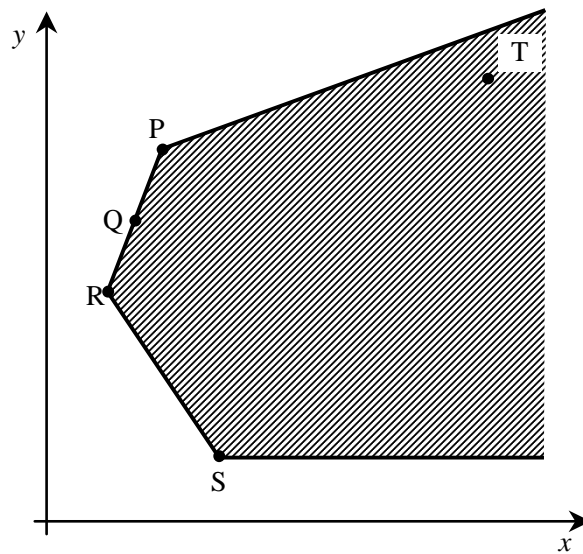


Name: _____

Math 5CST - Midyear Review

- 1 The following polygon of constraints represents the solutions to the constraints associated with an optimization problem. The coordinates of points P, Q, R, S and T are whole numbers.



Given the above, which of the following statements is true?

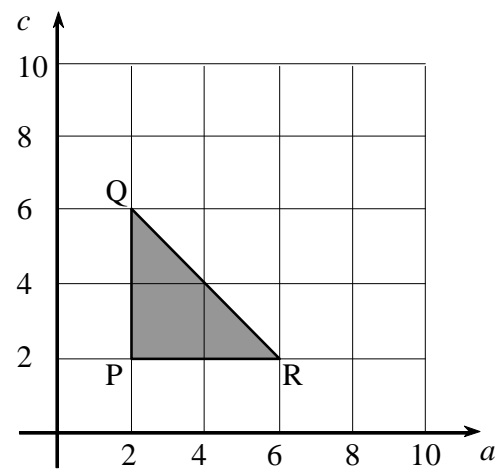
- A) Point T could represent an optimal solution for this optimization problem.
- B) Vertex S necessarily represents the solution that minimizes the function to be optimized.
- C) This optimization problem has at least two optimal solutions: one minimizes the function to be optimized and the other maximizes it.
- D) If vertices P and R represent optimal solutions, then point Q also represents an optimal solution for this optimization problem.

2

Allan drives a minibus. He transports passengers to and from a festival. The following polygon represents the constraints he must respect. In the graph, a represents the number of adults in the minibus and c , the number of children.

Which of the following constraints corresponds with one of the sides of the polygon?

- A) He must have at least three times as many adults as children.
- B) He must have at least three times as many children as adults.
- C) The minibus can transport a maximum of 6 passengers.
- D) The minibus can transport a maximum of 8 passengers.



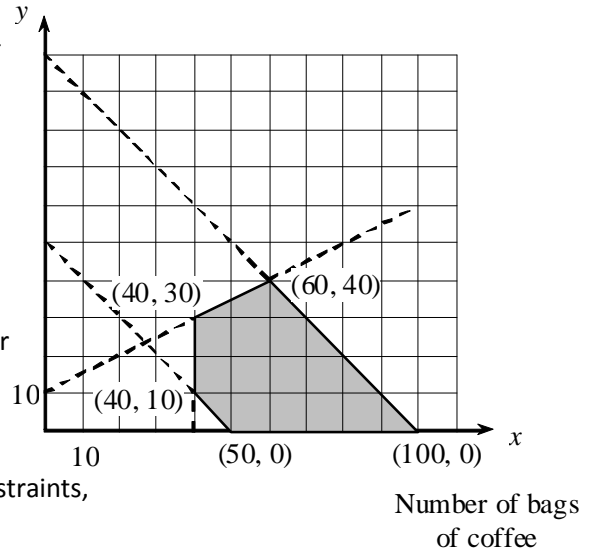
3

Michael sells boxes of chocolates and bags of coffee to raise money for his graduation activities.

The constraints of his fundraising are represented in the graph below:

Michael should make a profit of \$1 for each bag of coffee sold, and \$3 for each box of chocolate, such that the function to be optimized is $P = x + 3y$.

Number of boxes of chocolate



The amount of coffee and chocolate Michael sells respects all of the constraints, while maximizing his earnings.

Unfortunately, Michael makes a profit of \$1 for each bag of coffee sold and only \$2 for each box of chocolate, thus decreasing his total earnings.

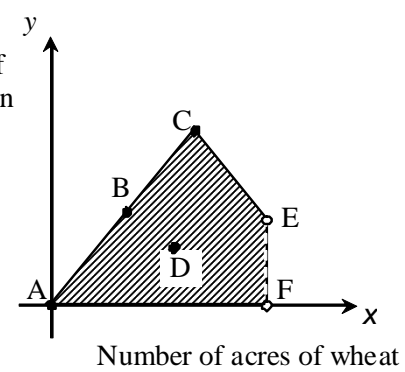
By how much did Michael's total earnings decrease because of this change in profit?

Michael's total earnings decreased by \$ _____ because of this change.

4 Mrs. Thomas owns 40 acres of farmland. She produces less than 30 acres of wheat and a maximum of 20 acres of corn. She grows at least as much wheat as corn. The wheat is sold at a profit of \$40 per acre, while the corn is sold at a profit of \$20 per acre.

x : number of acres of wheat
 y : number of acres of corn

The polygon of constraints is represented by a quadrilateral, shown on the right.



- a) What is the rule of the objective function?
- b) Which points, represented by letters, are in the solution set?

a) Rule: _____
 b) Points: _____

5 Mont Tremblant offers gondola rides to the top of the mountain. Children's tickets cost \$6.50 each and adult tickets cost \$10 each. On any given ride there will be at least 10 children and 5 adults. The gondola has room for a maximum of 45 passengers and can hold up to 1680 kg. The average mass of a child is 28 kg and that of an adult is 70 kg.

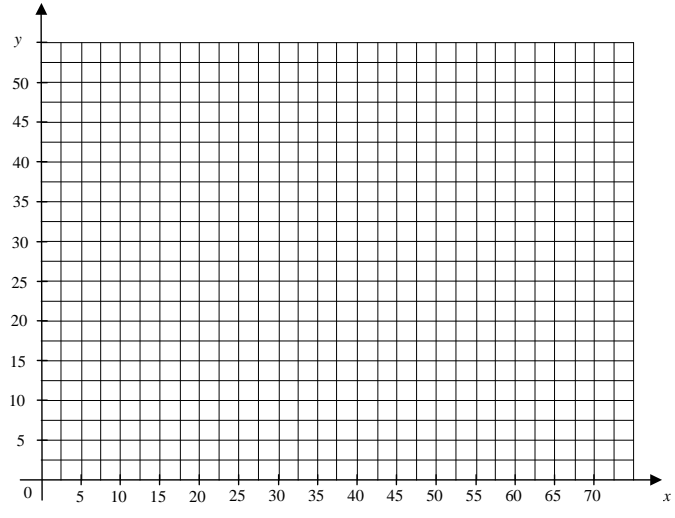
x : number of children tickets sold
 y : number of adult tickets sold

Given:

$$x \geq 10 \quad x + y \leq 45$$

$$y \geq 5 \quad 28x + 70y \leq 1680$$

What is the maximum revenue the gondola service can make in one trip?



