8.5 Proving Triangles are Similar

What *you should learn*

GOAL 1 Use similarity theorems to prove that two triangles are similar.

GOAL 2 Use similar triangles to solve **real-life** problems, such as finding the height of a climbing wall in **Example 5**.

Why *you should learn it*

▼ To solve **real-life** problems, such as estimating the height of the Unisphere in **Ex. 29**.

REAL LIFE

GOAL 1 Using Similarity Theorems

In this lesson, you will study two additional ways to prove that two triangles are similar: the Side-Side-Side (SSS) Similarity Theorem and the Side-Angle-Side (SAS) Similarity Theorem. The first theorem is proved in Example 1 and you are asked to prove the second theorem in Exercise 31.

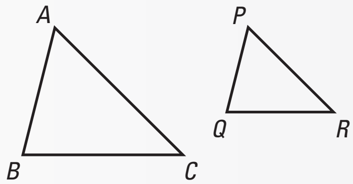
THEOREMS

THEOREM 8.2 ***Side-Side-Side (SSS) Similarity Theorem***

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$,

then $\triangle ABC \sim \triangle PQR$.

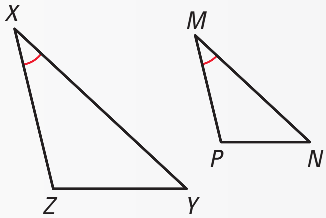


THEOREM 8.3 ***Side-Angle-Side (SAS) Similarity Theorem***

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$,

then $\triangle XYZ \sim \triangle MNP$.

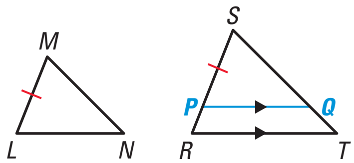


EXAMPLE 1 *Proof of Theorem 8.2*

Proof

GIVEN ▶ $\frac{RS}{LM} = \frac{ST}{MN} = \frac{TR}{NL}$

PROVE ▶ $\triangle RST \sim \triangle LMN$



Solution

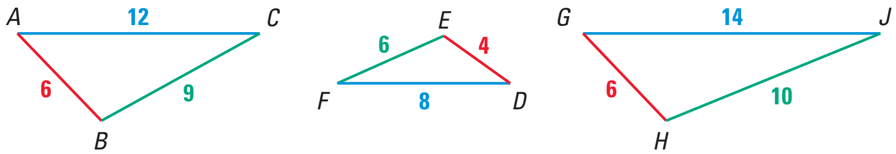
Paragraph Proof Locate P on $\overline{RS}$ so that $PS = LM$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$, by the AA Similarity Postulate, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$.

Because $PS = LM$, you can substitute in the given proportion and find that $SQ = MN$ and $QP = NL$. By the SSS Congruence Theorem, it follows that $\triangle PSQ \cong \triangle LMN$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle LMN$.

Logical Reasoning

EXAMPLE 2 *Using the SSS Similarity Theorem*

Which of the following three triangles are similar?



STUDENT HELP

Study Tip
Note that when using the SSS Similarity Theorem it is useful to compare the shortest sides, the longest sides, and then the remaining sides.

SOLUTION

To decide which, if any, of the triangles are similar, you need to consider the ratios of the lengths of corresponding sides.

Ratios of Side Lengths of $\triangle ABC$ and $\triangle DEF$

$$\frac{AB}{DE} = \frac{6}{4} = \frac{3}{2}, \qquad \frac{CA}{FD} = \frac{12}{8} = \frac{3}{2}, \qquad \frac{BC}{EF} = \frac{9}{6} = \frac{3}{2}$$

Shortest sides | **Longest sides** | **Remaining sides**

▶ Because all of the ratios are equal, $\triangle ABC \sim \triangle DEF$.

Ratios of Side Lengths of $\triangle ABC$ and $\triangle GHJ$

$$\frac{AB}{GH} = \frac{6}{6} = 1, \qquad \frac{CA}{JG} = \frac{12}{14} = \frac{6}{7}, \qquad \frac{BC}{HJ} = \frac{9}{10}$$

Shortest sides | **Longest sides** | **Remaining sides**

▶ Because the ratios are not equal, $\triangle ABC$ and $\triangle GHJ$ are not similar.

