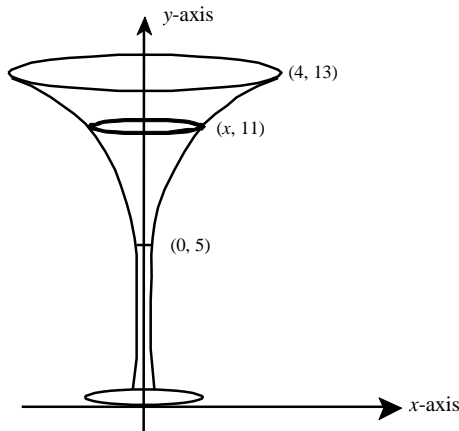


# SQUARE ROOT FUNCTION

3 Example of an appropriate solution



The square root function must be in the form:  $y = a\sqrt{x} + k$

Substituting (0, 5) we get:  $5 = a\sqrt{0} + k$       So  $k = 5$

Substituting (4, 13) we get:  $13 = a\sqrt{4} + 5$       So  $a = 4$

So the function is:  $y = 4\sqrt{x} + 4$

At the ring,  $y = 11$        $11 = 4\sqrt{x} + 5$       So  $x = 2.25$

But 2.25 represents the radius of the gold ring in centimetres.

So the circumference is:  $C = 2\pi(2.25) \approx 14.13$  cm

Therefore the gold ring will cost:  $14.13 \times 2 = 28.26$  cents.

Answer      Rounded to the nearest cent, the cost of the gold ring is 28 cents.

**Note** Do not deduct any marks if the student wrote 29 cents.

4 Which of the following is the inverse of:  $f(x) = 3\sqrt{x} + 8, x \geq 0$ ?

A)  $f^{-1}(x) = \left(\frac{x-8}{3}\right)^2, x \geq 8$

C)  $f^{-1}(x) = \frac{x^2}{3} - 8, x \geq 8$

B)

D)

$$f^{-1}(x) = \frac{(x-8)^2}{3}, x \geq 8$$

$$f^{-1}(x) = \frac{x^2}{9} - 8, x \geq 8$$

5

Rule of the function

$x$  : time in seconds

$f(x)$  : altitude in metres

$$f(x) = a\sqrt{x-h} + k$$

$$f(x) = a\sqrt{x-0} + 10$$

$$f(x) = a\sqrt{x} + 10$$

$$f(25) = 0 \quad \text{therefore} \quad 0 = a\sqrt{25} + 10$$

$$-10 = 5a$$

$$-2 = a$$

$$f(x) = -2\sqrt{x} + 10$$

Time at which Caroline entered the tunnel

$$f(x) = 6 \text{ therefore} \quad 6 = -2\sqrt{x} + 10$$

$$-4 = -2\sqrt{x}$$

$$2 = \sqrt{x}$$

$$4 = x$$

After 4 seconds, Caroline entered the tunnel.

Time at which Caroline exited the tunnel

$$f(x) = 4 \text{ therefore} \quad 4 = -2\sqrt{x} + 10$$

$$-6 = -2\sqrt{x}$$

$$3 = \sqrt{x}$$

$$9 = x$$

After 9 seconds, Caroline exited the tunnel.

Time during which Caroline was in the tunnel

$$9 - 4 = 5 \text{ seconds}$$

Answer: Caroline was in the tunnel for **5** seconds.

**Note:** Students who used an appropriate method in order to determine the rule of the function have shown that they have a partial understanding of the problem.

