STATISTICS 466

1 C

2 B

3 Example of an appropriate method

Since these three figures are equivalent, they are equal in area.

Area of the square, the triangle and the rectangle

Are of square LMNO: $(63)^2 = 3969$ cm^2

Area of triangle ONP = Area of rectangle NRQP = Area of square LMNO = 3969 cm^2

Length of segment NP

Area of right triangle ONP: 3969 cm^2

$$\frac{m\,\overline{NP} \times m\,\overline{ON}}{2} = 3969$$

$$\frac{m\,\overline{NP} \times 63}{2} = 3969$$

$$m\,\overline{NP} = 126 \text{ cm}$$

Length of segment PQ

Area of rectangle NRQP: 3969 cm^2

$$m\,\overline{NP} \times m\,\overline{PQ} = 3969$$

$$126 \times m\,\overline{PQ} = 3969$$

$$m\,\overline{PQ} = 31.5 \text{ cm}$$

Answer The length of segment PQ is **31.5** cm.

Note: Students who use an appropriate method in order to determine the length of segment NP have shown that they have a partial understanding of the problem.

4 Example of an appropriate solution

Since the polygons are equivalent, their surface areas are equal.

Area of the rectangle = area of the square

$(2x - 8)(x - 3) = x^2$

$2x^2 - 14x + 24 = x^2$

$x^2 - 14x + 24 = 0$

$(x - 12)(x - 2) = 0$

$x = 12$ or $x = 2$

Dimensions of the rectangle

$(2x - 8)$ cm and $(x - 3)$ cm.

If $x = 12$, the dimensions are 16 cm and 9 cm.

If $x = 2$, the dimensions are -4 and -1 (to be rejected).

Answer The actual dimensions of the rectangle are 16 cm and 9 cm.

5 A

6 Example of an appropriate method

Value of x

$$x^2 + (3x - 12)^2 = 52^2 \qquad \text{(Pythagorean theorem)}$$
$$x^2 + 9x^2 - 72x + 144 = 2704$$
$$10x^2 - 72x - 2560 = 0$$
$$5x^2 - 36x - 1280 = 0$$
$$5x^2 - 100x + 64x - 1280 = 0$$
$$5x(x - 20) + 64(x - 20) = 0$$
$$(x - 20)(5x + 64) = 0$$
$$x = 20 \qquad \text{or} \qquad x = \frac{-64}{5}$$
$$\text{impossible}$$

Area of the triangle

$$\frac{x(3x - 12)}{2} = \frac{20(3(20) - 12)}{2} = \frac{20(48)}{2} = 480 \text{ cm}^2$$

Length of the base of the rectangle

Since the rectangle and the triangle are equivalent, they are equal in area.

Area of the rectangle

480 cm^2

Length of the base of the rectangle

$$\frac{480\ \text{cm}^2}{15\ \text{cm}} = 32\ \text{cm}$$

Answer: The numerical length of the base of the rectangle is **32** cm.

Note: Students who use an appropriate method in order to determine the value of *x* have shown that they have a partial understanding of the problem.

7 D

8 D

9 Example of an appropriate method

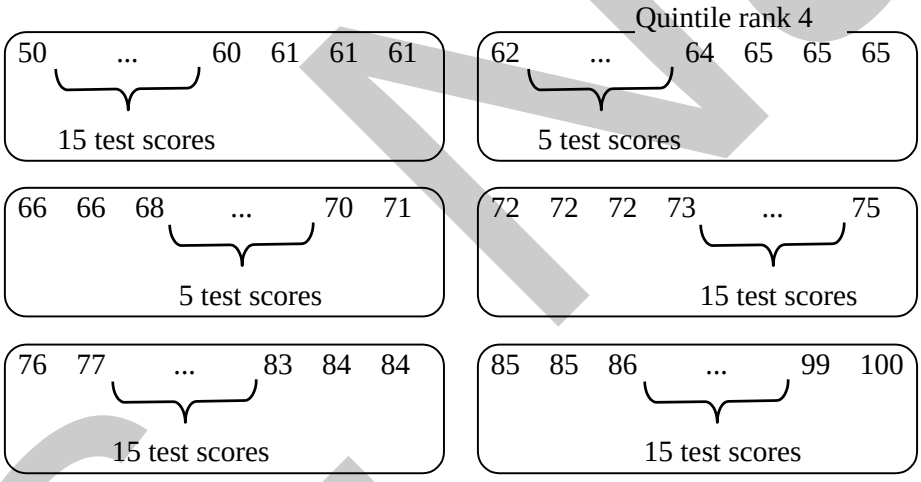
John's score when compared with the scores obtained by the other students in the school board who wrote the test

The 54th percentile rank is between Q_2 and Q_3.

