QUADRATIC

1 Example of an appropriate method

x-coordinate of the location of the basket

y-coordinate of the location of the basket: 3

$$-0.2(x-5)^2 + 3.45 = 3$$

$$-0.2(x-5)^2 = -0.45$$

$$(x-5)^2 = 2.25$$

$$x - 5 = -1.5 \quad \text{or} \quad x - 5 = 1.5$$

$$x = 3.5 \qquad\qquad x = 6.5$$

Since the basket is located to the right of the vertex of the parabola, $x = 6.5$.

x-coordinate of the location of the basket: 6.5

y-coordinate of the location of the ball at the moment Caroline throws it

x-coordinate of the location of the ball at the moment Caroline throws it: $6.5 - 4.5 = 2$

$$f(2) = -0.2(2-5)^2 + 3.45 = 1.65$$

y-coordinate of the location of the ball at the moment Caroline throws it: 1.65

Answer: At the moment that Caroline throws the ball, the distance between the ball and the ground is **1.65** m.

Note: Students who use an appropriate method in order to determine the *x*-coordinate of the location of the basket have shown that they have a partial understanding of the problem.

2 Example of an appropriate method

Rule of the function

x: time in minutes

f(x) = altitude in metres

$f(x) = a(x - h)^2 + k$

$f(x) = a(x - 3)^2 + 10$

$f(8) = 0$ then $0 = a(8 - 3)^2 + 10$

$0 = a(25) + 10$

$\frac{-10}{25} = a$

$-0.4 = a$

$f(x) = -0.4(x - 3)^2 + 10$

***y*-intercept**

$f(0) = -0.4(0 - 3)^2 + 10 = 6.4$

Answer The balcony is located 6.4 m off the ground.

Note Students who correctly or incorrectly determine the rule of the function depicted in the graph have shown that they have a partial understanding of the problem.

3 Example of an appropriate solution

Equation of the parabola

According to the table of values, the coordinates of the vertex of the parabola are S(29, 150).

$y = a(x - h)^2 + k$

$y = a(x - 29)^2 + 150$

$54 = a(9 - 29)^2 + 150$

$-96 = 400a$

$-0.24 = a$

The equation of the parabola is $y = -0.24(x - 29)^2 + 150$.

Launching point

If $y = 0$, then $0 = -0,24(x - 29)^2 + 150$ Hence, $x = 4$ and $x = 54$

Since the launching point is to the left of the vertex of the parabola, the coordinates of the launching point are $x = 4$ and $y = 0$.

Position of the rocket when it exploded

If $y = 96$, then $96 = -0.24(x - 29)^2 + 150$ Hence, $x = 14$ or $x = 44$

Since the position of the rocket when it exploded is the right of the vertex of the parabola, the coordinates of the position of the rocket when it exploded are $x = 44$ and $y = 96$.

Position of the fountain

Since the rocket exploded 96 m above the fountain, the coordinates of the position of the fountain are $x = 44$ and $y = 0$.

Distance between the launching point and the fountain

$44 - 4 = 40$ m

Answer The distance between the point from which the rocket was launched and the fountain is 40 m.

4 C

5 C

6 D

7 C

8 B

9 The rule of correspondence that defines the parabola is $(x - 8)^2 = 6(y + 6)$

or $x^2 - 16x - 6y + 28 = 0$

or $y = \frac{1}{6}\left(x^2 - 16x + 28\right)$

or $y = \frac{1}{2}x^2 - \frac{8}{3}x + \frac{14}{3}$

or any equivalent rule of correspondence.

10 C

11 The width AB of the tunnel is 6 m.

12 The rule of correspondence of the trajectory is $y = \frac{-4}{9}\left(x^2 - 6x\right)$

or any other equivalent rule of correspondence.

Example : $y = \frac{-4}{9}x^2 + \frac{8}{3}x$

13 B

14 D

15 B

16 The rule of correspondence that defines the parabola is $y = \frac{-1}{9}(x - 6)^2 + 4$

or any equivalent rule of correspondence.

17 B

18 C

19 B

20 A

21 Example of an appropriate solution

Equation of the parabola

$y = a(x - h)^2 + k$

$y = a(x - 9)^2 + 3$

$1 = a(5 - 9)^2 + 3$

$\frac{-1}{8} = a$

Hence, $y = \frac{-1}{8}(x - 9)^2 + 3$

Find the zeros

$0 = \frac{-1}{8}(x - 9)^2 + 3$

$\pm \sqrt{24} + 9 = x$

The zeros are $9 - \sqrt{24} \approx 4.101$ and $9 + \sqrt{24} \approx 13.899$

Distance between the net and the point where the volleyball hits the ground

Distance $\approx 13.899 - 9 = 4.899$

Answer The volleyball will hit the ground approximately 4.9 m from the net.