Since $\triangle ABC$ is similar to $\triangle DEF$ and $\triangle ABC$ is not similar to $\triangle GHJ$, $\triangle DEF$ is not similar to $\triangle GHJ$.

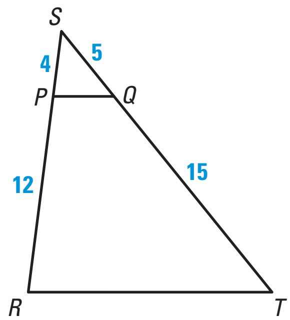
EXAMPLE 3 *Using the SAS Similarity Theorem*

Use the given lengths to prove that $\triangle RST \sim \triangle PSQ$.

SOLUTION

GIVEN ▶ $SP = 4$, $PR = 12$, $SQ = 5$, $QT = 15$

PROVE ▶ $\triangle RST \sim \triangle PSQ$



Paragraph Proof Use the SAS Similarity Theorem. Begin by finding the ratios of the lengths of the corresponding sides.

$$\frac{SR}{SP} = \frac{SP + PR}{SP} = \frac{4 + 12}{4} = \frac{16}{4} = 4$$

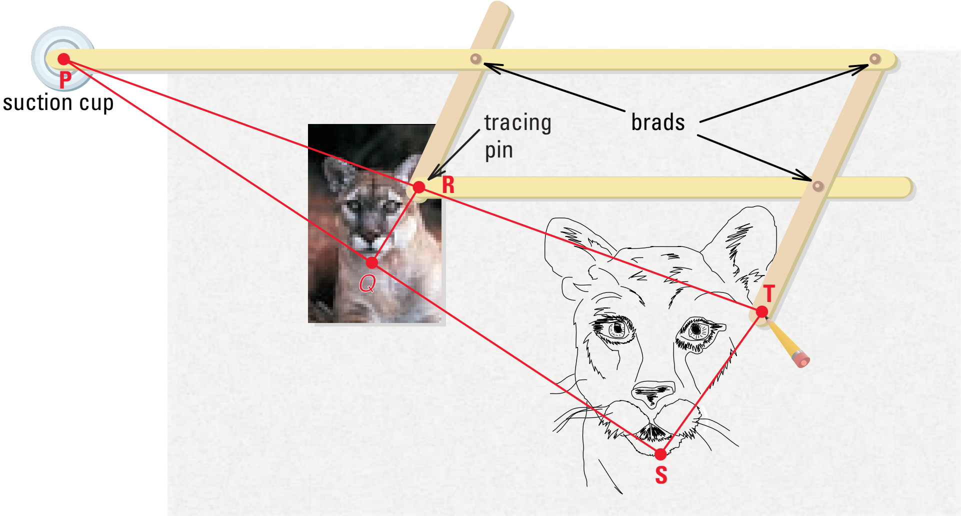
$$\frac{ST}{SQ} = \frac{SQ + QT}{SQ} = \frac{5 + 15}{5} = \frac{20}{5} = 4$$

So, the lengths of sides $\overline{SR}$ and $\overline{ST}$ are proportional to the lengths of the corresponding sides of $\triangle PSQ$. Because $\angle S$ is the included angle in both triangles, use the SAS Similarity Theorem to conclude that $\triangle RST \sim \triangle PSQ$.

GOAL 2 USING SIMILAR TRIANGLES IN REAL LIFE

EXAMPLE 4 *Using a Pantograph*

SCALE DRAWING As you move the tracing pin of a *pantograph* along a figure, the pencil attached to the far end draws an enlargement. As the pantograph expands and contracts, the three brads and the tracing pin always form the vertices of a parallelogram. The ratio of PR to PT is always equal to the ratio of PQ to PS. Also, the suction cup, the tracing pin, and the pencil remain collinear.



a. How can you show that $\triangle PRQ \sim \triangle PTS$?

b. In the diagram, PR is 10 inches and RT is 10 inches. The length of the cat, RQ, in the original print is 2.4 inches. Find the length TS in the enlargement.

