§4-2   QUADRATIC INEQUALITIES

Definition

> **Quadratic inequalities** in one variable are inequalities which can be written in one of the following forms: $ax^2 + bx + c > 0$,
> $$ax^2 + bx + c < 0,$$
> $$ax^2 + bx + c \geq 0 \text{ or}$$
> $$ax^2 + bx + c \leq 0 \text{ where } a, b \text{ and } c \text{ are real numbers.}$$

Procedure

> **Solving Quadratic Inequalities**
> 1. Move all terms to one side.
> 2. Simplify and factor the quadratic expression.
> 3. Find the roots of the corresponding quadratic equation.
> 4. Use the roots to divide the number line into regions.
> 5. Test each region using the inequality.

Example 1   Solve the inequality, $x^2 > x + 2$.

Solution
$$x^2 > x + 2$$
$$x^2 - x - 2 > 0$$
$$(x - 2)(x + 1) > 0$$

The corresponding equation is $(x - 2)(x + 1) = 0$ so…

$$x - 2 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 2 \qquad\qquad x = -1$$
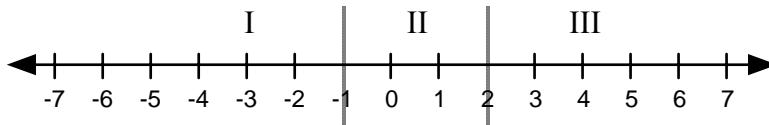
Now we test one point in each region.

Region	Test Point	Inequality	Status
I	$x = -2$	$(x - 2)(x + 1) = (-2 - 2)(-2 + 1) = 4 > 0$	True
II	$x = 0$	$(x - 2)(x + 1) = (0 - 2)(0 + 1) = -2 > 0$	False
III	$x = 3$	$(x - 2)(x + 1) = (3 - 2)(3 + 1) = 4 > 0$	True
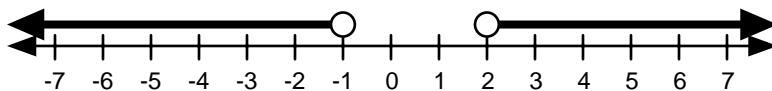
So the solution to this inequality is $x < -1$ or $x > 2$.

**Example 2**   Solve the inequality, $(x + 3)^2 \geq 2(x^2 + 7)$.

Solution

$$(x + 3)^2 \geq 2(x^2 + 7)$$
$$x^2 + 6x + 9 \geq 2x^2 + 14$$
$$-x^2 + 6x - 5 \geq 0$$
$$-(x^2 - 6x + 5) \geq 0$$

$$\frac{-(x^2 - 6x + 5)}{-1} \leq \frac{0}{-1}$$

$$x^2 - 6x + 5 \leq 0$$
$$(x - 1)(x - 5) \leq 0$$

The corresponding equation is $(x - 1)(x - 5) = 0$ so…

$$x - 1 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = 1 \qquad\qquad x = 5$$
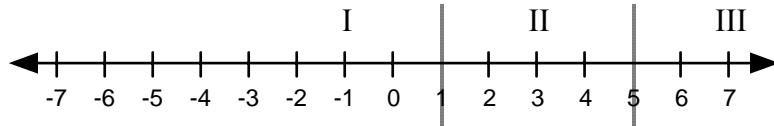
Now we check one point in each region.

Region	Test Point	Inequality	Status
I	$x = 0$	$(x - 1)(x - 5) = (0 - 1)(0 - 5) = 5 < 0$	False
II	$x = 2$	$(x - 1)(x - 5) = (2 - 1)(2 - 5) = -3 < 0$	True
III	$x = 6$	$(x - 1)(x - 5) = (6 - 1)(6 - 5) = 5 < 0$	False
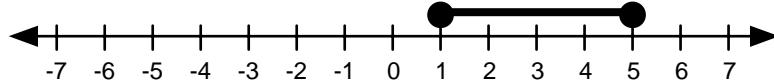
So the solution to this inequality is $1 \leq x \leq 5$.

Solve each quadratic inequality, and graph the solution on a number line.