22 C

23 D

24 At the moment the player hits the ball, the distance between the ball and the wall is 6.4 m.

25 Example of an appropriate method

Coordinates of point B

The axis of symmetry of the parabola representing f is $x = 3$.

Since the coordinates of A are A(0, 0), the coordinates of B are B(6, 0).

Rule of g

Since the zeros of function g are 6 and 10, the equation of the axis of symmetry of the parabola representing g is $x = 8$.

The coordinates of the vertex are h = 8 and k = 4.

$g(x) = a(x - 8)^2 + 4$

$0 = a(6 - 8)^2 + 4$

$0 = 4a + 4$

$-4 = 4a$

$-1 = a$

$g(x) = -1(x - 8)^2 + 4$

Answer: The rule of the function g is $g(x) = -(x - 8)^2 + 4$.

Note: Students who use an appropriate method in order to determine the value of the x-coordinate of the vertex of the parabola representing g have shown that they have a partial understanding of the problem.

26 The rule of function g is $g(x) = \mathbf{5(x - 8)^2 - 80}$.

Note: Accept any equivalent rule.

Name : _____

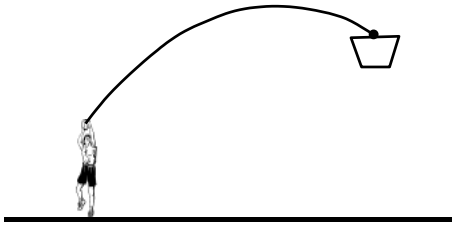
Group : _____

Date : _____

568436 - Mathematics

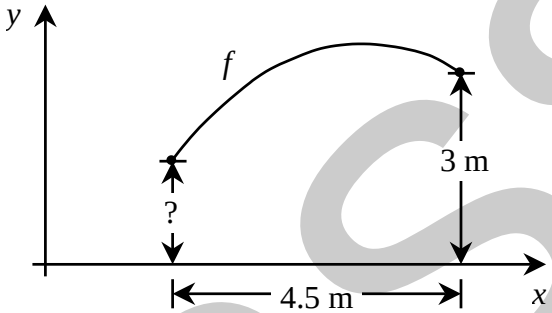
Question Booklet

1 Caroline throws a ball toward a basket located 3 m above the ground.



The ball reaches a maximum height. On its way down, it enters the basket.

In the Cartesian plane on the right, the side view of the ball's trajectory is represented by function *f*. The scale of this graph is in metres.



The rule associated with function *f* is

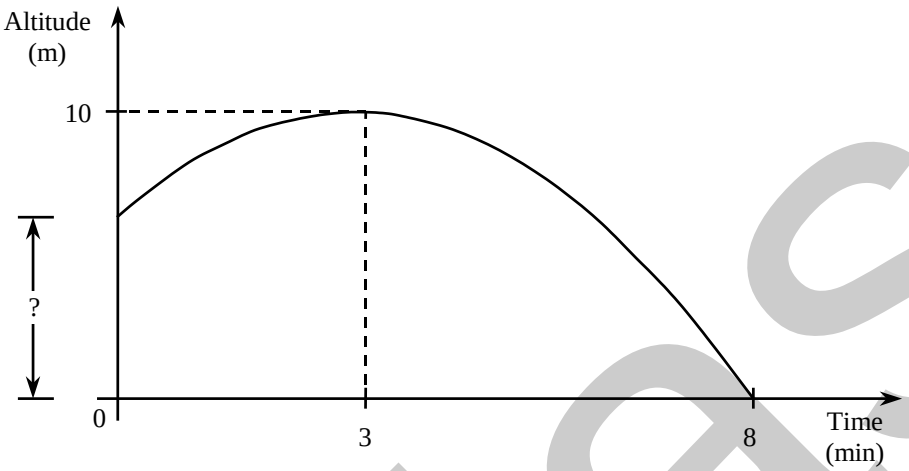
$f(x) = -0.2(x - 5)^2 + 3.45$.

The horizontal distance between Caroline and the location of the basket is 4.5 m.