6

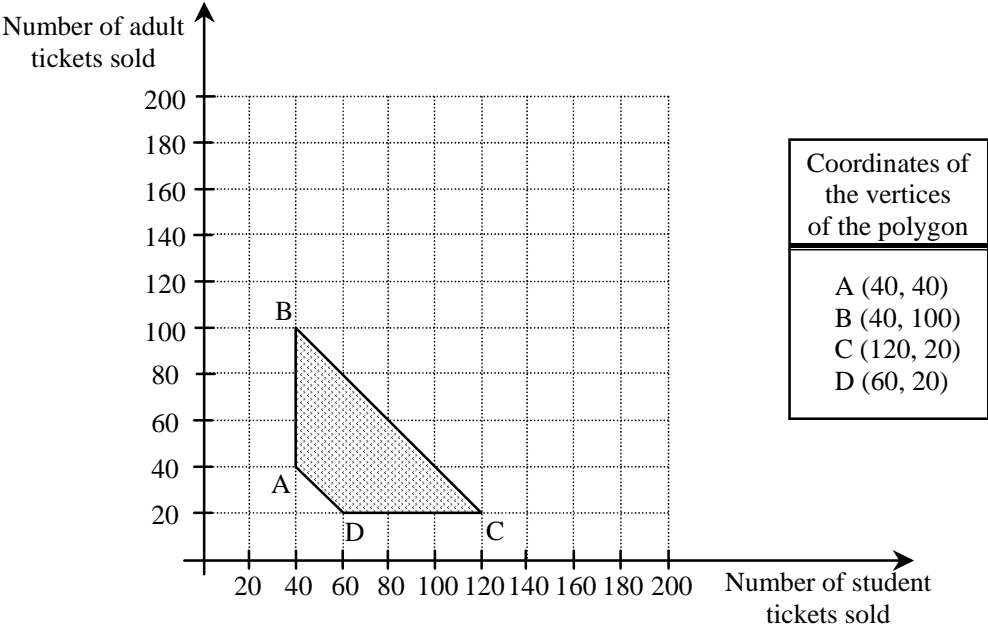
The student council in a school is organizing a fashion show. Ticket sales for this event will make it possible to fund student activities during the school year. The student council faces different constraints. These constraints and their related inequalities are given in the table below.

CONSTRAINTS	RELATED INEQUALITIES
We must sell at least 40 student tickets.	$x \geq 40$
We must sell at least 20 adult tickets.	$y \geq 20$
We must sell a minimum of 80 tickets.	$x + y \geq 80$
We can sell no more than 140 tickets because the school auditorium can hold a maximum of 140 people.	$x + y \leq 140$

x is the number of student tickets sold

y is the number of adult tickets sold

The polygon of constraints for this situation is given below.



Because of the demand for tickets to the fashion show, the student council has decided to hold the event in the school gymnasium rather than in the auditorium. Since the gymnasium can seat more people than the auditorium, the maximum number of tickets that can be sold will rise from 140 to 180.

The student council will earn a revenue of \$4 per student ticket sold and a revenue of \$8 per adult ticket sold.

By how much will the maximum possible revenue increase because the event is being held in the gymnasium?

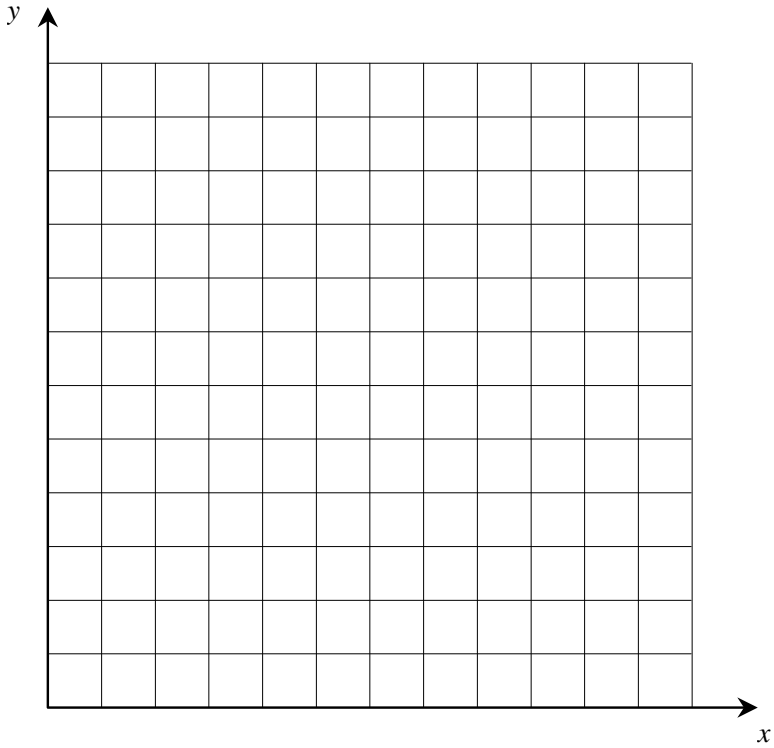
7

James has an aquarium large enough to hold 10 fish, maximum. He wants red fish and blue fish. At the pet store, James realizes that triple the number of blue fish plus the number of red fish is at most 18. He learns his aquarium must contain at least two blue fish.

Each red fish costs \$1 and each blue fish, \$2.

At the most, how much will James have to pay for his fish?

Let x : number of red fish
 y : number of blue fish



Answer: James will have to pay, at most, _____ to buy his fish.

8 A company makes two types of lawnmowers: gas and electric powered. The following table gives the necessary time, in hours, for assembly and inspection for one of each type of lawnmower.

Work	Gas	Electric
Assembly	2	5
Inspection	1	1

The employees working on assembly can work a maximum of 200 hours per week, and those working on inspection can work a maximum of 70 hours per week. There must be at least 10 electric mowers produced weekly. This situation can be represented by the following system of inequalities:

Let x : number of gas mowers
 y : number of electric mowers

$$x \geq 0, y \geq 0$$

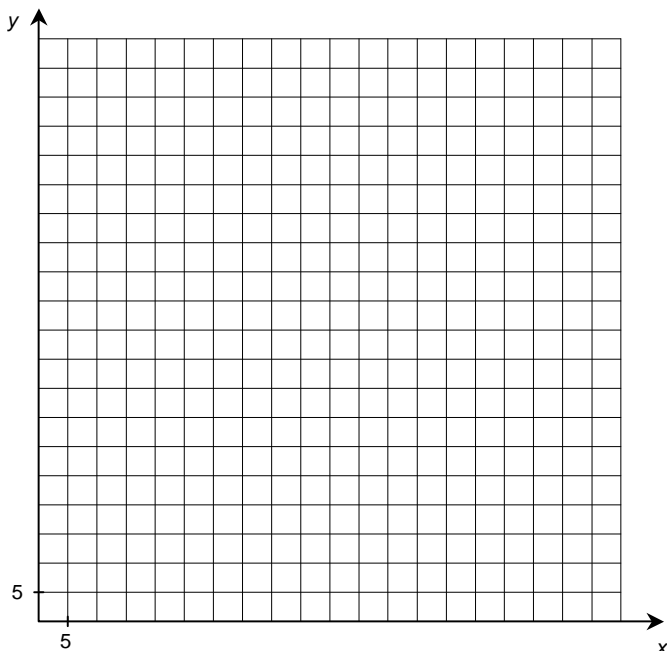
$$2x + 5y \leq 200$$

$$x + y \leq 70$$

$$y \geq 10$$

Gas powered and electric powered lawnmowers are sold for \$400 and \$420 respectively.