6

A

7

Example of an appropriate method

Finding the rules of the form

$$y = a\sqrt{b(x - h)} + k$$

Function 1:  $(h, k) = (0, 3)$

$$(x, y) = (4, 1)$$

Let  $b = 1$

$$1 = a\sqrt{1(4 - 0)} + 3$$

$$a\sqrt{4} = -2$$

$$a = -1$$

$$y = -\sqrt{x} + 3$$

Function 2:  $(h, k) = (2, 0)$

$$(x, y) = (6, 2)$$

Let  $b = 1$

$$2 = a\sqrt{1(6 - 2)} + 0$$

$$a\sqrt{4} = 2$$

$$a = 1$$

$$y = \sqrt{x - 2}$$

Finding the time at the intersection

$$-\sqrt{x} + 3 = \sqrt{x - 2}$$

$$(-\sqrt{x} + 3)^2 = (\sqrt{x - 2})^2$$

$$x - 6\sqrt{x} + 9 = x - 2$$

$$-6\sqrt{x} = -11$$

$$\sqrt{x} = \frac{11}{6}$$

$$x \approx 3.6$$

Time wanted

$$3.36 - 2 = 1.36$$

Answer: 1.36 seconds after it has been launched, the 2nd projectile will be higher than the 1st projectile.

8

C

9

C

10

C

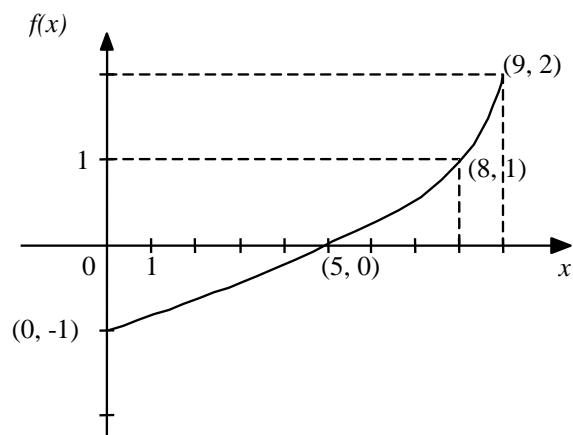
11

B

12

C

13 The graph of the real function is



14

C

15

Example of an appropriate solution

Equation of square root function

$$R(x) = a\sqrt{x-h} + k$$

$$5 = a\sqrt{10-1} - 4$$

$$5 = 3a - 4$$

$$5 + 4 = 3a$$

$$3 = a$$

$$R(x) = 3\sqrt{x-1} - 4$$

Zero of the function

$$R(x) = 3\sqrt{x-1} - 4$$

$$0 = 3\sqrt{x-1} - 4$$

$$4 = 3\sqrt{x-1}$$

$$\frac{4}{3} = \sqrt{x-1}$$

$$\frac{16}{9} = x-1$$

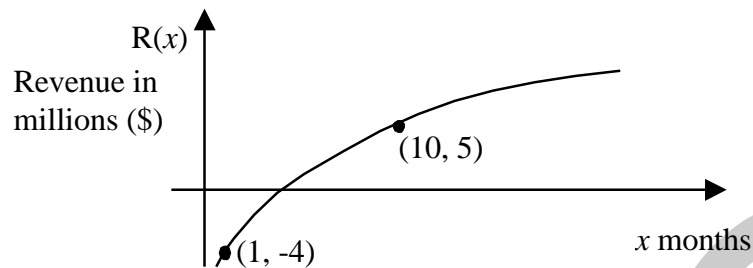
$$x = \frac{25}{9}$$

$$x \approx 2.8$$

Answer : The company began to make a profit after 2.8 months.

Accept an answer in [2.7; 3].

After one month in operation, a company's revenue has grown according to the formula  $R(x) = a\sqrt{x-h} + k$  where  $R(x)$  is the profit in millions of dollars after a period of  $x$  months.



Losses amounted to \$400 the first month. After ten months, \$5000 in profits was recorded.

After how many months in operation did the company begin to make a profit?

