

**Review:** Find the equation in **standard form** of the quadratic function with zeros  $5 + \sqrt{2}$  and  $5 - \sqrt{2}$ , and passes through the point  $(8, -3)$ . Factored form:  $y = a(x - r)(x - s)$ .

$$r = (5 + \sqrt{2}) \quad \text{and} \quad s = (5 - \sqrt{2})$$

$(8, -3)$   
x ↙      ↘ y

$$y = a(x - r)(x - s)$$

$$-3 = a[8 - (5 + \sqrt{2})][8 - (5 - \sqrt{2})]$$

$$-3 = a(8 - 5 - \sqrt{2})(8 - 5 + \sqrt{2})$$

$$-3 = a(3 - \sqrt{2})(3 + \sqrt{2})$$

$$-3 = a(9 + 3\sqrt{2} - 3\sqrt{2} - 2)$$

$$-3 = a(7)$$

$$\boxed{-3/7 = a}$$

$$y = \frac{-3}{7} (x - 5 - \sqrt{2})(x - 5 + \sqrt{2})$$

$$= \frac{-3}{7} (x^2 - 5x + x\sqrt{2} - 5x + 25 - 5\sqrt{2} - x\sqrt{2} + 5\sqrt{2} - 2)$$

$$\boxed{y = \frac{-3}{7} (x^2 - 10x + 23)}$$

### LESSON: SOLVING QUADRATIC EQUATIONS

**Solve** means to find numerical value(s) for "x".

### METHODS

There are 3 widely used methods for solving quadratic equations. Quadratic equations generally have 2 solutions (roots, x-intercepts).

#### 1) ISOLATION/SQUARE ROOT PROPERTY

**Solve** for x:  $x^2 - 16 = 0$

1	Isolate the term that contains the squared variable.	$x^2 = 16$
2.	Take the square root of both sides and solve for the variable.	$\sqrt{x^2} = \sqrt{16}$
3.	Remember the possibility of two roots for every square root, one positive and one negative. Place a $\pm$ sign in front of the side containing the constant before you take the square root of that side.	$x = \pm 4$

Solve the following function using the method of isolation:

$$(2m + 4)^2 - 1 = 7$$

$$\sqrt{(2m + 4)^2} = \sqrt{8}$$

$$(2m + 4) = \sqrt{2 \cdot 2 \cdot 2}$$

$$2m + 4 = \pm 2\sqrt{2}$$

$$2m + 4 = 2\sqrt{2} - 4$$

$$\frac{2m}{2} = \frac{2\sqrt{2} - 4}{2}$$

$$m = \sqrt{2} - 2$$

or

$$2m + 4 = -2\sqrt{2} - 4$$

$$\frac{2m}{2} = \frac{-2\sqrt{2} - 4}{2}$$

$$m = -\sqrt{2} - 2$$

$\therefore$  The solutions are  $\sqrt{2} - 2$  or  $-\sqrt{2} - 2$

Method 2: **FACTORIZING** Solve for x:

$$x^2 - x = 6$$

1.	Move all terms to the same side of the equal sign, so the equation is set equal to 0.	$x^2 - x - 6 = 0$ This places the equation in <b>standard form</b> .
2.	Factor the algebraic expression.	$(x - 3)(x + 2) = 0$ $(x + 3)$ and $(x + 2)$ are called <b>factors</b> . These are factors of the <b>expression</b> $x^2 - x - 6$ .
3.	Set each factor equal to 0. (This process is called the "zero product property". If the product of two factors equals 0, then either one or both of the factors must be 0.)	$x - 3 = 0$ ; $x + 2 = 0$
4.	Solve each resulting equation.	$x = 3$ ; $x = -2$ $x = 3$ and $x = -2$ are called <b>roots</b> . These are roots of the <b>equation</b> $x^2 - x - 6 = 0$ .

Try:  $2x(x-1) = -x+15$

$$2x^2 - 2x = -x + 15$$

$$2x^2 - 2x + x - 15 = 0$$

$$2x^2 - x - 15 = 0$$

$$\frac{(2x-6)(2x+5)}{2} = 0$$

$$\frac{2(x-3)(2x+5)}{2} = 0$$

$$(x-3)(2x+5) = 0$$

M	A	N
-30	-1	5, -6

$$x - 3 = 0 \quad \text{OR} \quad 2x + 5 = 0$$

$$x = 3 \qquad \qquad \qquad 2x = -5$$

$$\qquad \qquad \qquad \qquad \qquad x = -5/2$$

∴ The solutions are -2.5 or 3.

