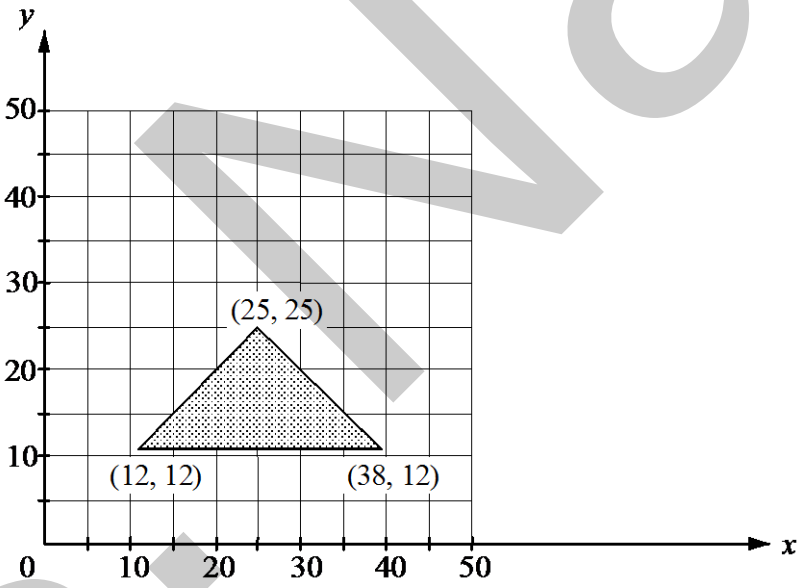
Mathematizing the constraints

$x + y \leq 50$

$y \geq 12$

$x \geq y$

Polygon of constraints and coordinates of vertices



Mathematizing the function to be optimized

$C = 30x + 18y + 10\,500$

The value of the function to be optimized for each of the vertices

Vertex	$C = 30x + 18y + 10\,500$
(12, 12)	11 076
(25, 25)	11 700
(38, 12)	11 856

Result : The minimal cost of the renovations will be $11 076.

6 B

7 Work : (example)

Let x : number of Thermex models sold

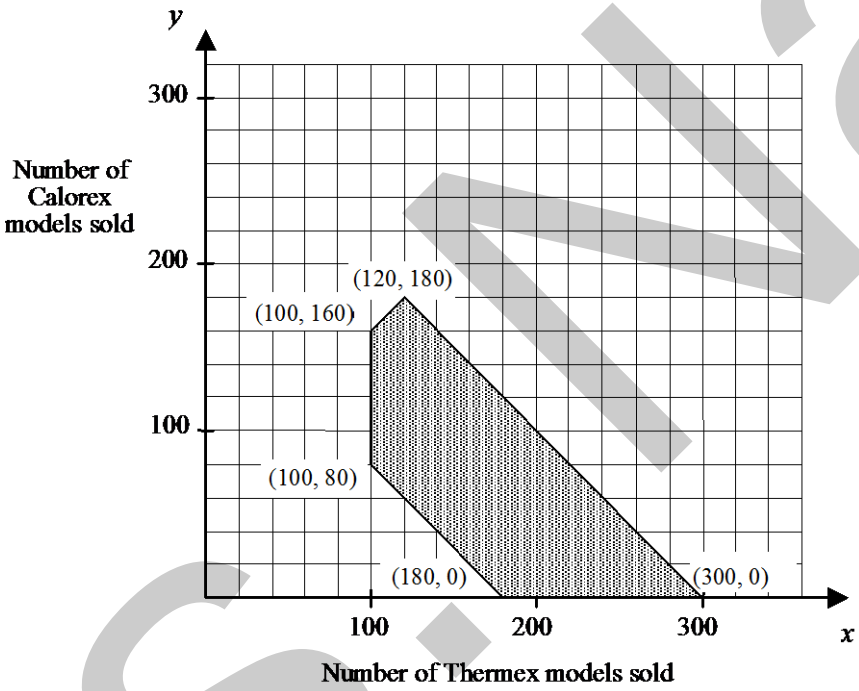
y : number of Calorex models sold

Constraints

$x + y \geq 180$ $x \geq 100$ $x \geq 0$

$x + y \leq 300$ $y \leq x + 60$ $y \geq 0$

Polygon of constraints and coordinates of vertices



Function to be optimized

$P = 525x + 700y$

Value of the function to be optimized at each vertex

Vertex	$P = 525x + 700y$
(180, 0)	94 500
(100, 80)	108 500
(100, 160)	164 500
(120, 180)	189 000
(300, 0)	157 500

Result : The store should sell 120 Thermex models and 180 Calorex models.