1. $y^2 - 17y + 70 < 0$

2. $x^2 + 9x + 13 > -7$

3. $x(x+1) > 112 - 5x$

4. $a^2 + 3a + 2 < -3(a+2)$

5. $2x^2 \le 5x - 2$

6. $10 - 9y \ge -2y^2$

7. $b(b+3) \ge -2$

8. $a^2 \le 4(2a-3)$

9. $y^2 - 17y + 70 < 0$

10. $x^2 + 9x + 13 > -7$

11. $x(x+1) > 112 - 5x$

12. $a^2 + 25 < 10a$

13. $2d^2 + 5d \le 12$

14. $a^2 + 3a + 2 \ge -3(a+2)$

15. $10 - 9y \ge -2y^2$

16. $2x^2 \le 5x - 2$

17. $c(c+4) < 3 + 3(9+c)$

18. $2a(a+6) > 5 - a(a+2)$

19. $b(b+3) > -2$

20. $a^2 < 4(2a-3)$

21. $(x+3)^2 \le 6(x+15)$

22. $2x^2 + 7 \ge 9x$

23. $7x^2 \ge 4(1+3x)$

24. $3x^2 + 7x \le -2$

25. $-8 < 4(x - x^2)$

26. $x^2 - x - 2 > 0$

27. $2k^2 + 3k - 2 > 0$

28. $t^2 + 2t - 3 < 0$

29. $4x^2 + 8 \le 33x$

30. $x^2 \ge 4(x-5)$

31. $x^2 + 4 \ge 2x^2 - 3x$

32. $10 - 3x \le x^2$

33. $4 < 13x - 3x^2$

34. $6(x^2 + 1) > -13$

35. $6x - x^2 > 8$

36. $20a^2 < 1 - a$

37. $8x \le -3(1 - x^2)$

38. $y^2 \ge 25$

39. $t^2 + 18 \ge 11t$

40. $3x(x+1) \le x(x+5)$

41. $x^2 < 8$

42. $x^2 + 3x > 12$

43. $2t^2 > 9t + 18$

44. $4x^2 - 9x + 2 < 0$

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1. $7 < y < 10$

2. $x < -5$ or $x > -4$

3. $x < -14$ or $x > 8$

4. $-4 < a < -2$

5. $\frac{1}{2} \le x \le 2$

6. $y \le 2$ or $y \ge \frac{5}{2}$

7. $b \le -2$ or $b \ge -1$

8. $2 \le a \le 6$

9. $7 < y < 10$

10. $x < -5$ or $x > -$

11. $x < -14$ or $x > 8$

12. no solution

13. $-4 \le d \le \frac{3}{2}$

14. $x \le -4$ or $x \ge -2$

15. $y \le 2$ or $y \ge \frac{5}{2}$

16. $\frac{1}{2} \le x \le 2$

17. $-6 < c < 5$

18. $a < -5$ or $a > \frac{1}{3}$

19. $b \le -2$ or $b \ge -1$

20. $2 < a < 6$

21. $-9 \le x \le 9$

22. $x < 1$ or $x \ge \frac{7}{2}$

23. $x \le -\frac{2}{7}$ or $x \ge 2$

24. $-2 \le x \le -\frac{1}{3}$

25. $-1 < x < 2$

26. $x < -1$ or $x > 2$

27. $k < -2$ or $k > \frac{1}{2}$

28. $-3 < t < 1$

29. $\frac{1}{4} \le x \le 8$

30. all real numbers

31. $-1 < x < 4$

32. $x \le -5$ or $x \ge 2$

33. $\frac{1}{3} < x < 4$

34. no real solutions

35. $2 < x < 4$

36. $-\frac{1}{4} < x < \frac{1}{5}$

37. $x \le -\frac{1}{3}$ or $x \ge 3$

38. $x \le -5$ or $x \ge 5$

39. $t \le 2$ or $t \ge 9$

40. $0 \le x \le 1$

41. $-2\sqrt{2} < x < 2\sqrt{2}$

42. $x < \frac{-3 - \sqrt{57}}{2}$ or $x > \frac{-3 + \sqrt{57}}{2}$

43. $t < -\frac{3}{2}$ or $t > 6$

44. $\frac{1}{4} < x < 2$