SOLUTION

a. You know that $\frac{PR}{PT} = \frac{PQ}{PS}$. Because $\angle P \cong \angle P$, you can apply the SAS Similarity Theorem to conclude that $\triangle PRQ \sim \triangle PTS$.

b. Because the triangles are similar, you can set up a proportion to find the length of the cat in the enlarged drawing.

$\frac{PR}{PT} = \frac{RQ}{TS}$ **Write proportion.**

$\frac{10}{20} = \frac{2.4}{TS}$ **Substitute.**

$TS = 4.8$ **Solve for *TS*.**

▶ So, the length of the cat in the enlarged drawing is 4.8 inches.

FOCUS ON APPLICATIONS

PANTOGRAPH Before photocopiers, people used pantographs to make enlargements. As the tracing pin is guided over the figure, the pencil draws an enlargement.

Similar triangles can be used to find distances that are difficult to measure directly. One technique is called *Thales' shadow method* (page 486), named after the Greek geometer Thales who used it to calculate the height of the Great Pyramid.

FOCUS ON APPLICATIONS

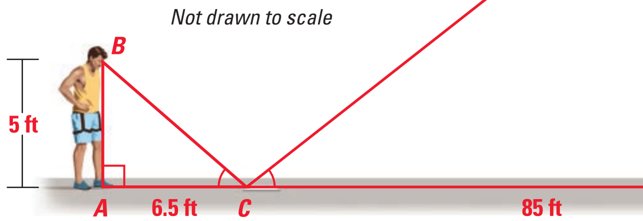
ROCK CLIMBING Interest in rock climbing appears to be growing. From 1988 to 1998, over 700 indoor rock climbing gyms opened in the United States.

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 5 *Finding Distance Indirectly*

ROCK CLIMBING You are at an indoor climbing wall. To estimate the height of the wall, you place a mirror on the floor 85 feet from the base of the wall. Then you walk backward until you can see the top of the wall centered in the mirror. You are 6.5 feet from the mirror and your eyes are 5 feet above the ground. Use similar triangles to estimate the height of the wall.



SOLUTION

Due to the reflective property of mirrors, you can reason that $\angle ACB \cong \angle ECD$. Using the fact that $\triangle ABC$ and $\triangle EDC$ are right triangles, you can apply the AA Similarity Postulate to conclude that these two triangles are similar.

$\frac{DE}{BA} = \frac{EC}{AC}$ — **Ratios of lengths of corresponding sides are equal.**

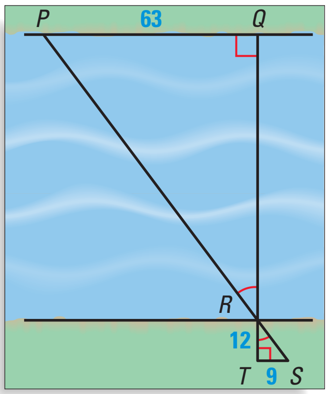
$\frac{DE}{5} = \frac{85}{6.5}$ — **Substitute.**

$65.38 \approx DE$ — **Multiply each side by 5 and simplify.**

▶ So, the height of the wall is about 65 feet.

EXAMPLE 6 *Finding Distance Indirectly*

INDIRECT MEASUREMENT To measure the width of a river, you use a surveying technique, as shown in the diagram. Use the given lengths (measured in feet) to find RQ.



SOLUTION

By the AA Similarity Postulate, $\triangle PQR \sim \triangle STR$.

$\frac{RQ}{RT} = \frac{PQ}{ST}$ — **Write proportion.**

$\frac{RQ}{12} = \frac{63}{9}$ — **Substitute.**

$RQ = 12 \cdot 7$ — **Multiply each side by 12.**

$RQ = 84$ — **Simplify.**

▶ So, the river is 84 feet wide.