John's score is therefore greater than or equal to 70.5 but less than or equal to 76.

Assigning quintile ranks to the scores obtained by the 100 students in the school

If 100 test scores are divided into 5 groups, there are 20 scores per quintile rank.



John's score is greater than or equal to 62 but less than or equal to 71.

John's score

Of the test scores obtained by the 100 students in the school, the only one between 70.5 and 76 that was assigned a quintile rank of 4 was 71.

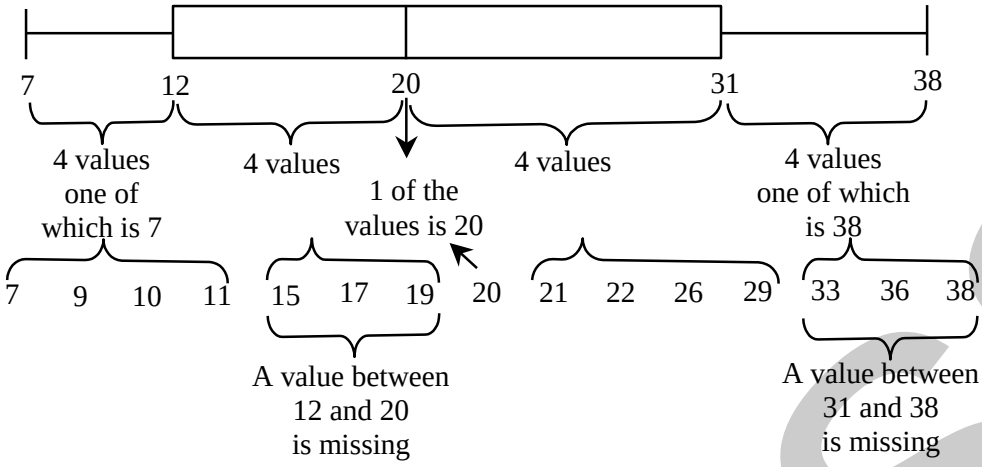
John's score is therefore 71.

Answer: John obtained a score of **71**.

Note: Students have shown that they have a partial understanding of the problem if they use an appropriate method in order to determine that John's score is between 70.5 and 76 **or** to determine which scores are assigned to quintile rank 4.

10 D

11 Example of an appropriate method



The first quartile is halfway between the 4th lowest value and the 5th lowest value in the distribution. The 4th lowest value is 11. The 5th lowest value must therefore be 13.

One of the missing values is 13.

The mean of the missing values is 25.

$$\frac{\text{Sum of the two missing values}}{2} = 25$$

Sum of the two missing values = 50

13 + ? = 50

The other missing value is 37.

Answer: These two missing values are **13** and **37**.

Note: Students have shown that they have a partial understanding of the problem if they use an appropriate method in order to determine that one of the missing values is between 12 and 20 **or** that one of the missing values is between 31 and 38.

12 Carl's score is in the **83rd** percentile.

Note: Accept **82nd** as an answer.

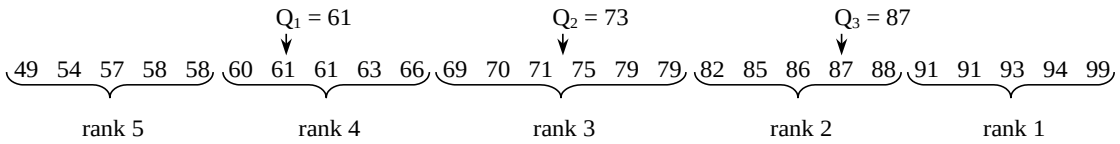
13 D

14 D

15 A

16 78 women aged 30 to 45 should be in the sample.

17 Example of an appropriate method



Sarah's mark

Sarah received a mark of 87, since it is the only quartile that appears only once in the distribution.