3) **QUADRATIC FORMULA**

Solve the following using quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Determine the x - intercepts of the function  $f(x) = 5x^2 + 2x - 1$  using the **quadratic formula**. Leave answers in exact reduced radical form (not decimal form).

$$5x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(5)(-1)}}{10}$$

$$x = \frac{-2 \pm \sqrt{24}}{10}$$

$$= \frac{-2 \pm \sqrt{6 \cdot 4}}{10}$$

$$= \frac{-2 \pm 2\sqrt{6}}{10}$$

$$x_1 = \frac{-2 + 2\sqrt{6}}{10}$$

$$= \frac{2(1 + \sqrt{6})}{5 \cancel{10}}$$

$$= \frac{-1 + \sqrt{6}}{5}$$

$$x_2 = \frac{-2 - 2\sqrt{6}}{10}$$

$$= \frac{2(-1 - \sqrt{6})}{10}$$

$$= \frac{-1 - \sqrt{6}}{5}$$

∴ The solutions are  $\frac{-1 + \sqrt{6}}{5}$  or  $\frac{-1 - \sqrt{6}}{5}$ .

**PRACTICE**

1. The town decides to build a rectangular fence around a playground. The playground without the fence measures 60 m by 40 m; however after the building of the fence, the area gets **doubled**. The designers put the fence around the playground with a uniform distance. Calculate the distance between the playground and the fence.

$$\text{Area}_{\text{new}} = (2x+40)(2x+60) \quad \text{Area}_{\text{old}} = 40 \times 60 = 2400$$

$$4800 = 2(x+20)(2)(x+30)$$

$$\frac{4800}{4} = \frac{4(x+20)(x+30)}{4}$$

$$1200 = (x+20)(x+30) \quad \text{FOIL}$$

$$1200 = x^2 + 30x + 20x + 600$$

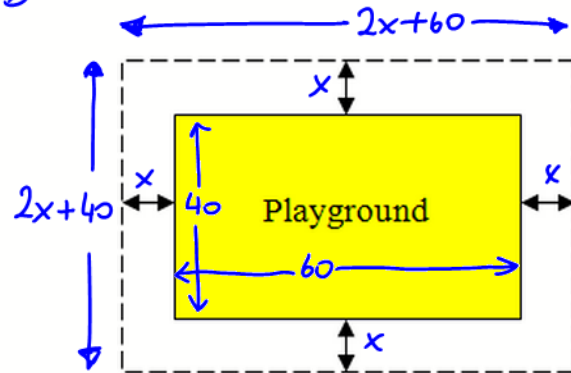
$$0 = x^2 + 50x - 600$$

M	A	N
-600	50	60, -10

$$0 = (x+60)(x-10)$$

$$x = -60 \quad x = 10$$

∴ The distance between the fence and playground is 10 m.



2. A factory is to be built on lot that measures 90 m by 70 m. A lawn of uniform width and with an area of 3900 m<sup>2</sup> must surround the factory. What dimensions must the factory have?

$$(90-2x)(70-2x) = 90 \times 70 - 3900$$

$$2(45-x)(2)(35-x) = 6300 - 3900$$

$$4(45-x)(35-x) = 2400$$

$$(45-x)(35-x) = 600$$

$$1575 - 45x - 35x + x^2 = 600$$

$$x^2 - 80x + 975 = 0$$

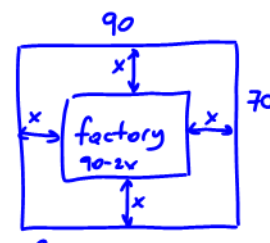
$$x = \frac{-(-80) \pm \sqrt{(-80)^2 - 4(1)(975)}}{2(1)}$$

$$= \frac{80 \pm \sqrt{2500}}{2}$$

$$= \frac{80 \pm 50}{2}$$

$$x_1 = \frac{80+50}{2} = 65$$

$$x_2 = \frac{80-50}{2} = 15$$



∴ 65m would not make sense in this context; therefore, the lawn's width is 15m. The factory's dimensions are 60 and 40m.

2. Determine the x-intercepts of the function  $f(x) = 5x^2 + 2x - 1$  using the quadratic equation. Leave answers in exact reduced radical form (not decimal form).