At the moment that Caroline throws the ball, what is the distance between the ball and the ground?

Show all your work.

2 Melanie was playing with a remote-controlled toy airplane. The plane took off from a balcony and landed on the ground 8 minutes later. Three minutes after taking off, the plane reached a maximum altitude of 10 metres. In the graph below, the plane's altitude as a function of time is represented by a portion of a parabola.

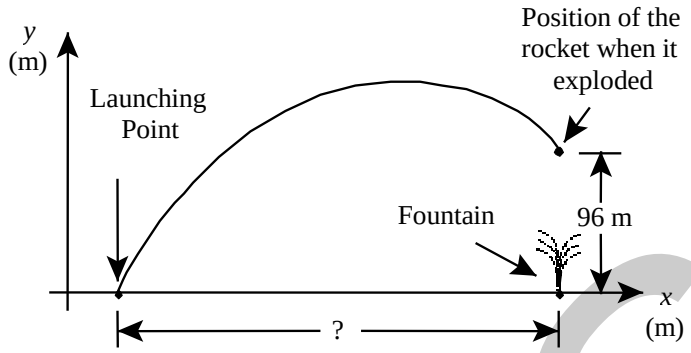


How high off the ground is the balcony located?

Show all your work.

3 A rocket was launched during a fireworks show. The side view of the rocket's parabolic trajectory is represented by the following table of values and graph.

x (metres)	*y* (metres)
9	54
19	126
29	150
39	126



The rocket exploded 96 m above a fountain.

What is the distance between the point from which the rocket was launched and the fountain?

Show all your work.

4 Which function has a range of $-\infty, 4]$ and is positive for $x \in]-1, 3[$?

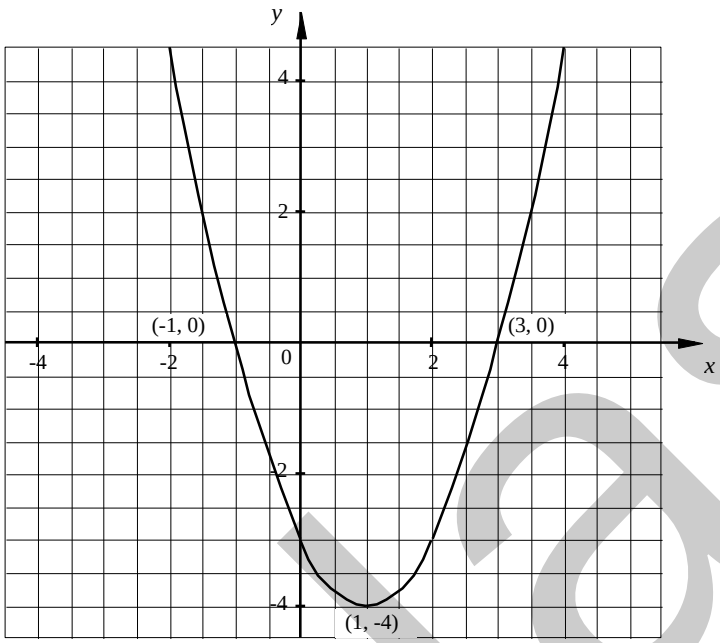
A) $f(x) = 4(x + 1)^2 + 3$

B) $g(x) = -4(x + 1)^2 + 3$

C) $k(x) = -x^2 + 2x + 3$

D) $r(x) = x^2 - 2x + 3$

5 The parabola represented below crosses the x-axis at the points (-1, 0) and (3, 0) and its vertex is the point P(1, -4).



Among the following equations, which one represents the parabola graphed above?

A) $y = 2x^2 + 2x - 3$

B) $y = x^2 + 2x - 3$

C) $y = x^2 - 2x - 3$

D) $y = -x^2 - 2x + 3$

6 A mountain bike manufacturer employs 10 workers. The company's monthly production B(x) can be expressed by the following function :

$$B(x) = -x^2 + 60x + 1000$$

where B(x) represents the number of mountain bikes produced in one month and x represents the average number of years of experience of the workers in the company.

What is the maximum number of mountain bikes that this manufacturer can hope to produce in a month?