How many lawnmowers of each type must this company sell per week to maximize its revenue?



9

An activity day for Secondary 4 and 5 students is scheduled for late spring.

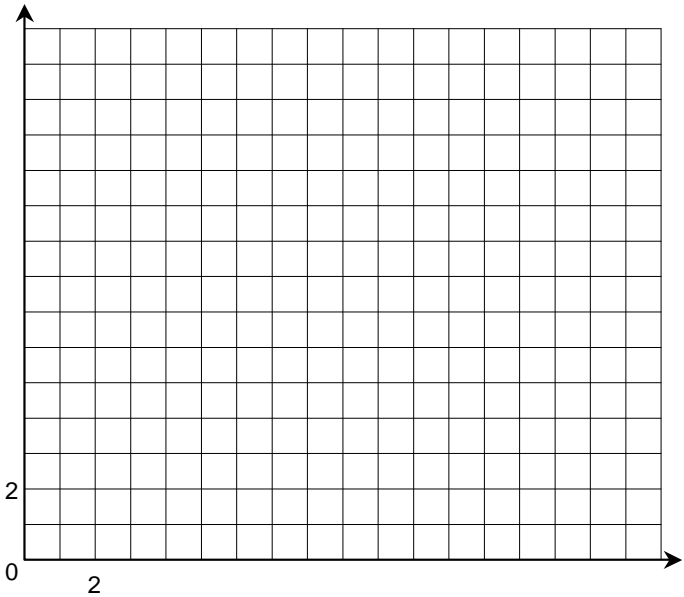
The activity director must reserve two types of buses for transportation. The first type costs \$200 a day and can carry 48 passengers, while the second costs \$140 a day and can carry 32 passengers.

The director must reserve a minimum of two 48-passenger buses and two 32-passenger buses in order to get the buses at this price.

Only 6 of the 48-passenger buses are available. At least 384 students, but at most 480 students, will need transportation.

How many buses of each type must the director order to keep costs to a minimum?

x: number of 48-passenger buses
y: number of 32-passenger buses



Answer: The activity director should order _____ 48-passenger buses and _____ 32-passenger buses.

10

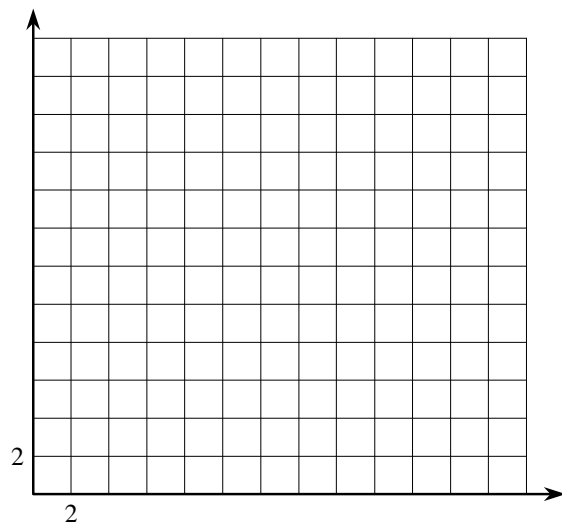
A restaurant manager must determine how many students are needed to be hired this summer to work the lunchtime and the dinnertime shifts.

There must be at least 18 employees working during the two shifts. However, the restaurant's operating budget requires that the number of employees working during the two shifts cannot be more than 22.

The lunchtime rush requires a minimum of 6 employees. To handle the dinnertime rush, the number of employees must be at least two more than the number who work the lunchtime shift.

Employees who work the lunchtime shift earn \$8 an hour, while employees working the dinnertime shift earn \$10 an hour.

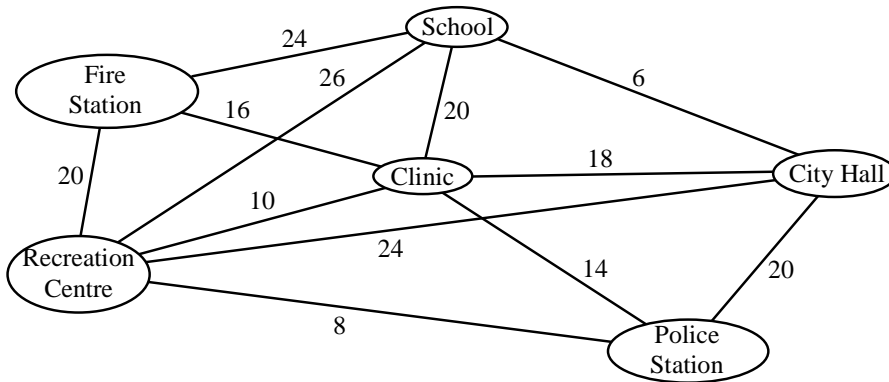
How many employees should the manager hire for each shift in order to satisfy the constraints and minimize the cost of employee salaries?



Answer: To minimize costs, the manager must hire _____ employees for the lunchtime shift and _____ employees for the dinnertime shift.

11

A piping system must be installed to supply water to public buildings in a city. Each edge in the following graph represents a possible section of this piping system. The number of each edge indicates the installation cost in thousands of dollars for that section of the system.



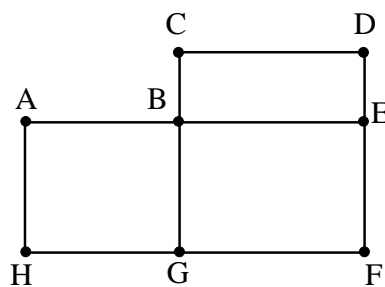
What is the minimum cost of installing this system?

- A) \$54 000
- B) \$58 000
- C) \$72 000
- D) \$88 000

12

The edges of the graph on the right represent the streets in a residential area.

While on patrol, a police officer takes different routes.



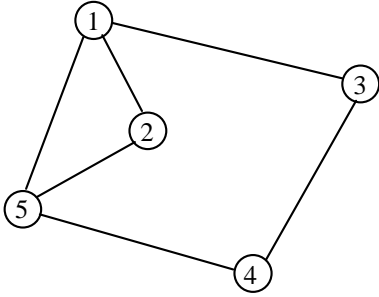
Which of the following routes describes a circuit?