8 Example of an appropriate method

Let x: the number of Colomb hats

y: the number of Cartier hats

System of inequalities

$x \geq 0$

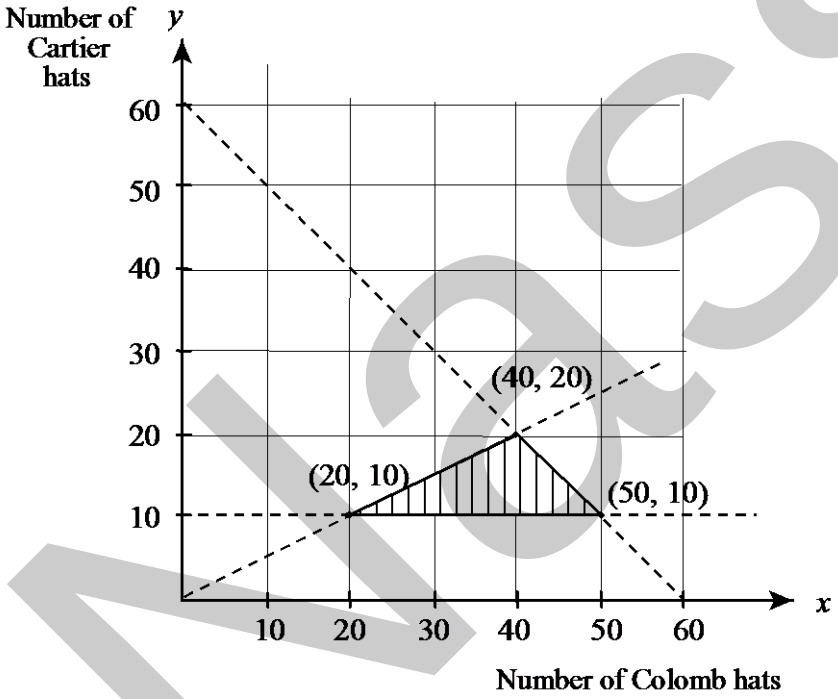
$y \geq 0$

$x \geq 10$

$y \geq 10$

$x \geq 2y$

$x + y \leq 60$



Function to be optimized

$P = 15x + 25y$

Evaluation of the objective function

Vertices	$P = 15x + 25y$
(20, 10)	550
(40, 20)	1100

(50, 10)	1000

Answers: The owner must sell 40 Colomb hats and 20 Cartier hats.

9 Example of an appropriate method

Coordinates of the vertices

P: intersection of $y = 4$ and $x = 1$

P(1, 4)

Q: intersection of $y = 4 + 2x$ and $x = 1$

If $x = 1$ then $y = 4 + 2(1) = 6$

Q(1, 6)

R: intersection of $y = 4 + 2x$ and $x + y = 16$

$y = 4 + 2x$ and $y = 16 - x$

$4 + 2x = 16 - x$

$x = 4$

If $x = 4$ then $y = 4 + 2(4) = 12$

R(4, 12)

S: intersection of $y = 4$ and $x + y = 16$

If $y = 4$ then $x + 4 = 16$ therefore $x = 12$

S(12, 4)

Maximum income

Vertices	Income	
	Option A $13x + 8y$	Option B $9x + 9y$
P(1, 4)	45	45
Q(1, 6)	61	63
R(4, 12)	148	144
S(12, 4)	188	144

Maximum income

Answer Option A will allow Oliver to maximize his income.

Note Students who correctly or incorrectly determine the optimization function for each option **or** who correctly or incorrectly determine the coordinates of the vertices using an algebraic or graphic method have shown that they have a partial understanding of the problem.