A) 30 bikes

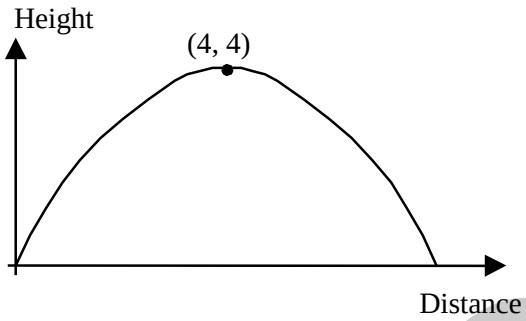
B) 100 bikes

C) 1000 bikes

D) 1900 bikes

7 The parabolic trajectory (path) of a ball thrown from Pat to Chris is illustrated in the Cartesian diagram below.

The distance between Pat, who is standing at the origin, and Chris is 8 m. The maximum height reached by the ball is 4 m.



Which of the following rules of correspondence correctly defines this parabola?

A) $x^2 - 8x - 4y = 0$

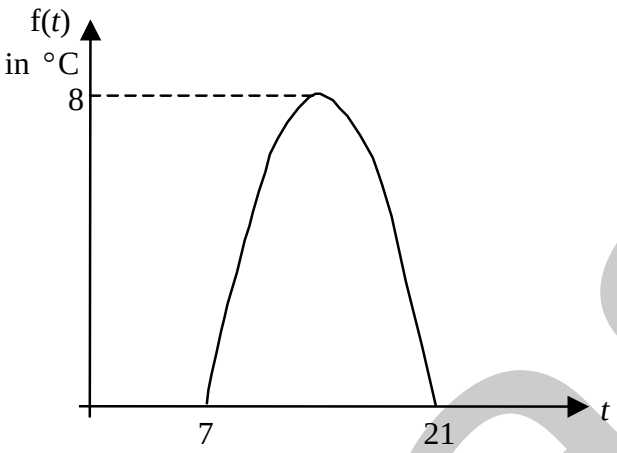
B) $y^2 - 8y - 4x + 32 = 0$

C) $x^2 - 8x + 4y = 0$

D) $y^2 - 8y - 4x = 0$

8 Temperature changes recorded on a given day in April can be represented by a quadratic function.

The function is graphed below.



Which of the following rules of correspondence defines this function?

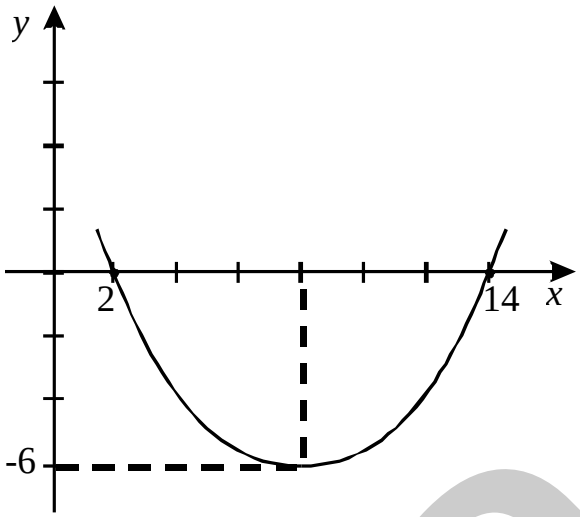
A) $f(t) = -(t-14)^2 + 8$

B) $f(t) = \frac{-8}{49}(t-14)^2 + 8$

C) $f(t) = -(t+14)^2 + 8$

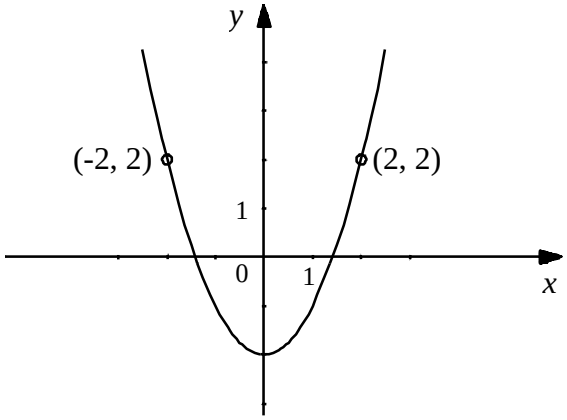
D) $f(t) = \frac{-8}{49}(t+14)^2 + 8$

9 An engineer sketched a parabola in the Cartesian plane.



Which rule of correspondence defines this parabola?