- A) D, E, F, G, D
- B) B, G, F, E, D, C
- C) B, A, H, G, F, E, B
- D) F, E, B, A, H, G, F, E

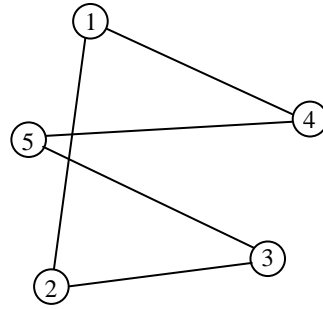
13

Which of the following graphs represents an Euler circuit?

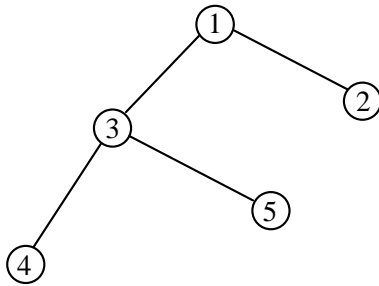
A)



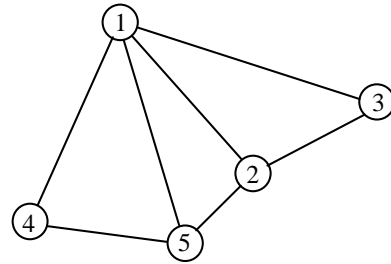
C)



B)



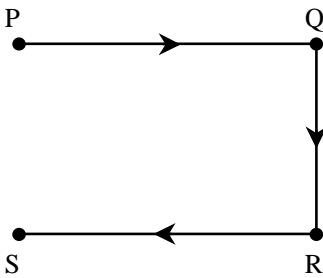
D)



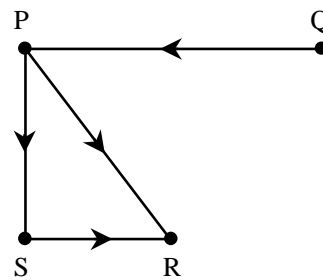
14

Which one of the following graphs contains a circuit?

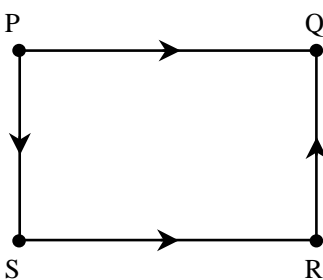
A)



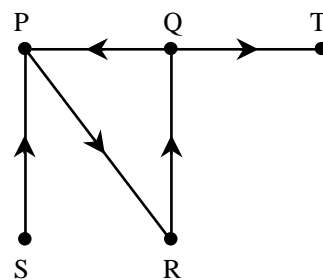
C)



B)

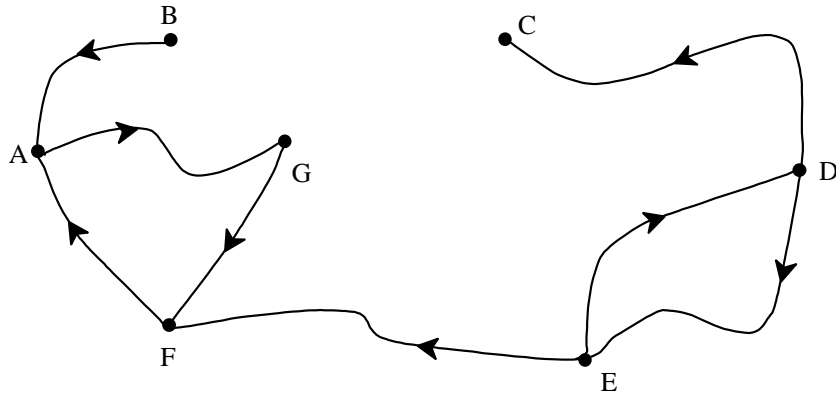


D)



15

Consider the graph below.

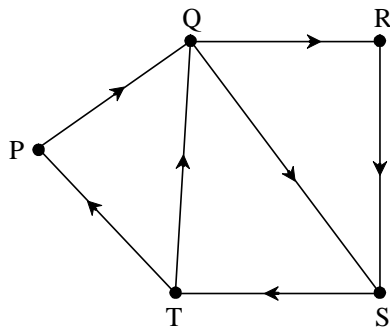


Which of the following statements is true?

- A) There is a path that begins at vertex A and ends at vertex D.
- B) There is a path that begins at vertex B and ends at vertex F.
- C) There is a path that begins at vertex E and ends at vertex B.
- D) There is a path that begins at vertex G and ends at vertex C.

16

Consider the graph below.

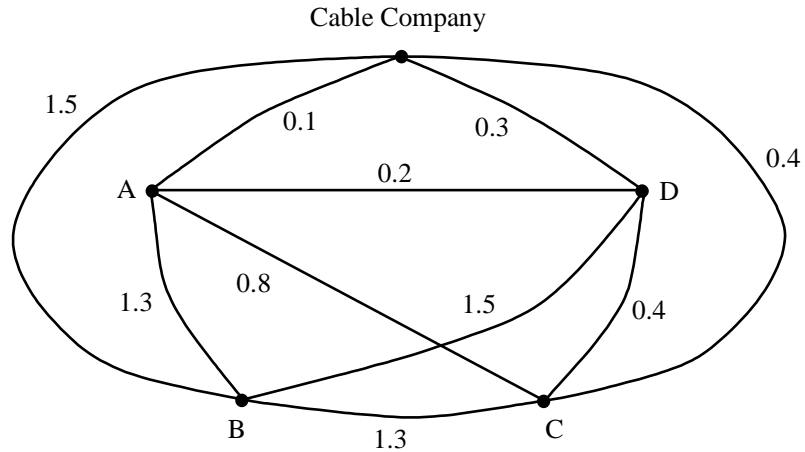


Which one of the following statements is true?

- A) P, Q, R, S, T represents a circuit that passes through all the vertices of the graph.
- B) P, Q, R, S, Q, T, P represents a circuit that passes through all the vertices of the graph.
- C) T, P, Q, S, T represents a circuit that passes through all the vertices of the graph.
- D) T, P, Q, R, S, T represents a circuit that passes through all the vertices of the graph.

17

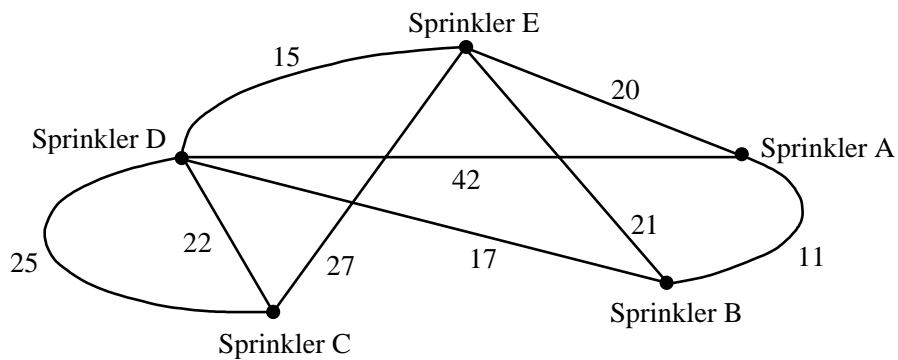
A cable TV company wants to connect cities A, B, C and D to its distribution network. In the following graph, the number on each edge indicates the installation cost (in millions of dollars) for each possible section of this network.