10 Example of an appropriate method

Let x: the number of type A containers

y: the number of type B containers

System of inequalities

$x \geq 0$

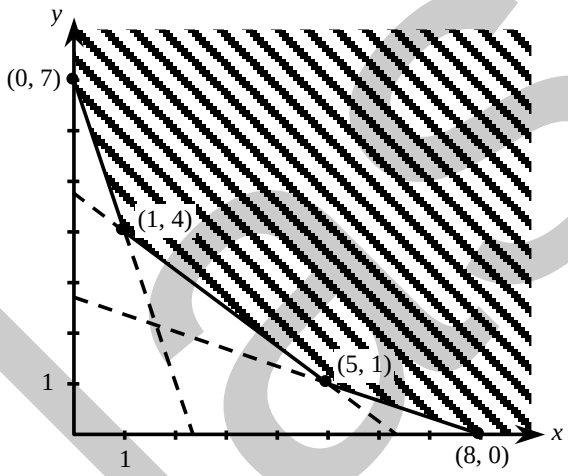
$y \geq 0$

$3x + y \geq 7$

$3x + 4y \geq 19$

$x + 3y \geq 8$

The polygon of constraints



The rule to be minimized

Transportation costs = $6000x + 5000y$

Value of the rule to be minimized

Vertex	$6000x + 5000y$
(0, 7)	35 000
(1, 4)	26 000
(5, 1)	35 000
(8, 0)	48 000

Answer The company must use 1 type A container and 4 type B containers.

11 Example of an appropriate method

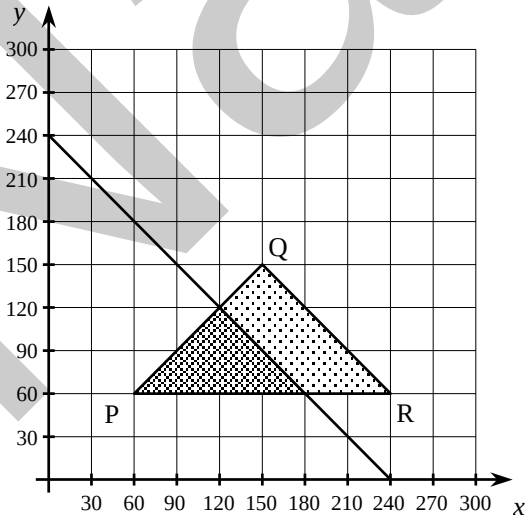
Maximum revenue before the runway is built

Vertex	Revenue: $4x + 6y$
P(60, 60)	\$600
Q(150, 150)	\$1500
R(240, 60)	\$1320

The maximum revenue is \$1500.

Additional constraint

$x + y \leq 240$



Maximum revenue after the runway is built

Vertex	Revenue: $4x + 6y$
(60, 60)	\$600

(120, 120)	$1200
(180, 60)	$1080

The maximum revenue is $1200.

Decrease in maximum revenue

$1500 – $1200 = $300

Answer: The maximum possible revenue will decrease by $300 once the runway is built.

Note: Students who use an appropriate method in order to draw the new polygon of constraints **or** to determine the maximum revenue before the runway is built have shown that they have a partial understanding of the problem.

12 Example of an appropriate method

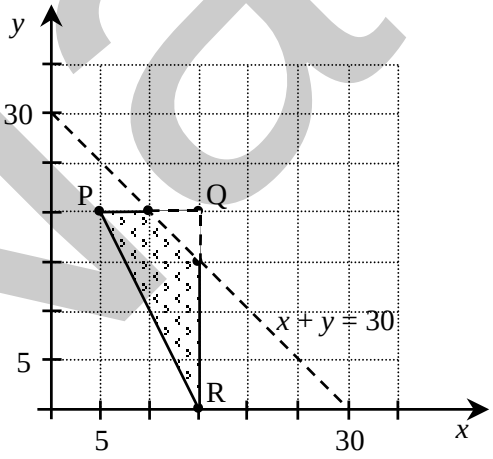
Maximum possible income before the tournament

Vertices	Income: $9x + 8y$
P (5, 20)	\$205
Q (15, 20)	\$295 ← maximum
R (15, 0)	\$135

Additional constraint

$x + y \leq 30$

2 new vertices (10, 20) and (15, 15)



Maximum possible income after the new constraint was imposed

Vertices	Income: $9x + 8y$
P (5, 20)	\$205
(10, 20)	\$250

(15, 15)	\$255 ← maximum
R (15, 0)	\$135

Difference between the maximum incomes

$295 - 255 = \$40$

Answer: Jenny's maximum possible income will decrease by \$**40** because of this additional constraint.

Note: Students who use an appropriate method in order to determine the maximum possible income before **or** after the additional constraint was imposed have shown that they have a partial understanding of the problem.