GUIDED PRACTICE

Vocabulary Check ✓

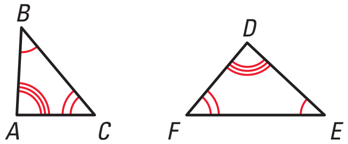
1. You want to prove that $\triangle FHG$ is similar to $\triangle RXS$ by the SSS Similarity Theorem. Complete the proportion that is needed to use this theorem.

$$\frac{FH}{?} = \frac{?}{XS} = \frac{FG}{?}$$

Concept Check ✓

Name a postulate or theorem that can be used to prove that the two triangles are similar. Then, write a similarity statement.

2.



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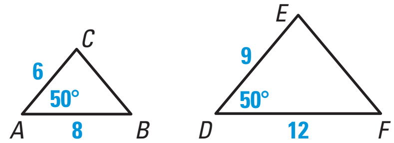
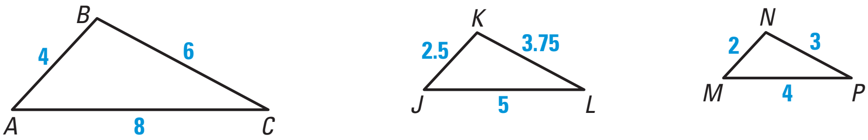


Skill Check ✓

4. Which triangles are similar to $\triangle ABC$? Explain.



5. The side lengths of $\triangle ABC$ are 2, 5, and 6, and $\triangle DEF$ has side lengths of 12, 30, and 36. Find the ratios of the lengths of the corresponding sides of $\triangle ABC$ to $\triangle DEF$. Are the two triangles similar? Explain.

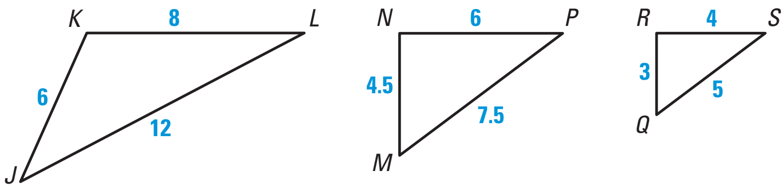
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 818.

DETERMINING SIMILARITY **In Exercises 6–8, determine which two of the three given triangles are similar. Find the scale factor for the pair.**

6.



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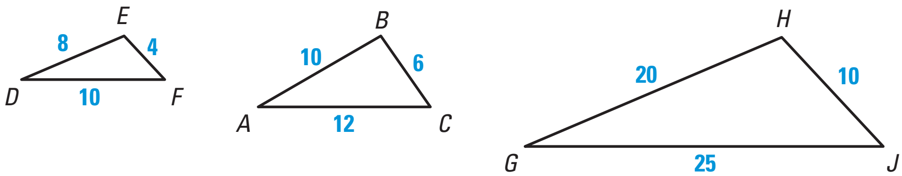
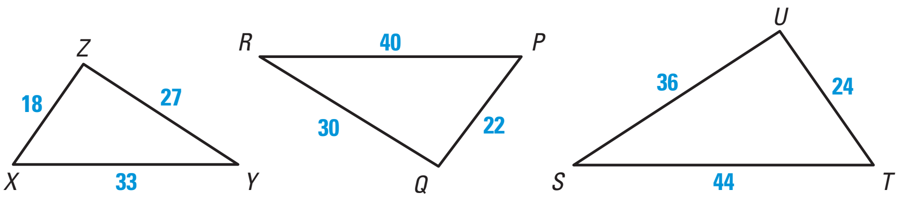


STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 30, 31
Example 2: Exs. 6–18

8.

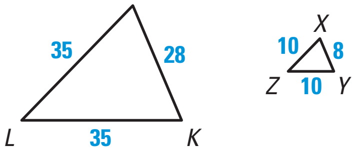


STUDENT HELP

HOMEWORK HELP
Example 3: Exs. 6–18, 30, 31
Example 4: Exs. 19–26, 29, 32–35
Example 5: Exs. 29, 32–35
Example 6: Exs. 29, 32–35

DETERMINING SIMILARITY **Are the triangles similar? If so, state the similarity and the postulate or theorem that justifies your answer.**

9.