What is the minimum cost of connecting cities A, B, C and D to this network?

18

Steve wants to install a sprinkler system on his property. The vertices of the following graph represent the required sprinklers. Each edge represents a possible water pipe. The number on each edge indicates the length of the pipe in metres.



Steve wants to connect all the sprinklers so as to minimize the total combined length of the pipes.

What is the shortest length of piping required to connect all the sprinklers?

The production of a high school yearbook involves many different tasks. Some can be carried out at the same time, while others cannot be performed until one or more preceding tasks have been completed.

The following table lists these tasks, the time required to complete each, and the task(s) that must be carried out before performing each task.

Tasks		Time Required (weeks)	Preceding Task
A	Writing the text	1	none
B	Typing the text on the computer	2	A
C	Taking photographs	3	none
D	Editing the text	2	B
E	Typesetting the text and photographs	2	C and D
F	Selling advertising space	8	none
G	Typesetting the advertisements	2	F
H	Designing the cover page	1	none
I	Assembling the pages of the yearbook	1	E, G and H
J	Printing the yearbook	6	I

The yearbook production committee has decided that the time allotted to selling advertising space will be reduced to 4 weeks.

How will this reduction affect the total time required to produce the yearbook?

20

For a horseback riding competition, seven fences were set up on a course divided into five areas. The following table indicates where the fences are located. For example, the check mark in the upper left-hand corner of the table indicates that a fence is located between areas 1 and 2.

	Area 2	Area 3	Area 4	Area 5	
Area 1	✓		✓		
Area 2		✓	✓		
Area 3			✓	✓	
Area 4				✓	

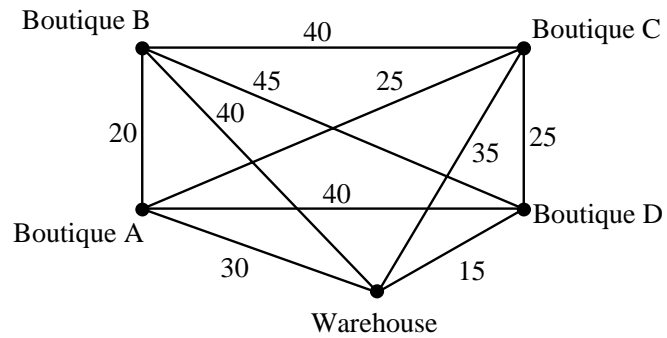
Competitors must jump over each of the seven fences only once.

What route can a competitor take in this case?

21

Louise delivers merchandise for a chain of boutiques.

The edges of the following graph represent the different routes Louise can take. The number on each edge indicates the time in minutes needed to get from one place to another.



In choosing her route, Louise must take the following constraints into account:

- Her route must begin and end at the warehouse.
- She is required to visit each of the 4 boutiques only once.
- She must go to Boutiques B and C before going to Boutique A.
- She must go to Boutique C before going to Boutique D.
- She wants to minimize the time it takes to make all her deliveries.

Which route should Louise take?

22

Patricia is planning a trip during which she will visit cities A, B, C, D, E and F. She will drive from one city to the other. The following table shows the distance she must travel to get from one city to another.

Travel between cities	Distance (km)
A and D	30
A and E	65
A and F	25
B and C	90
B and D	40
C and D	45
C and E	50
E and F	60

When preparing her itinerary, Patricia takes certain constraints into account. She wants to:

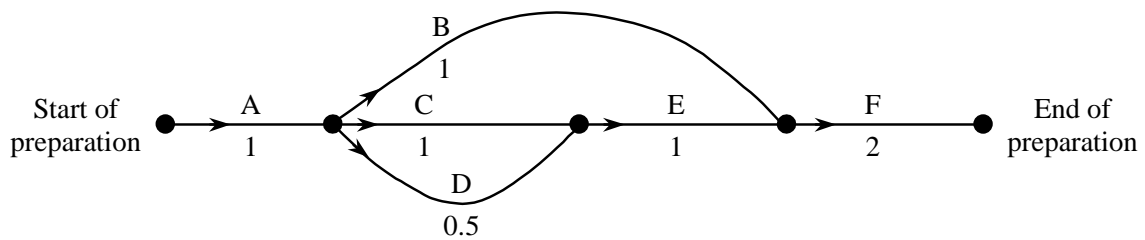
- visit cities A, B, C, D, E and F
- minimize the total distance travelled
- start her trip in city E
- visit city B before city A
- finish her trip in city E

What should be Patricia's itinerary?

23

Bernard is in charge of organizing a group of adults who are preparing an elementary school gym for a students' party.

The graph below represents the different tasks involved in preparing the gym. The value on each edge indicates the number of hours needed to finish the corresponding task. The direction of the arrows indicates the order in which the tasks must be completed. Some tasks may be carried out at the same time.



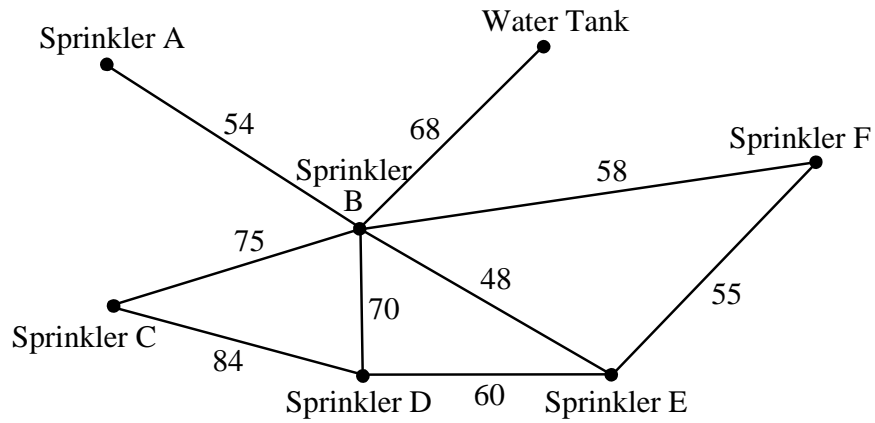
If they complete the tasks in the minimum required time, Bernard figures that they need to start preparing the gym at 1 p.m. to finish just in time for the party.

Bernard reviews the situation and realizes that task B will take 3 hours to complete instead of 1 hour. This means that he will have to change the time at which they start preparing the gym so that everything will be ready just in time for the party.

In view of this new situation, what time, at the latest, must they start preparing the gym?

24

A sprinkler system is installed in a park. The edges of the following graph represent the different water pipes that could be installed to connect the sprinklers and the water tank. The number on each edge indicates the length of the corresponding pipe in metres.