10 What function is represented by the parabola shown below?



A) $x = y^2 - 2$

B) $x = -y^2 + 2$

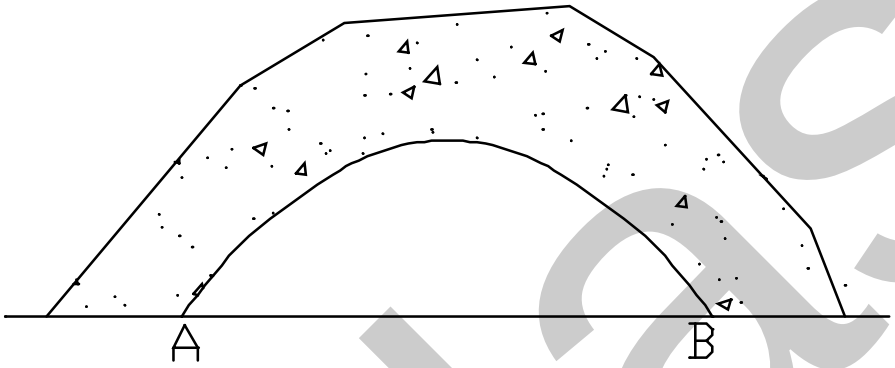
C) $y = x^2 - 2$

D) $y = -x^2 + 2$

11 The cross section of a tunnel has the shape of a parabola defined by this rule of correspondence

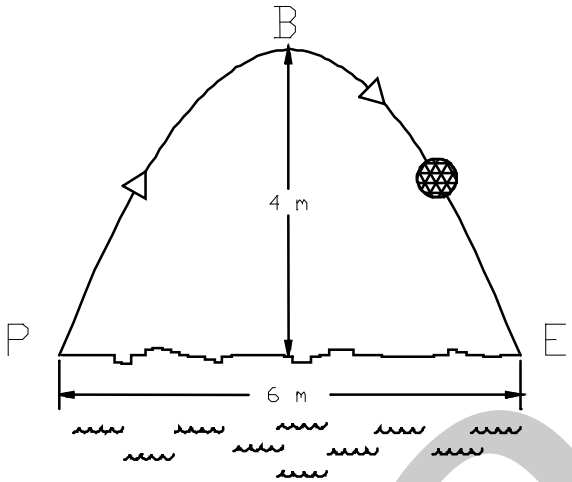
$$x^2 = -\frac{5}{2}(y - 2)$$

where *x* and *y* represent distances in metres.



If segment AB coincides with the *x*-axis, what is the width of the tunnel?

12 Pascal (P) and Elaine (E) are playing with a ball in the swimming pool. The following diagram shows the parabolic trajectory of the ball thrown by Pascal.



What is the rule of correspondence of this trajectory if point P is at the origin of the coordinate system, point B is the maximum height of the ball and point E has coordinates (6, 0)?

13 Students from the school's science club observed that the outdoor temperature recorded at five o'clock between the 1st and the 20th of May was determined by the rule.

$$t(x) = \frac{1}{16}x^2 - x + 3$$

where x is the number of days elapsed since the 1st of May.

What was the minimum outdoor temperature recorded during this period?

A) -2 °C

B) -1 °C

C) 1 °C

D) 8 °C

14 The altitude of a remote-controlled toy airplane is expressed by the following equation :

$$f(t) = \frac{-1}{4}t^2 + 3t + 4$$

where t represents the time of the flight expressed in minutes and $f(t)$ is in metres.

In which of the following intervals of time is the altitude decreasing?

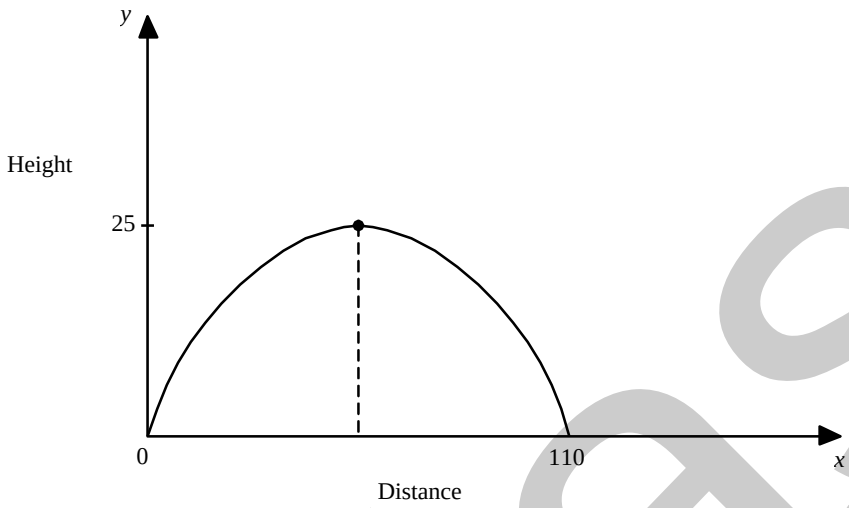
A) [0, 3]

B) [2, 6]

C) [5, 11]

D) [10, 13]

15 The path of a ball hit during a baseball game is approximated by a parabola. This path is shown in the Cartesian plane below.