BIM : Correction

Example of an appropriate solution

Equation of square root function

$$R(x) = a\sqrt{x-h} + k$$

$$5 = a\sqrt{10-1} - 4$$

$$5 = 3a - 4$$

$$5 + 4 = 3a$$

$$3 = a$$

$$R(x) = 3\sqrt{x-1} - 4$$

Zero of the function

$$R(x) = 3\sqrt{x-1} - 4$$

$$0 = 3\sqrt{x-1} - 4$$

$$4 = 3\sqrt{x-1}$$

$$\frac{4}{3} = \sqrt{x-1}$$

$$x-1$$

$$\frac{16}{9} =$$

$$x = \frac{25}{9}$$

$$x \approx 2.8$$

Answer : The company began to make a profit after 2.8 months.

16

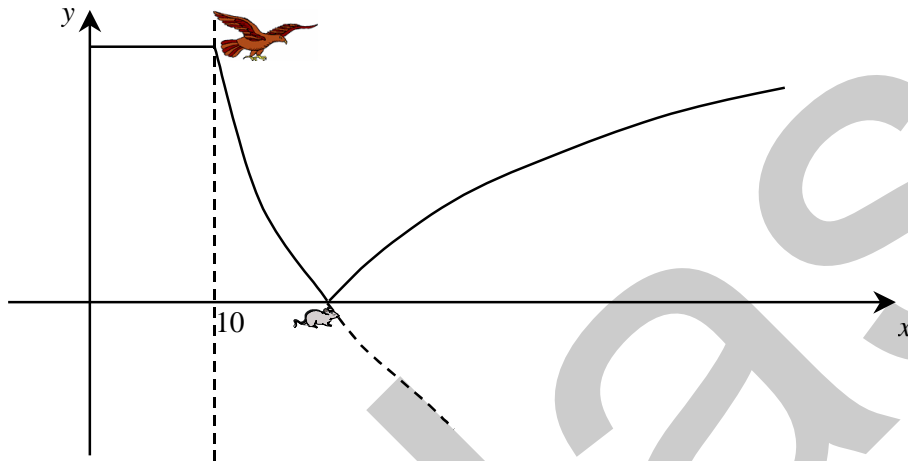
D

## SQAURE ROOT FUNCTION

- 1 After flying for 10 seconds, a hawk swoops down, catches a mouse, and immediately takes off with its prey.

The path of the hawk's descent has been determined to be in the shape of a rational function and the path of its ascent is in the shape of a square root function. The point of ascent corresponds to the vertex of the square root function. Four seconds after the hawk catches the mouse, it is 8 m above the ground.

The rational function is  $f(x) = \frac{4}{x - 10} - 2$ .



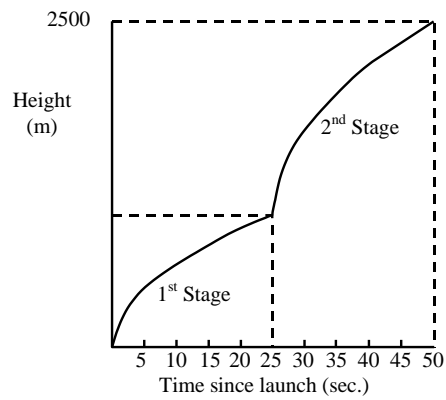
How many metres above the ground will the hawk be 9 seconds after it catches its prey?

- 2 A two-stage model rocket is launched from ground level. Its 1<sup>st</sup> stage (engine) powers the rocket vertically, according to the rule  $H(t) = 200\sqrt{t}$ ,

where  $H(t)$  is height, in metres,  
and  $t$  is time, in seconds, after launch.

At 25 seconds, the exhausted 1<sup>st</sup> stage is ejected, and the 2<sup>nd</sup> stage fires. The height of the rocket after the first 25 seconds can be expressed according to a new square root function of the form  $y = a\sqrt{(x - h)} + k$ .

50 seconds after the initial launch the rocket reaches a height of 2500 m.