The shortest length of piping required to connect the sprinklers and the water tank costs \$1728.

How much does one metre of piping cost?

Answers

1 D

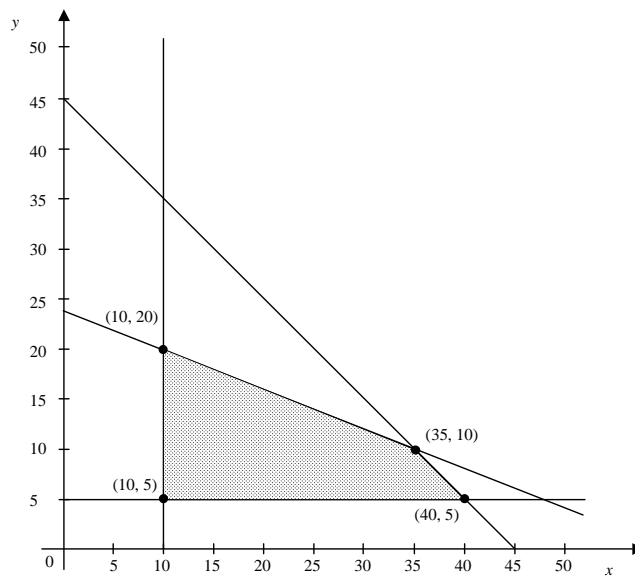
2 D

3 Michael's total earnings decreased by \$ **40** because of this change.

4 a) Rule: $P = 40x + 20y$

b) Points: A, B, C, D

5 Example of an appropriate method



2 marks for graph drawn correctly

Ext.	Profit = $6.5x + 10y$
(10, 5)	\$115.00
(10, 20)	\$265.00
(40, 5)	\$310.00
(35, 10)	\$327.50 MAX PROFIT

Answer: The maximum revenue the gondola service can make is **\$327.50**.

6

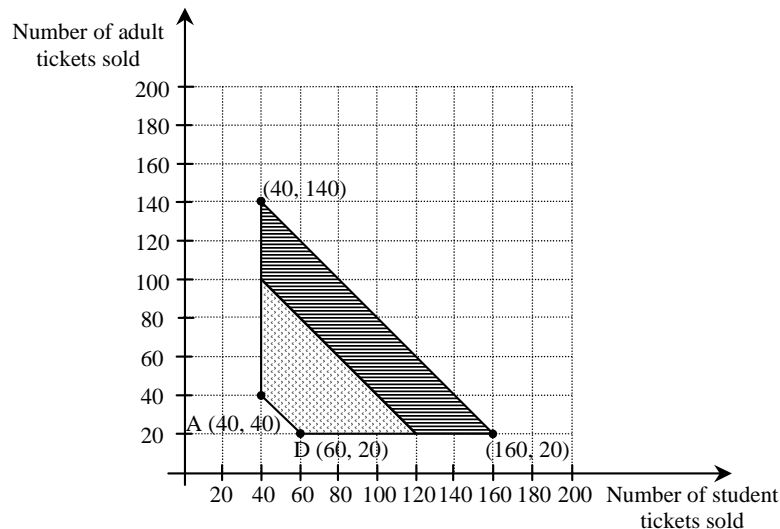
Maximum possible revenue before one of the constraints was changed

Vertex	Revenue: $4x + 8y$
A(40, 40)	$4(40) + 8(40) = \$480$
B(40, 100)	$4(40) + 8(100) = \$960$
C(120, 20)	$4(120) + 8(20) = \$640$
D(60, 20)	$4(60) + 8(20) = \$400$

The maximum possible revenue before one of the constraints was changed was \$960.

New constraint The constraint $x + y \leq 140$ is replaced by the new constraint $x + y \leq 180$.

Vertices of the new polygon of constraints



Maximum possible revenue after one of the constraints was changed

Vertex	Revenue: $4x + 8y$
A(40, 40)	$4(40) + 8(40) = \$480$
B(40, 140)	$4(40) + 8(140) = \$1\ 280$
C(160, 20)	$4(160) + 8(20) = \$800$
D(60, 20)	$4(60) + 8(20) = \$400$

The maximum possible revenue after one of the constraints was changed was \$1 280.

Increase in the maximum possible revenue

$$\$1\ 280 - \$960 = \$320$$

Answer: The maximum possible revenue will increase by **\$320** because the event is being held in the gymnasium.

7

Let x : number of red fish
 y : number of blue fish

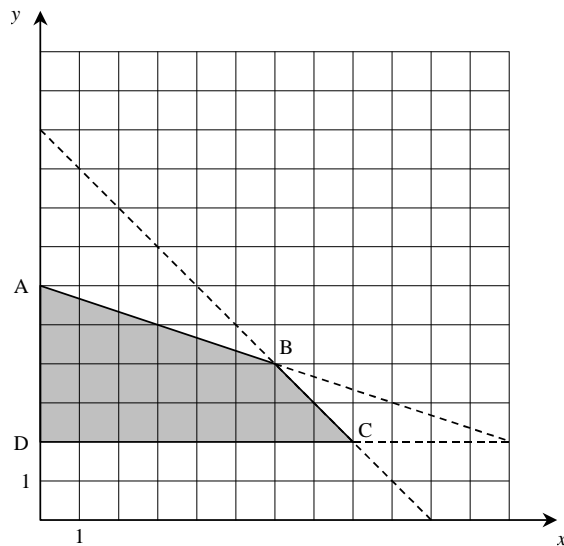
System of Inequalities

$$x \geq 0 \quad y \geq 2 \quad x + y \leq 10 \quad x + 3y \leq 18$$

Function to be optimized

$$M = x + 2y$$

Polygon of Constraints



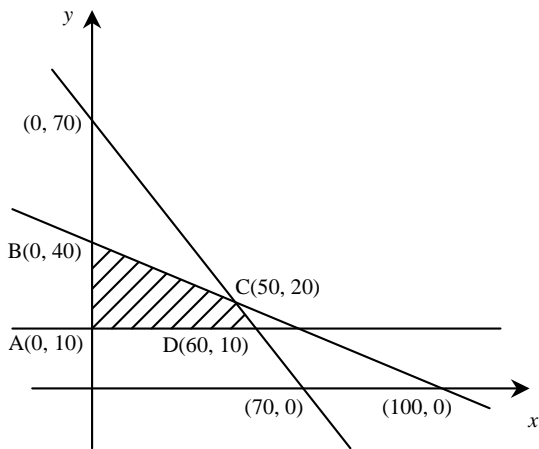
Optimization

Vertex	$M = x + 2y$
A(0, 6)	12
B(6, 4)	14
C(8, 2)	12
D(0, 2)	4

← maximum

Answer: James will have to pay, at most, **\$14** to buy his fish.

8



Vertex	Weekly Revenue = $400x + 420y$
A(0, 10)	\$4200
B(0, 40)	\$16 800
C(50, 20)	\$28 400
D(60, 10)	\$28 200

Answer: The company must sell of **50** gas and **20** electric mowers to maximize its revenue.