Carl's mark

Carl received a mark of 85, since it is the only odd-numbered mark in rank 2 that is different from Sarah's mark.

Answer Sara received a mark of 87.

Carl received a mark of 85.

Note Students who correctly or incorrectly determine Sarah's mark have shown that they have a partial understanding of the problem.

18 Four possible sources of bias with corresponding solutions

Source of bias Size of the sample

Solution: Survey many more than 50 people to get a representative sample of Québec retirees.

Source of bias All those surveyed live in Sherbrooke.

Solution: In the sample, include people from the different regions of Québec.

Source of bias All those surveyed live in a retirement home.

Solution: In the sample, include retirees with different living arrangements (e.g. retirees living in their own home).

Source of bias Age range of the respondents

Solution: Expand the age range of the respondents, since some people retire before age 75.

19 D

20 Example of an appropriate solution

EXPLANATION	MARKS
The maximum value must belong to the distribution.	93
The median is equal to 69.5. Since the distribution consists of 30 marks, the median is the mean of the 15^{th} and 16^{th} values.	69
Since the closet given value is 70, 69 is one of the missing marks.	
Since the distribution consists of 30 marks, Q_3 corresponds to the 8^{th} value if we count down from the maximum value. $Q_3 = 73$.	73
The distribution consists of 30 marks, 7 of which must be less than $Q_1 = 61$. As a result, we are missing a mark between 43 and 61 inclusive.	A value in the interval [43, 61] (e.g. 50).

21 A child whose height is 85 cm should be assigned a percentile rank of 30 or 31.

22 B

23 36 girls from the second cycle should be in the sample.

24 The percentile rank is 78.

25 B

26 D

27 Example of an appropriate solution

Quintile rank:

Group A	Group B
1st rank : 98 97 97 96	1st rank : 98 96 95 94
2nd rank : 91 91 90 **89**	2nd rank : **89** 88 88 87
3rd rank : 87 87 87 86	3rd rank : 85 85 84 79
4th rank : 85 84 83 81	4th rank : 78 78 77 77
5th rank : 79 77 76 75	5th rank : 76 75 75 75

Oliver and George are both in the same quintile.

Percentile rank :

George : $100\left(\frac{12+\frac{1}{2}\times 1}{20}\right)=62.5$ hence 62

Oliver : $100\left(\frac{15+\frac{1}{2}\times 1}{20}\right)=77.5$ hence 77

Final answer Oliver has the better chance of being awarded a bursary.

Name : _____

Group : _____

Date : _____

568436 - Mathematics

Question Booklet

1 Which of the following statements are TRUE?

1. All cubes are similar.

2. All right pyramids whose base is 100 cm^2 and whose height is 8 cm are isometric.

3. If the dimensions of two right prisms with rectangular bases are respectively 4 cm by 5 cm by 6 cm and 3 cm by 4 cm by 10 cm, these prisms are equivalent.

A) 1, 2 and 3

B) 1 and 2 only

C) 1 and 3 only

D) 2 and 3 only

2 Which of the following statements is true?

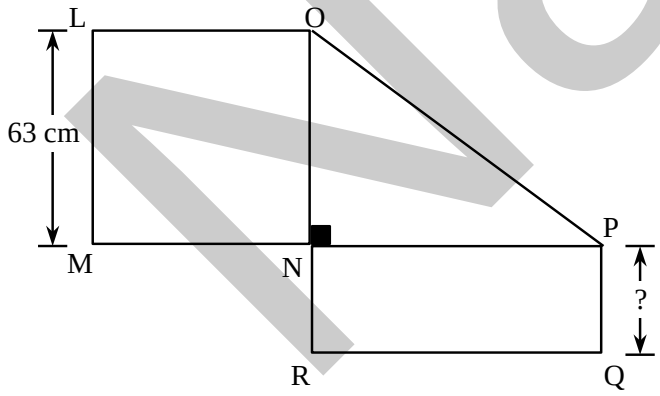
A) Two similar squares are always isometric.

B) Two equivalent squares are always isometric.

C) Two similar triangles are always isometric.

D) Two equivalent triangles are always isometric.