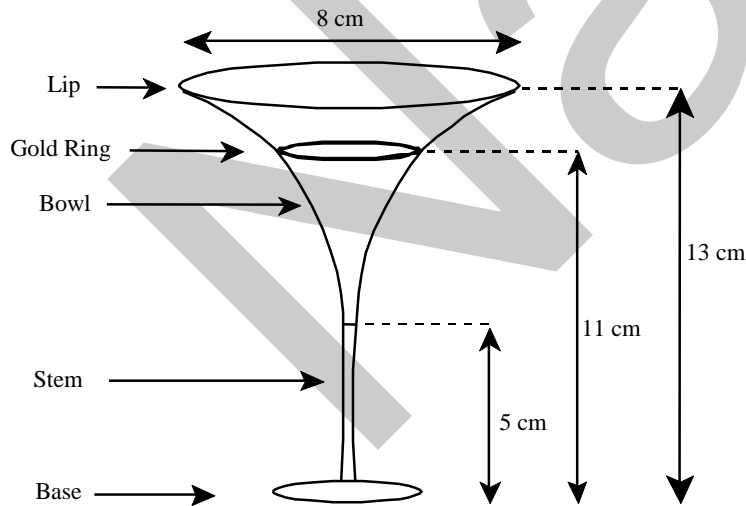


Rounded to the nearest metre, what is the height of the rocket 11 seconds after the firing of the 2<sup>nd</sup> stage?

- 3 A new glass has been designed by rotating part of a graph of a square root function about the axis containing the stem of the glass. (Assume the width of the stem to be zero.)

As illustrated in the diagram, the diameter of the lip of the bowl is 8 centimetres. The glass stands 13 centimetres in height and the top of the stem of the glass is 5 centimetres high.

A decorative gold ring is to be painted around the bowl 11 centimetres from the bottom of the glass, at a cost of 2 cents per centimetre.

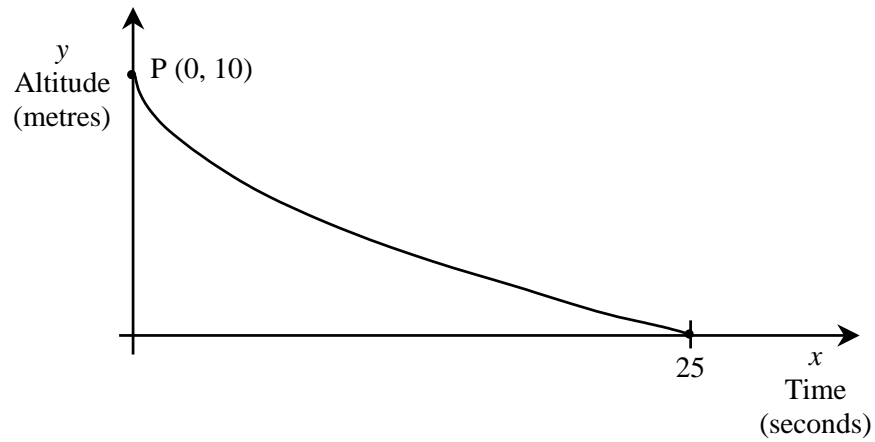


How much will it cost to paint the gold ring around the bowl?

Round your answer to the nearest cent.

- 5 Caroline slides down a waterslide at an aquatic park. The following graph represents Caroline's altitude in relation to her sliding time. This curve represents a square root function whose vertex is point P(0, 10).