The ball reaches a maximum height of 25 m and hits the ground 110 m from home plate.

Which rule of correspondence defines the parabolic path travelled by the baseball?

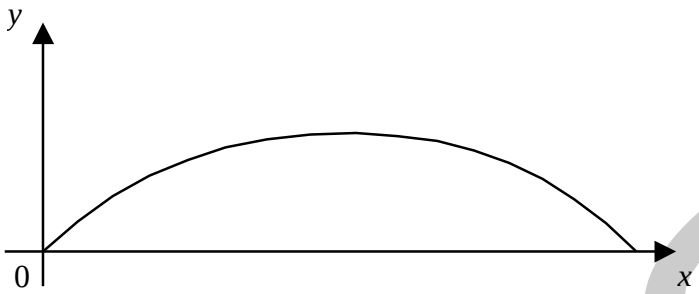
A) $y = -x^2 + 110x$

B) $y = \frac{-x^2}{121} + \frac{110x}{121}$

C) $y = \frac{x^2}{121} - \frac{110x}{121}$

D) $y = x^2 - 110x + 3050$

16 The trajectory of a ball thrown from Marie to Louise is parabolic. This trajectory is illustrated in the Cartesian plane below.



The distance between Marie and Louise is 12 m and the maximum height reached by the ball is 4 m.

What rule of correspondence defines the parabola?

17 The rule of correspondence of a real polynomial function of the second degree is $f(x) = ax^2 + bx + c$.

If $a > 0$ and $b^2 - 4ac < 0$, which graph corresponds to this function?

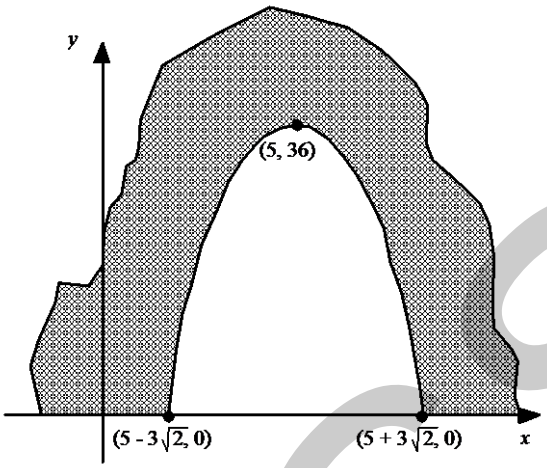
A) A parabola that opens upward and that is tangent to *x*-axis.

B) A parabola that opens upward and that is located above the *x*-axis.

C) A parabola that opens downward and that is located below the *x*-axis.

D) A parabola that opens downward and that is tangent to the *x*-axis.

18 The entrance of a mining shaft has the shape of a parabolic arch as shown to the right.



The zeros of the quadratic function defining the arch are the following numbers :

$5 - 3\sqrt{2}$ and $5 + 3\sqrt{2}$.

The computer of the company in charge of the entire shaft shows that there is a crack at the top of the arch at point (5, 36).

Which of the following rules of correspondence describes the shape of the parabolic arch?

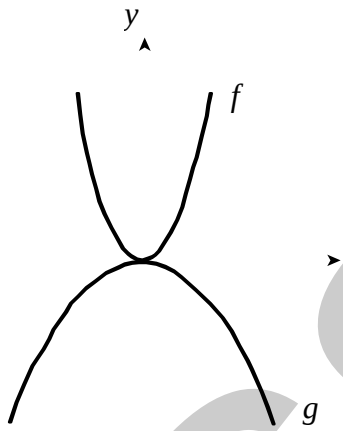
A) $y = -0.5x^2 + 5x - 3.5$

B) $y = -x^2 + 10x - 7$

C) $y = -2x^2 + 20x - 14$

D) $y = \frac{x^2}{2} - 5x + \frac{7}{2}$

19 Functions *f* and *g* are graphed on the right.



The Cartesian coordinate graph of function *g* is obtained by transforming the Cartesian coordinate graph of function *f*. The rule of correspondence of function *f* is $f(x) = x^2$. The rule of correspondence of function *g* is of the form $g(x) = ax^2$.