9

x: number of 48-passenger buses
y: number of 32-passenger buses

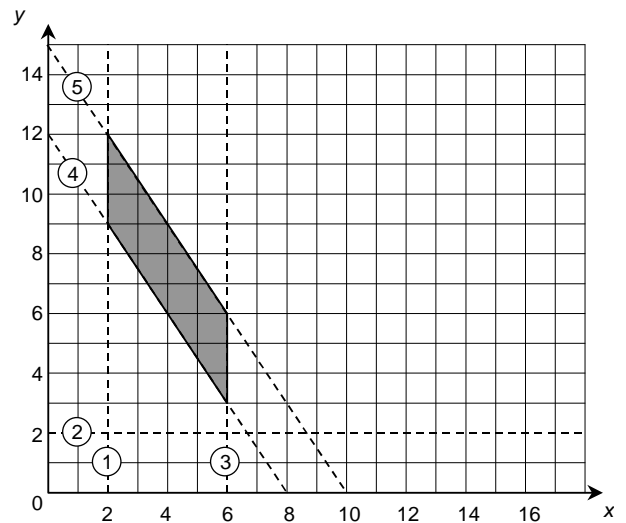
Constraints

1. $x \geq 2$
2. $y \geq 2$
3. $x \leq 6$
4. $48x + 32y \geq 384$
5. $48x + 32y \leq 480$

Objective Function

Minimize $C = 200x + 140y$

Corner Points	Objective Function	Cost
(2, 12)	$200(2) + 140(12)$	\$2080
(2, 9)	$200(2) + 140(9)$	\$1660
(6, 3)	$200(6) + 140(3)$	\$1620
(6, 6)	$200(6) + 140(6)$	\$2040



Answer: The activity director should order **six** 48-passenger buses and **three** 32-passenger buses.

10

x : number of employees working the lunchtime shift

y : number of employees working the dinnertime shift

System of constraints

$$\begin{aligned} x &\geq 6 \\ y &\geq 0 \\ x + y &\geq 18 \\ x + y &\leq 22 \\ y &\geq x + 2 \end{aligned}$$

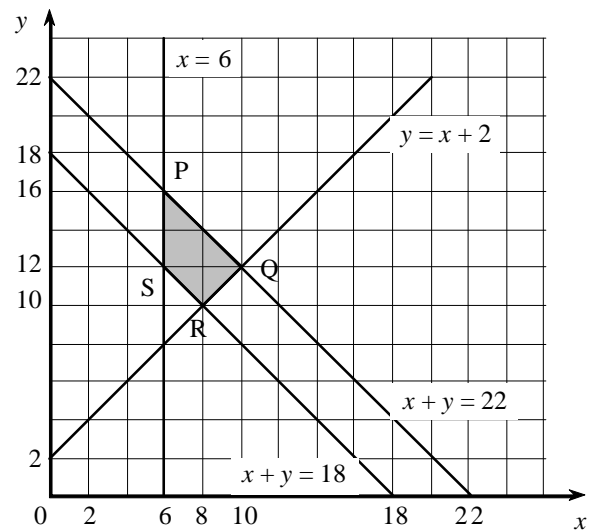
Objective rule

$$C = 8x + 10y$$

Optimum solution

Point	$8x + 10y$	Cost
(6, 16)	$48 + 160$	208
(10, 12)	$80 + 120$	200
(8, 10)	$64 + 100$	164 ← min.
(6, 12)	$48 + 120$	168

Minimum cost = \$164



Answer: To minimize costs, the manager must hire **8** employees for the lunchtime shift and **10** employees for the dinnertime shift.

11 B

12 C

13 C

14 D

15 B

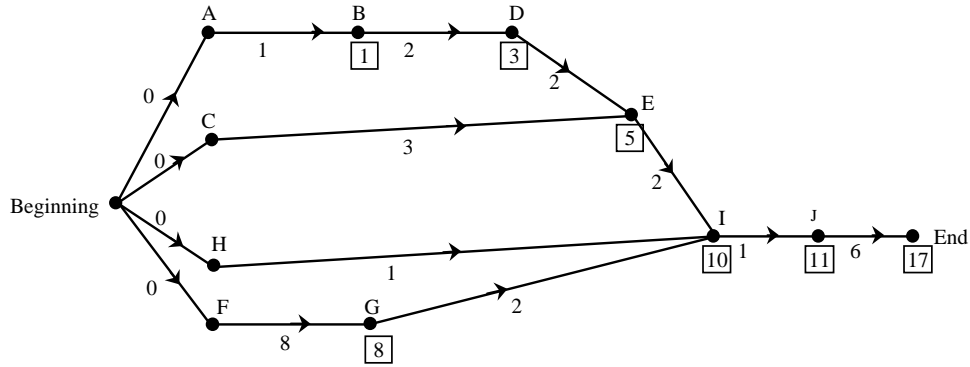
16 D

17 The minimum cost of connecting cities A, B, C and D to this network is 2 million dollars.

18 The shortest length of piping required to connect all the sprinklers is 65 m.

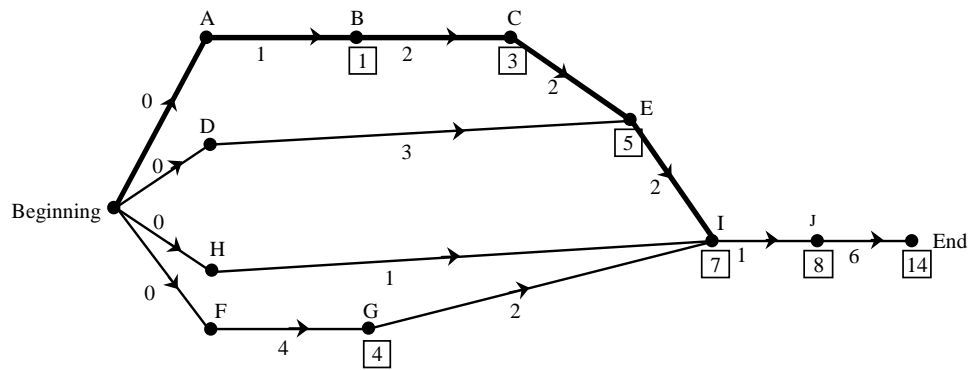
19

Graph of the initial situation



It takes 17 weeks to produce the yearbook.

Graph of the situation after the reduction in the time devoted to task F.



The reduction in the time allotted to task F changes the critical path.

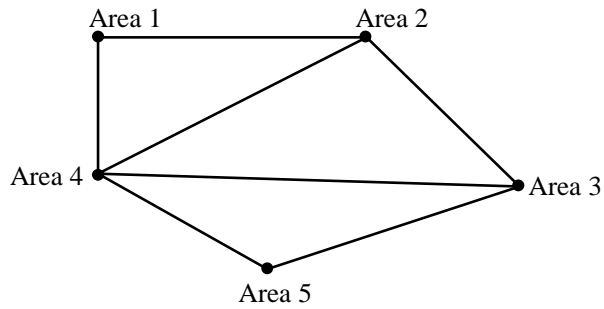
The yearbook can now be produced in 14 weeks.