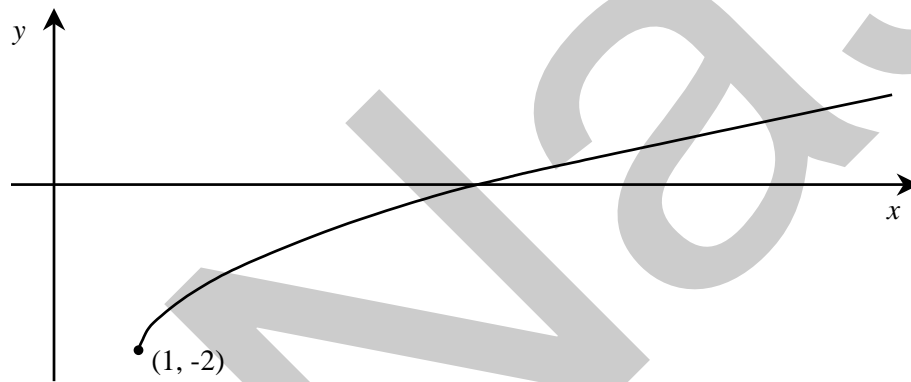




A section of the slide is covered by a tarpaulin, forming a tunnel. Caroline enters the tunnel when she is at an altitude of 6 m. She exits the tunnel at an altitude of 4 m.

How long was Caroline in the tunnel?

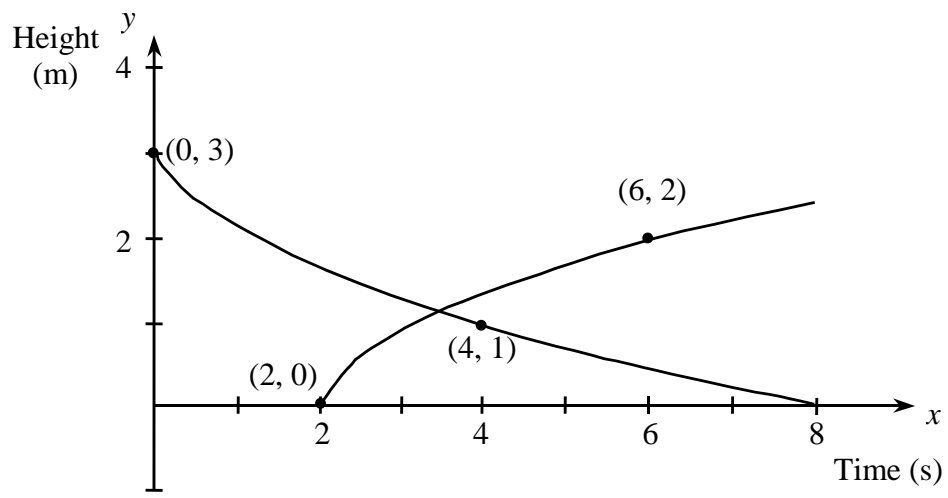
- 6 The rule of function  $f$  represented by the following graph is  $f(x) = \sqrt{x-1} - 2$ .



What is the rule of its inverse  $f^{-1}$ ?

- A)  $f^{-1}(x) = x^2 + 4x + 5$  where  $x \geq -2$
- B)  $f^{-1}(x) = x^2 + 4x + 5$  where  $x \geq 1$
- C)  $f^{-1}(x) = x^2 - 4x + 5$  where  $x \geq -2$
- D)  $f^{-1}(x) = x^2 - 4x + 5$  where  $x \geq 1$

- 7 Two missiles are launched 2 seconds apart. The paths they follow over a span of 8 seconds can be represented by two different square root functions, as illustrated below:



How many seconds after the 2<sup>nd</sup> projectile has been launched, will it be higher than the 1<sup>st</sup> projectile?

- 8 A missile is picked up by an airplane's radar. The path of the missile across the radar screen is represented by the following rule of correspondence:

$$A(s) = -10\sqrt{16(s-3)} + 5000$$

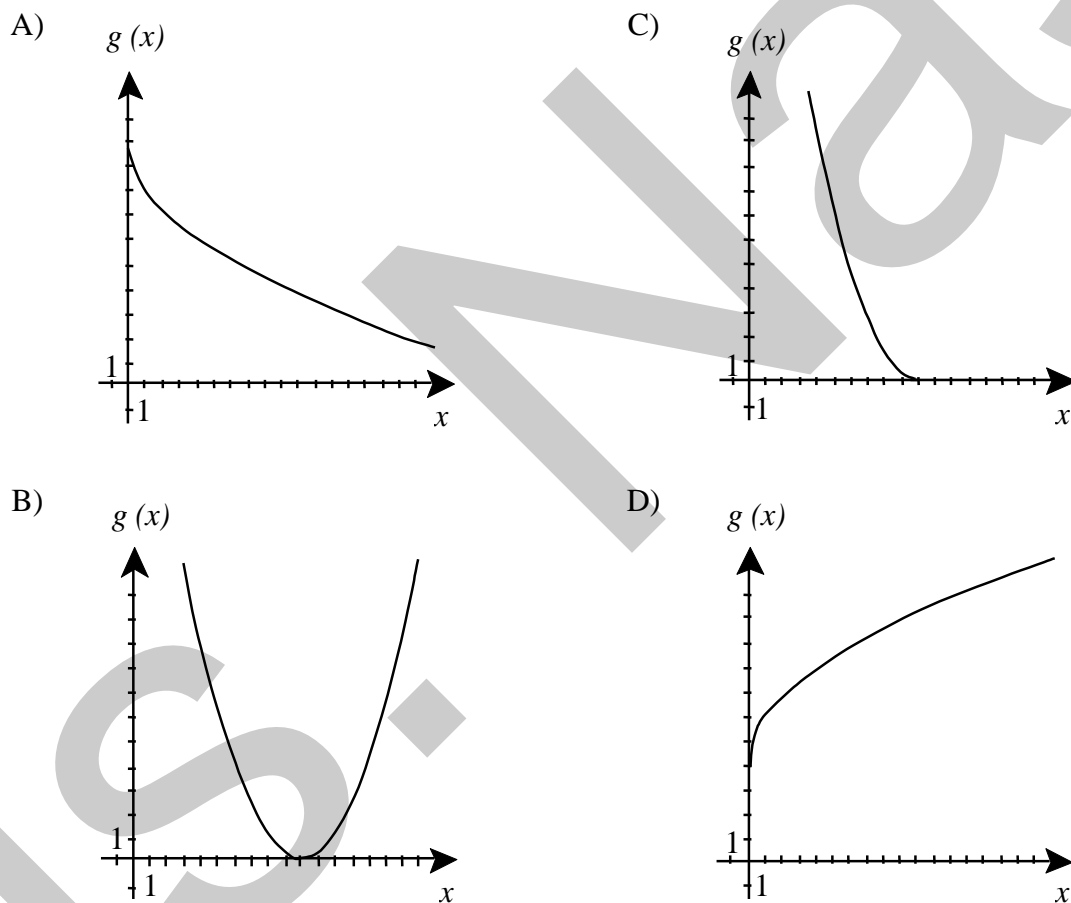
where  $A(s)$  represents the altitude in metres and  $s$ , the time in seconds.

Which statement is true?

- A) The domain of the function is  $[-3, +\infty[$ .
- B) On the radar screen, the path of the missile is directed left.
- C) According to the rule of correspondence, the function is decreasing.
- D) According to the rule of correspondence, the function is increasing between 0 and 3.

- 9 The function  $f(x) = -2\sqrt{x} + 10$  represents the function of the outside temperature, in degrees Celsius, in relation to the number of hours elapsed,  $x$ , since the beginning of observations.

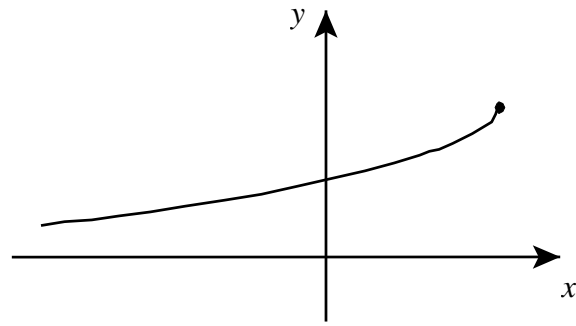
Which of the following graphs represents  $g(x)$ , the inverse of the function  $f(x)$ ?