Name : _____

Group : _____

Date : _____

568514 - Mathematics

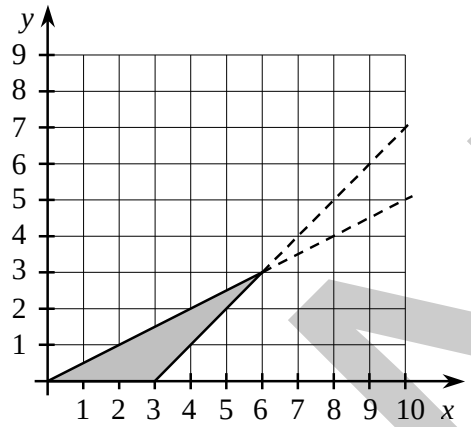
Question Booklet

1 The following system of inequalities represents the constraints associated with an optimization situation.

$$x \geq 0$$
$$y \geq 0$$
$$y \leq x - 3$$
$$x \leq 2y$$

Which one of the following polygons of constraints could represent this situation?

A)



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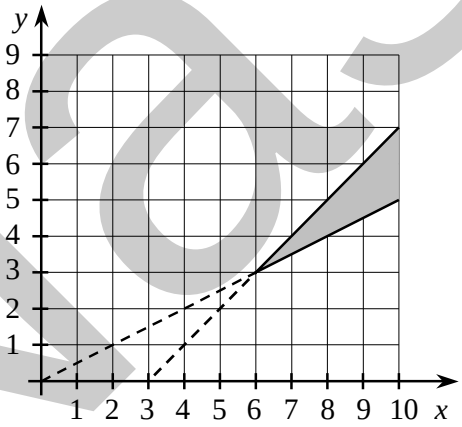
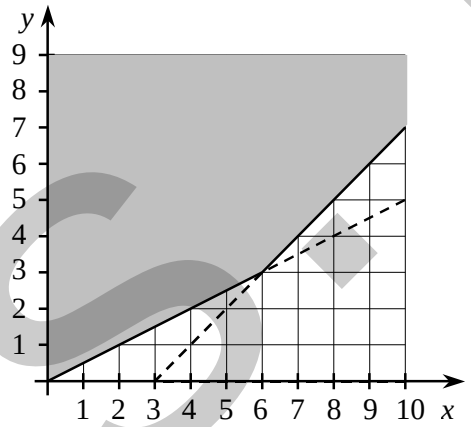
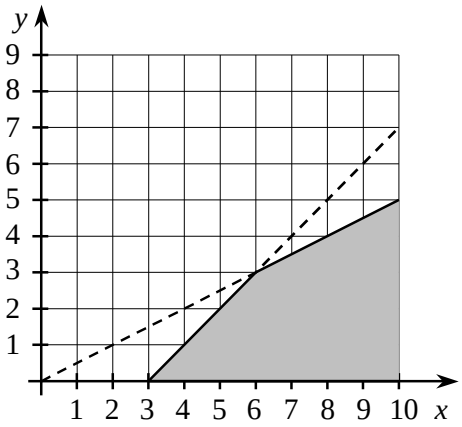


B)



D)



2 An isosceles triangle is to be constructed respecting the following constraints :

- its perimeter is less than or equal to 120 cm,
- its perimeter is greater than 60 cm,
- the measure of its third side is less than or equal to the sum of the measures of its congruent sides,
- the measure of its third side is greater than or equal to the measure of one of its congruent sides.

Furthermore, the height of the triangle is to be as great as possible.

The function that can be used to maximize the height is

$$h = \sqrt{x^2 - 0{,}25y^2}$$

where x is the measure of each of the congruent sides and y is the measure of the third side.

What is the maximum height of the triangle?

Show your work.

3 An automobile dealer holds an annual sale of both all-terrain vehicles and vans. Past experience has shown that his sales tend to respect the following constraints :

- The total number of all-terrain vehicles and vans is greater than or equal to 12 but less than or equal to 24.
- The number of all-terrain vehicles sold is at least twice the number of vans sold.

The dealer makes an average profit of $700 on each all-terrain vehicle and $1000 on each van.

What is the maximum profit the dealer can expect to make on his sales?

4 A landscape gardener has to transplant several oak trees and fir trees in a park according to the following constraints :

- He must transplant at least 30 trees but not more than 40.
- He must transplant at least 50 oak trees and 10 fir trees.

The landscaper makes a profit of $35 for each oak he transplants and $25 for each fir tree.