Answer The total time required to produce the yearbook will decrease by 3 weeks.

or

The total time required to produce the yearbook will decrease from 17 to 14 weeks.

20

Graph of the situation**Beginning and end of the route**

Since vertices "Area 2" and "Area 3" are of odd degree, the route must begin and end in these areas.

Answer A competitor can take the following route:

Area 2, Area 1, Area 4, Area 2, Area 3, Area 4, Area 5, Area 3.

21

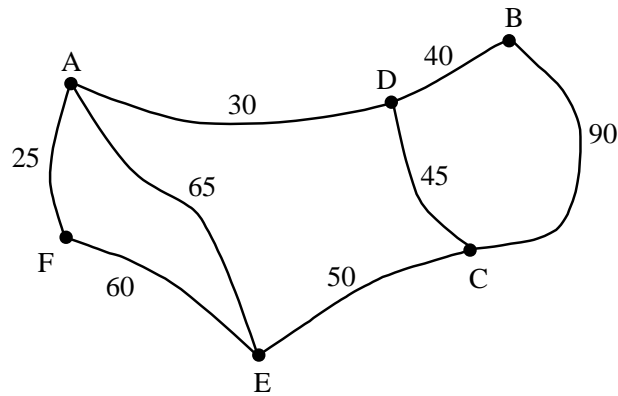
Listed below are the possible routes and the time required to complete them.

ROUTE	TIME (min)
Warehouse, Boutique B, Boutique C, Boutique A, Boutique D, Warehouse	160
Warehouse, Boutique B, Boutique C, Boutique D, Boutique A, Warehouse	175
Warehouse, Boutique C, Boutique B, Boutique A, Boutique D, Warehouse	150
Warehouse, Boutique C, Boutique B, Boutique D, Boutique A, Warehouse	190
Warehouse, Boutique C, Boutique D, Boutique B, Boutique A, Warehouse	155

Answer Louise should take the following route:

Warehouse, Boutique C, Boutique B, Boutique A, Boutique D, Warehouse

Weighted graph representing the situation



Circuit of minimum value passing through all the vertices

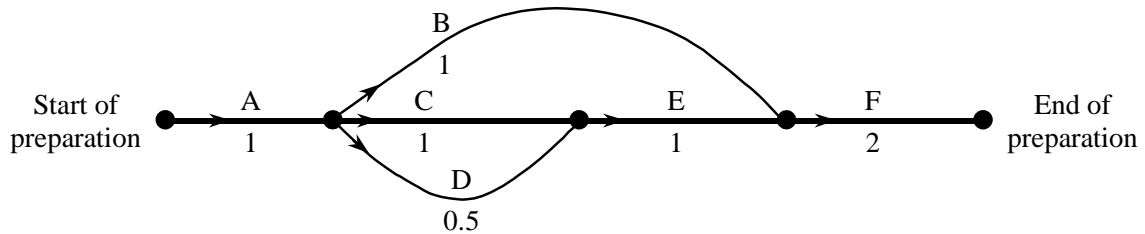
We are looking for the shortest circuit, without retracing our path unless we fulfill the given constraints by doing so.

Circuit	Value
E C B D A F E	$50 + 90 + 40 + 30 + 25 + 60 = 295$ km
E C D B D A F E	$50 + 45 + 40 + 40 + 30 + 25 + 60 = 290$ km ← Minimum

The circuit of minimum value is E C D B D A F E.

Answer: Patricia's itinerary should be as follows: **E C D B D A F E**.

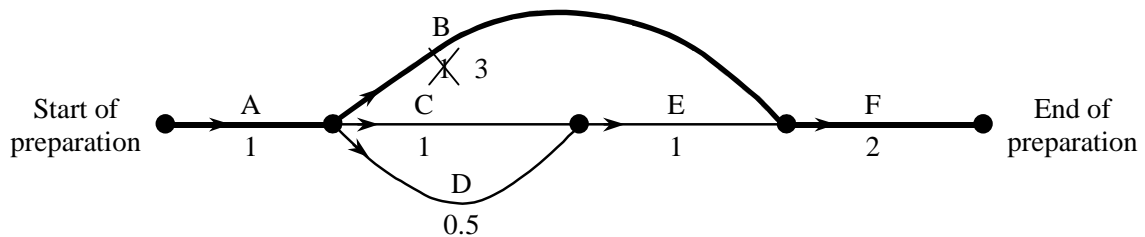
Minimum amount of time required to finish all the tasks before the change was made



Path	Total time
A, B, F	4 h
A, C, E, F	5 h
A, D, E, F	4.5 h

← The minimum amount of time required to finish all the tasks before the change was made is 5 hours.

Minimum amount of time required to finish all the tasks after the change was made



Path	Total time
A, B, F	6 h
A, C, E, F	5 h
A, D, E, F	4,5 h

← The minimum amount of time required to finish all the tasks after the change made is 6 hours.

Time at which preparation work should start

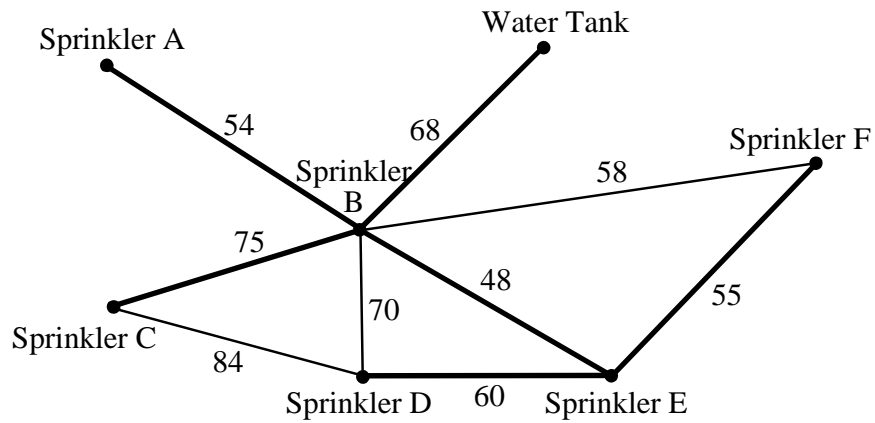
$$\text{Additional time required: } 6 \text{ h} - 5 \text{ h} = 1 \text{ h}$$

$$\text{Time at which preparation work begins: } 13 \text{ h} - 1 \text{ h} = 12 \text{ h}$$

Answer: In view of this new situation, they must start preparing the gym at **12:00 pm** at the latest.

Note: Students who use an appropriate method in order to determine the minimum amount of time required before **or** after the change was made have shown that they have a partial understanding of the problem.

- Tree of minimum value connecting all the vertices



- Minimum length of piping required

$$54 + 75 + 68 + 48 + 60 + 55 = 360 \text{ m}$$

- Cost of one metre of piping

$$\frac{\text{Total cost}}{\text{Minimum length of piping}} = \frac{\$1728}{360 \text{ m}} = \frac{\$4.80}{1 \text{ m}}$$

Answer: One metre of piping costs **\$4.80**.