In which interval does the value of parameter *a* fall?

A) $]-\infty, -1[$

B) $]-1, 0[$

C) $]0, 1[$

D) $]1, +\infty[$

20 The function f is defined by $f(x) = x^2 - 8x - 240$.

Which of the following statements are TRUE?

1. The graph of this function has an axis of symmetry whose equation is $x = 4$.

2. The sum of the zeros of this function is 8.

3. Range of $f = [-224, +\infty[$

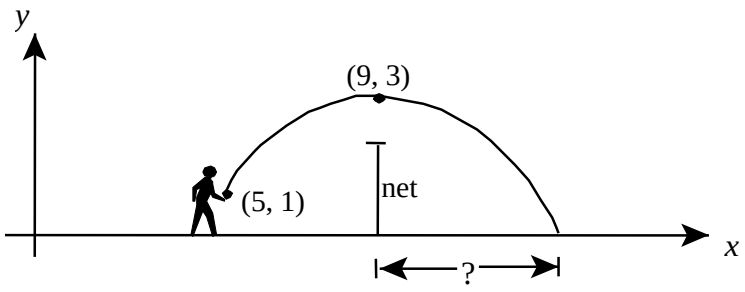
A) 1 and 2 only

B) 1 and 3 only

C) 2 and 3 only

D) 1, 2 and 3

21 The side view of the trajectory of a volleyball is represented in the following Cartesian plane.

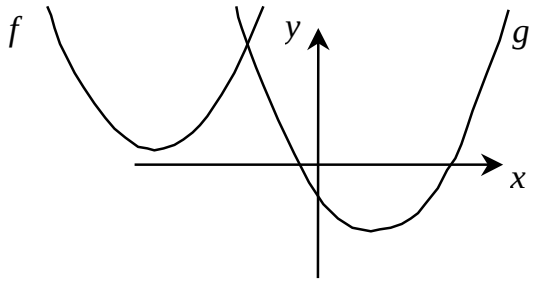


The trajectory is represented by a parabola whose vertex is (9, 3). The scale on the graph is given in metres.

How far from the net will the volleyball hit the ground? Round your answer to the nearest tenth.

Show all your work.

22 Functions *f* and *g* are represented by the parabolas shown below.



The rule of functions *f* and *g* are of the form $y = a(x - h)^2 + k$

The graph of function *g* is obtained by transforming the graph of function *f*.

Which of the following statements is true?

A) The value of h decreases and the value of k decreases.

B) The value of h decreases and the value of k increases.

C) The value of h increases and the value of k decreases.

D) The value of h increases and the value of k increases.

23 Function f is defined by $f(x) = -3(x - 5)^2 + 12$.

Which of the following statements is true?

A) If $x \in \mathfrak{R}$ then function f is positive

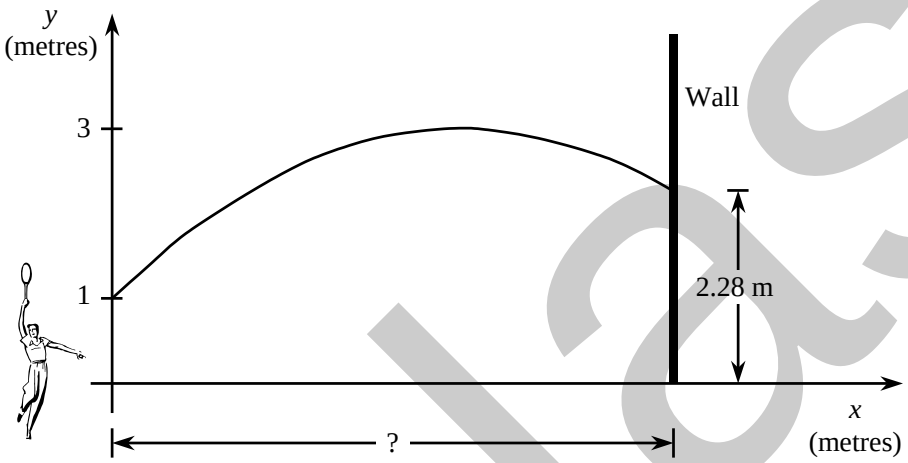
B) If $x \in \mathfrak{R}$ then function f is negative.