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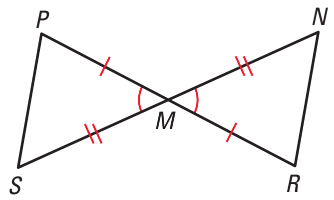

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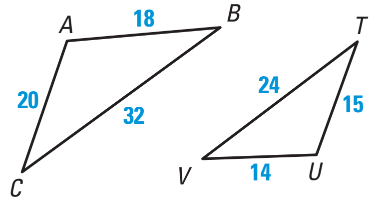

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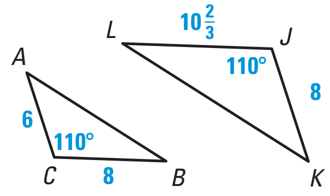

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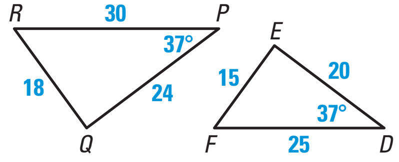
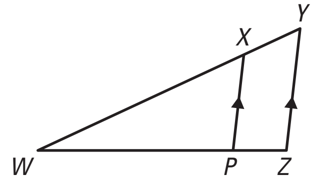


14.



LOGICAL REASONING **Draw the given triangles roughly to scale. Then, name a postulate or theorem that can be used to prove that the triangles are similar.**

15. The side lengths of $\triangle PQR$ are 16, 8, and 18, and the side lengths of $\triangle XYZ$ are 9, 8, and 4.

16. In $\triangle ABC$, $m\angle A = 28°$ and $m\angle B = 62°$. In $\triangle DEF$, $m\angle D = 28°$ and $m\angle F = 90°$.

17. In $\triangle STU$, the length of $\overline{ST}$ is 18, the length of $\overline{SU}$ is 24, and $m\angle S = 65°$. The length of $\overline{JK}$ is 6, $m\angle J = 65°$, and the length of $\overline{JL}$ is 8 in $\triangle JKL$.

18. The ratio of VW to MN is 6 to 1. In $\triangle VWX$, $m\angle W = 30°$, and in $\triangle MNP$, $m\angle N = 30°$. The ratio of WX to NP is 6 to 1.

FINDING MEASURES AND LENGTHS **Use the diagram shown to complete the statements.**

19. $m\angle CED =$ _?_.

20. $m\angle EDC =$ _?_.

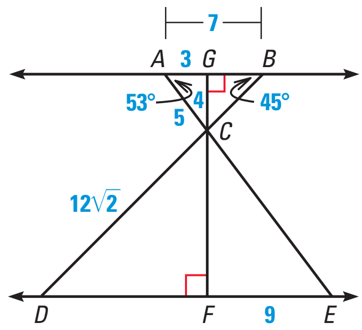
21. $m\angle DCE =$ _?_.

22. $FC =$ _?_.

23. $EC =$ _?_.

24. $DE =$ _?_.

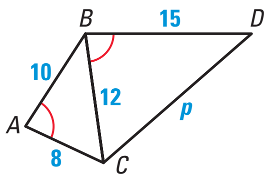
25. $CB =$ _?_.



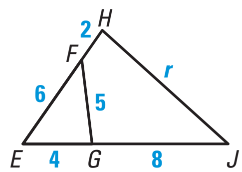
26. Name the three pairs of triangles that are similar in the figure.

DETERMINING SIMILARITY **Determine whether the triangles are similar. If they are, write a similarity statement and solve for the variable.**

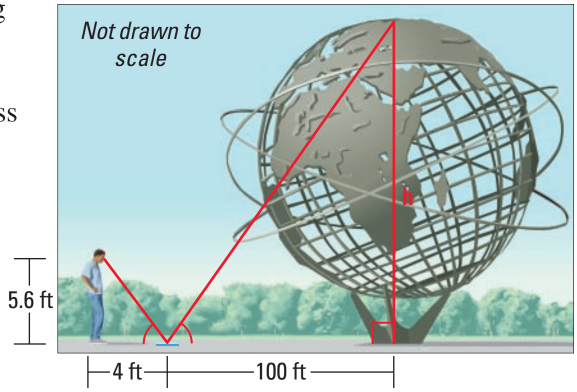
27.



28.