Given x : the number of oak trees transplanted

y : the number of fir trees transplanted

How many oak trees and fir trees does he have to plant to maximize his profit?

Show your work.

5 The owner of an arena plans to have the lighting in the skating rink improved. He has hired an electrician and his apprentice to do the work. He estimates that it will take a maximum of 50 hours to complete the job. The apprentice must work a minimum of 12 hours and the electrician has to work at least as many hours as the apprentice.

The material needed for the job costs $10 500. The hourly wage of the electrician is $30 while the apprentice earns $18 an hour.

Given x : number of work hours for electrician

y : number of work hours for apprentice

What will be the minimal cost for these renovations?

Show your work.

6 A company needs to purchase some printers.

A laser printer operating at a rate of 8 pages per minute costs $1000.

An ink jet printer operating at a rate of 3 pages per minute costs $500.

The company needs at least 5 laser printers and has set a budget limit of $19 500 to purchase all its printers. The computer room will hold a maximum of 25 printers.

Given x : the number of laser printers

y : the number of ink jet printers

These linear inequalities translate the constraints of this problem :

$x + y \leq 25$

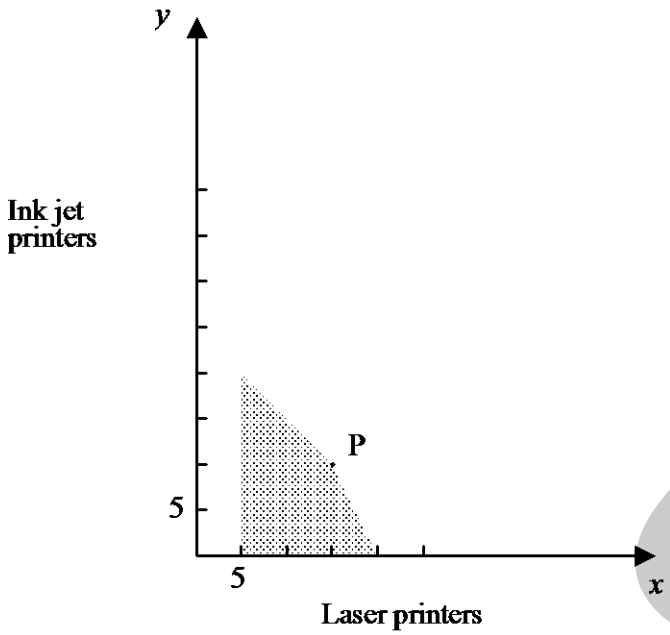
$1000x + 500y \leq 19\,500$

$x \geq 5$

$x \geq 0$

$y \geq 0$

The polygon below represents this situation :



What are the coordinates of vertex P of this polygon of constraints?

A) (11, 14)

B) (14, 11)

C) (15, 10)

D) (12, 13)

7 The manager of a warehouse store decided to offer his customers two models of a certain heat pump : the Thermex model and the Calorex model.

The marketing department gathered the following information :

- The annual demand for both models will be no more than 300 units.
- The annual demand for both models will be at least 180 units.
- A minimum of 100 Thermex models will likely be sold.
- At most there are 60 more Calorex models than Thermex models in the warehouse.

The company makes a profit of $525 on each Thermex model and $700 on each Calorex model.

Let x : represent the number of Thermex models sold

y : represent the number of Calorex models sold

How many heat pumps of each model should be sold in order to maximize the profit?

Show your work.

8 A speciality store sells two designer-style hats: Colomb and Cartier. The owner cannot keep any more than 60 hats in the store. However, he must always have at least 10 hats of each style. He sells at least twice many Colomb hat as Cartier hat.

His profit on a Colomb hat is $15 and on a Cartier hat is $25.

How many hats of each style must the owner sell to maximize his profit?

Show all your work.

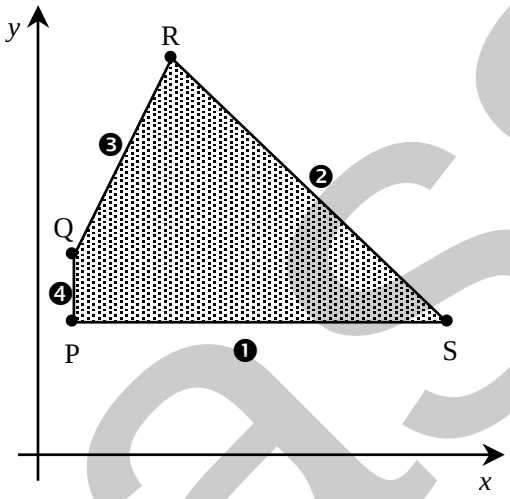
9 Oliver works as a cashier and wrapper at a grocery store. There are different constraints that limit the number of hours he can work per week. These constraints are represented by the inequalities and the polygon of constraints given below. Each side of the polygon and its corresponding inequality are identified by the same number.