- 11 The graph on the right represents function  $f$

The rule of this function is of the form

$$f(x) = a\sqrt{b(x-h)} + k.$$

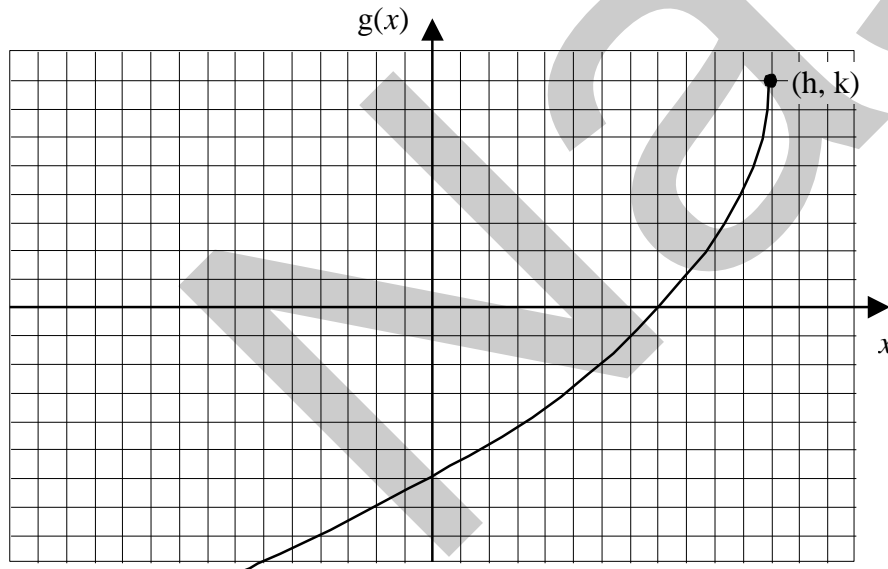


Which of the following statements is FALSE?

- A)  $a \in ]-\infty, 0[$                       C)  $h \in ]0, +\infty [$   
 B)  $b \in ]0, +\infty [$                       D)  $k \in ]0, +\infty [$

- 12 The standard equation of the function graphed below is of the form:

$$g(x) = a\sqrt{b(x-h)} + k.$$



Which of the following is true?

- A)  $a > 0$  and  $b > 0$                       C)  $a < 0$  and  $b < 0$   
 B)  $a > 0$  and  $b < 0$                       D)  $a < 0$  and  $b > 0$

- 13 A real function is defined in the interval  $[0, 9]$  by

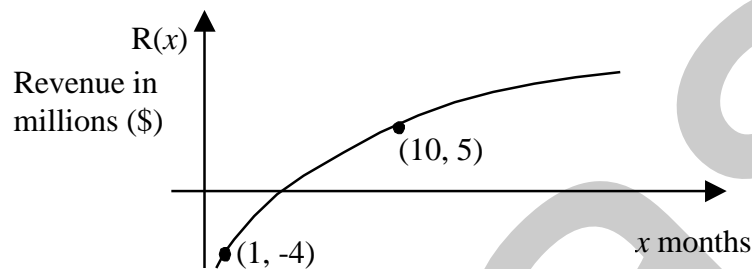
$$f(x) = 2 - \sqrt{9-x}.$$

Draw the graph of this function.

- 14 Given the function defined by the equation  $f(x) = 4 + 2\sqrt{3x + 6}$ .

What is the domain of this function?

- A)  $\mathfrak{R}^+$   
B)  $[-6, \infty)$   
C)  $[-2, \infty)$   
D)  $[4, \infty)$
- 15 After one month in operation, a company's revenue has grown according to the formula  $R(x) = a\sqrt{x - h} + k$  where  $R(x)$  is the profit in millions of dollars after a period of  $x$  months.



Losses amounted to \$400 the first month. After ten months, \$5000 in profits was recorded.

After how many months in operation did the company begin to make a profit?