29. **UNISPHERE** You are visiting the Unisphere at Flushing Meadow Park in New York. To estimate the height of the stainless steel model of Earth, you place a mirror on the ground and stand where you can see the top of the model in the mirror. Use the diagram shown to estimate the height of the model.

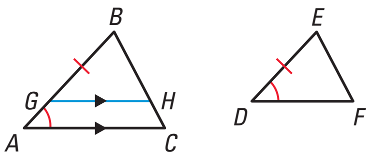


30. **PARAGRAPH PROOF** Two isosceles triangles are similar if the vertex angle of one triangle is congruent to the vertex angle of the other triangle. Write a paragraph proof of this statement and include a labeled figure.

31. **PARAGRAPH PROOF** Write a paragraph proof of Theorem 8.3.

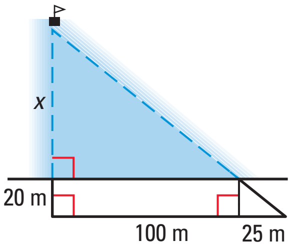
GIVEN ▶ $\angle A \cong \angle D, \dfrac{AB}{DE} = \dfrac{AC}{DF}$

PROVE ▶ $\triangle ABC \sim \triangle DEF$

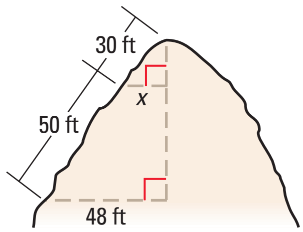


FINDING DISTANCES INDIRECTLY **Find the distance labeled *x*.**

32.

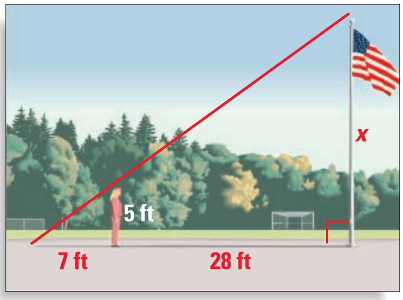


33.



FLAGPOLE HEIGHT **In Exercises 34 and 35, use the following information.**

Julia uses the shadow of the flagpole to estimate its height. She stands so that the tip of her shadow coincides with the tip of the flagpole's shadow as shown. Julia is 5 feet tall. The distance from the flagpole to Julia is 28 feet and the distance between the tip of the shadows and Julia is 7 feet.

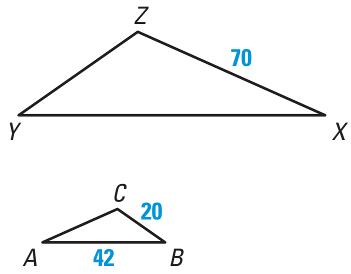


34. Calculate the height of the flagpole.

35. Explain why Julia's shadow method works.

Test Preparation

QUANTITATIVE COMPARISON In Exercises 36 and 37, use the diagram, in which △*ABC* ~ △*XYZ*, and the ratio *AB*:*XY* is 2:5. Choose the statement that is true about the given quantities.



Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

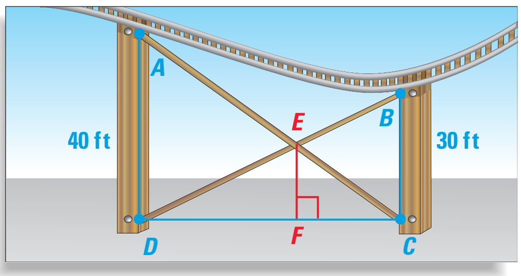
Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the given information.

	Column A	Column B
36.	The perimeter of △*ABC*	The length *XY*
37.	The distance *XY* + *BC*	The distance *XZ* + *YZ*

★ Challenge

38. **DESIGNING THE LOOP** A portion of an amusement park ride called the Loop is shown. Find the length of $\overline{EF}$. (*Hint:* Use similar triangles.)