❶ $y \geq 4$

❷ $x + y \leq 16$

❸ $y \leq 4 + 2x$

❹ $x \geq 1$



x: number of hours he can work per week as a cashier

y: number of hours he can work per week as a wrapper

Oliver's boss suggests two ways of paying him:

Option A: \$13 per hour as a cashier and \$8 an hour as a wrapper;

Option B: \$9 per hour, whether he works as a cashier or as a wrapper.

Which option will allow Oliver to maximize his income?

Show all your work.

10 A mining company receives an order for 7 tonnes of iron, 19 tonnes of lead and 8 tonnes of copper. This order can be delivered using two types of containers.

Each type A container holds 3 tonnes of iron, 3 tonnes of lead and 1 tonne of copper.

Each type B container holds 1 tonne of iron, 4 tonnes of lead and 3 tonnes of copper.

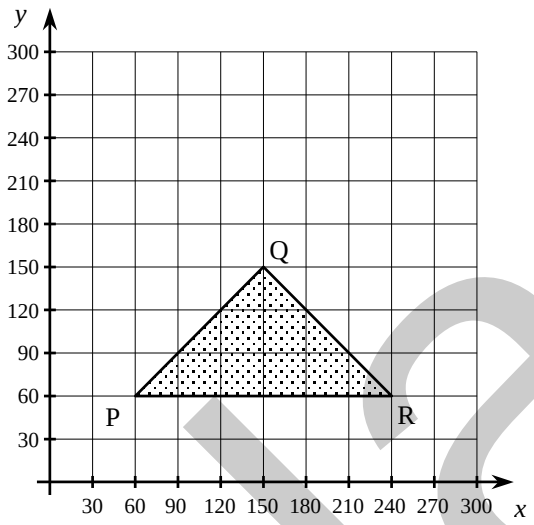
Its costs $6000 to transport one type A container and $5000 to transport one type B container.

How many containers of each type must the company use to minimize transportation costs?

Show all your work.

11 The student council of a school is organizing a fashion show, which will be held in the school auditorium. The tickets will cost $4 for students at the school and $6 for the general public.

The following polygon of constraints reflects the different constraints faced by the student council.



x: number of tickets for students at the school

y: number of tickets for the general public

A runway will be built so that the models can walk down the middle of the auditorium. The runway will reduce the seating capacity. As a result, only a maximum of 240 tickets can be sold.

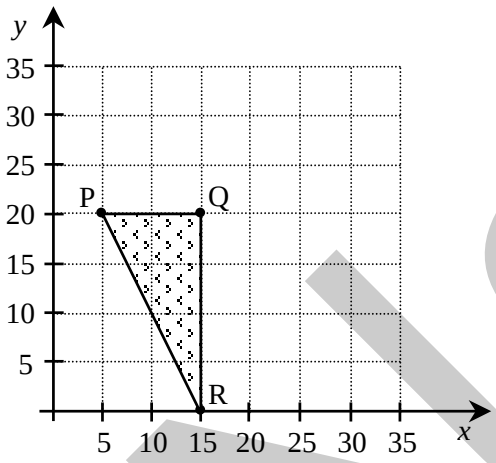
By how much will the maximum possible revenue decrease once the runway is built?

Show all your work.

12 Jenny has two part-time jobs for the summer. She cleans a senior citizens' centre and also works in a grocery store.

Jenny makes $9 an hour when she works at the senior citizens' centre and $8 an hour when she works at the grocery store.

Every week, there are different constraints on the number of hours she can devote to each job. This situation is represented by the following polygon of constraints:



Coordinates of the vertices of the polygon
P (5, 20)
Q (15, 20)
R (15,0)

x : number of hours worked in the senior citizens' centre

y : number of hours worked in the grocery store

Since Jenny will be playing in a soccer tournament this week, she can work at most 30 hours.

By how much will Jenny's maximum possible income decrease because of this additional constraint?

Show all your work.