C) If $x \in]-\infty, 12]$ then function f is increasing.

D) If $x \in [5, +\infty[$ then function f is decreasing.

24 A tennis player hits a ball against a wall. At the moment the player hits the ball, it is 1 m above the ground. The ball reaches a maximum height of 3 m. On its way down, the ball hits the wall at a point 2.28 m above the ground. The side view of the ball's trajectory is illustrated below.

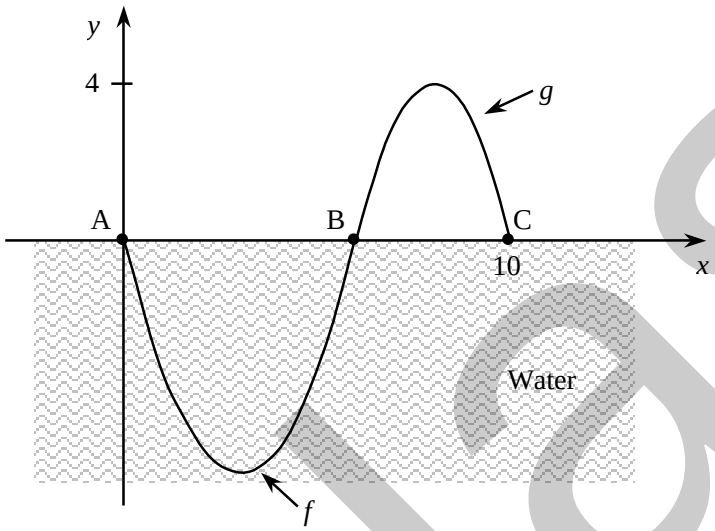
The rule representing this trajectory is $f(x) = -\frac{1}{8}(x-4)^2 + 3$.



At the moment the player hits the ball, what is the distance between the ball and the wall?

25 The following graph represents the side view of the path of a dolphin as it performs a trick during a show at an aquarium. This path is composed of portions of two parabolas associated with function *f* and *g* respectively.

The scale of the graph is in metres.



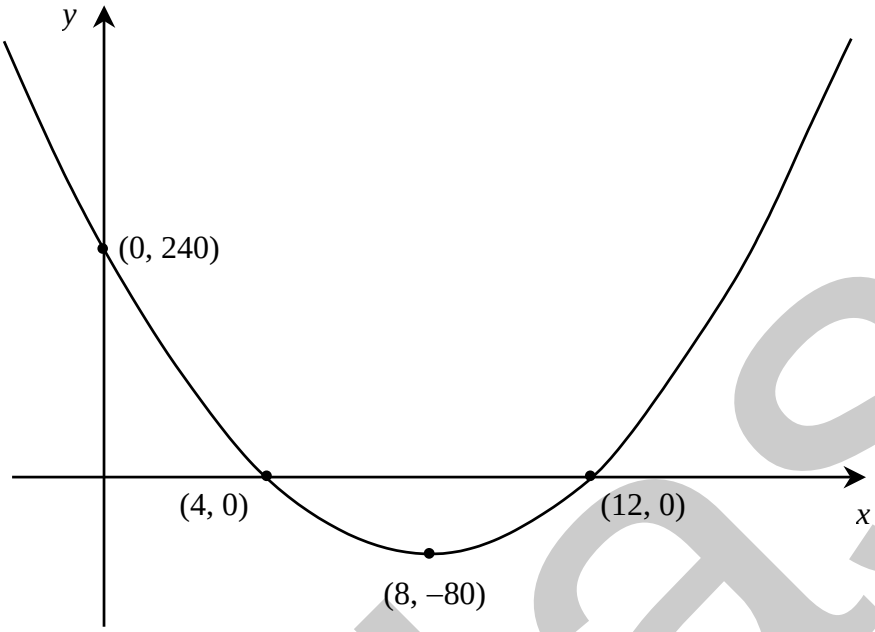
The rule $f(x) = \frac{5}{9}(x-3)^2 - 5$ represents the dolphin's path when it is in the water.

When it is out of the water, the dolphin reaches a maximum height of 4 metres. The distance between points A and C is 10 metres.

What is the rule of the function *g*?

Show all your work.

26 In the Cartesian plane below, function *g* is represented by a parabola. The graph indicates the coordinates of four points of the parabola.



What is the rule of function *g*?