EXTRA CHALLENGE
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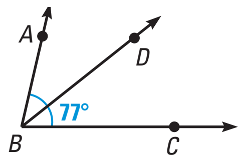
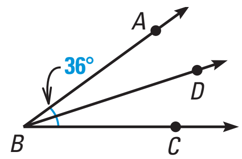
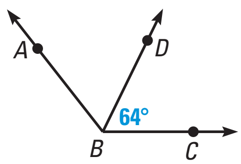
MIXED REVIEW

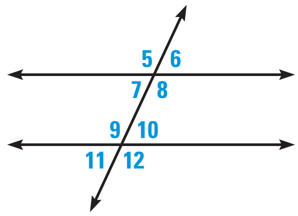
ANALYZING ANGLE BISECTORS $\overrightarrow{BD}$ **is the angle bisector of ∠*ABC*. Find any angle measures not given in the diagram.** (Review 1.5 for 8.6)

39. 

40. 

41. 

RECOGNIZING ANGLES Use the diagram shown to complete the statement. (Review 3.1 for 8.6)



42. ∠5 and __?__ are alternate exterior angles.

43. ∠8 and __?__ are consecutive interior angles.

44. ∠10 and __?__ are alternate interior angles.

45. ∠9 and __?__ are corresponding angles.

FINDING COORDINATES Find the coordinates of the image after the reflection without using a coordinate plane. (Review 7.2)

46. $T(0, 5)$ reflected in the x-axis

47. $P(-2, 7)$ reflected in the y-axis

48. $B(-3, -10)$ reflected in the y-axis

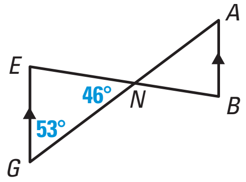
49. $C(-5, -1)$ reflected in the x-axis

QUIZ 2

Self-Test for Lessons 8.4 and 8.5

Determine whether you can show that the triangles are similar. State any angle measures that are not given. (Lesson 8.4)

1.



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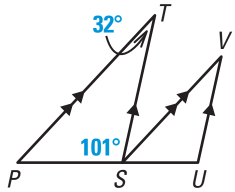

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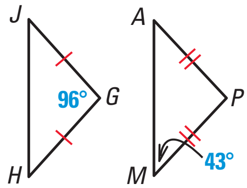


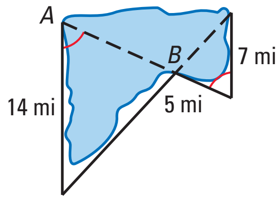
In Exercises 4–6, you are given the ratios of the lengths of the sides of $\triangle DEF$. If $\triangle ABC$ has sides of lengths 3, 6, and 7 units, are the triangles similar? (Lesson 8.5)

4. $4:7:8$

5. $6:12:14$

6. $1:2:\frac{7}{3}$

7. **DISTANCE ACROSS WATER** Use the known distances in the diagram to find the distance across the lake from A to B. (Lesson 8.5)



MATH & History

The Golden Rectangle

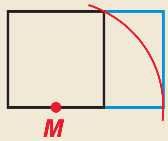
APPLICATION LINK www.mcdougallittell.com

THEN

THOUSANDS OF YEARS AGO, Greek mathematicians became interested in the *golden ratio*, a ratio of about $1:1.618$. A rectangle whose side lengths are in the golden ratio is called a *golden rectangle*. Such rectangles are believed to be especially pleasing to look at.

NOW

THE GOLDEN RATIO has been found in the proportions of many works of art and architecture, including the works shown in the timeline below.



1. Follow the steps below to construct a golden rectangle. When you are done, check to see whether the ratio of the width to the length is $1:1.618$.

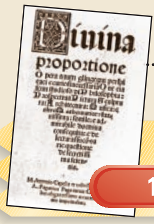
- Construct a square. Mark the midpoint M of the bottom side.
- Place the compass point at M and draw an arc through the upper right corner of the square.
- Extend the bottom side of the square to intersect with the arc. The intersection point is the corner of a golden rectangle. Complete the rectangle.

c. 1300 B.C.

The Osirion (underground Egyptian temple)

c. 440 B.C.

The Parthenon, Athens, Greece



1509

Leonardo da Vinci illustrates Luca Pacioli's book on the golden ratio.

1956

Le Corbusier uses golden ratios based on this human figure in his architecture.