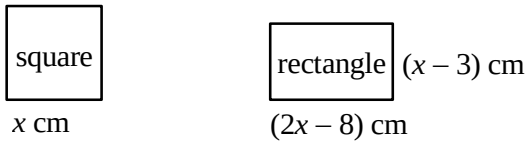
3 Square LMNO, right triangle ONP, and rectangle NRQP given below are equivalent. Segment LM measures 63 cm.



What is the length of segment PQ?

Show all your work.

4 The following square and rectangle are equivalent.



What are the actual dimensions of the rectangle?

Show all your work.

5 Which of the following statements is always true?

A) If two spheres are equivalent, then they are congruent.

B) If two right circular cylinders are equivalent, then they are congruent.

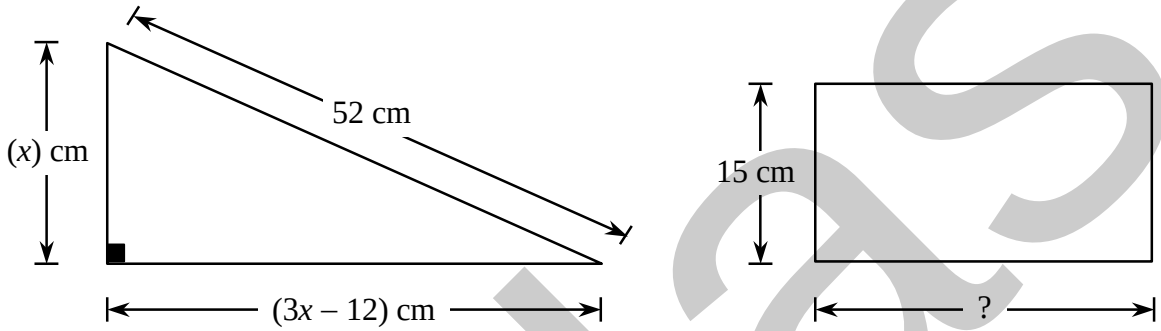
C) If two right pyramids with square bases are equivalent, then they are congruent.

D) If two right rectangular prisms are equivalent, then they are congruent.

6 The right triangle and the rectangle given below are equivalent.

The hypotenuse of the triangle measures 52 cm. The sides of the right angle of the triangle measure (x) cm and ($3x - 12$) cm respectively.

The height of the rectangle is 15 cm.



What is the numerical length of the base of the rectangle?

Show all your work.

7 A school administered a placement test.

Of the 270 candidates who wrote the test:

- no one obtained the same test score as Martin did
- 65 candidates obtained a higher test score than Martin did
- 204 candidates obtained a lower test score than Martin did

In what percentile does Martin's test score fall?

A) 25^{th}

B) 33^{rd}

C) 69^{th}

D) 76^{th}

8 The following table shows the distribution of the 700 people enrolled in three courses offered at a community centre.

	Adolescents	Adults
Cartoon drawing	110	40
Dance	150	120
Photography	140	140

A poll is conducted to determine the extent to which the students are satisfied with these courses. A sample of 210 people is formed. This sample must be representative of the strata shown in the table.

Which of the following statements is true?

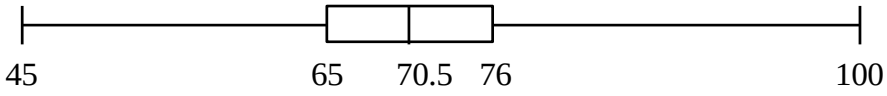
A) There are 105 adults in this sample.

B) There are 35 adolescents from the dance course in this sample.

C) There are 140 people from the photography course in this sample.

D) There are 45 people from the cartoon-drawing course in this sample.

9 In a school board, 600 students wrote a test. The following box-and-whisker plot represents the distribution of their test scores.



The 100 students in a school within this school board wrote this test. The table below shows their test scores in increasing order.

50	...	60	61	61	61	62	...	64	65	65	65			
15 test scores						5 test scores								
66	66	68	...	70	71	72	72	72	73	...	75			
5 test scores									15 test scores					
76	77	...	83	84	84	85	85	86	...	99	100			
15 test scores								15 test scores						

John is a student in this school.

When compared with the scores obtained by the 100 students in his school, John's score was assigned a quintile rank of 4.

When compared with the scores obtained by the 600 students in his school board who wrote the test, John's score is in the 54^{th} percentile rank.

What score did John obtain?

Show all your work.

10 Which of the following describes a census?

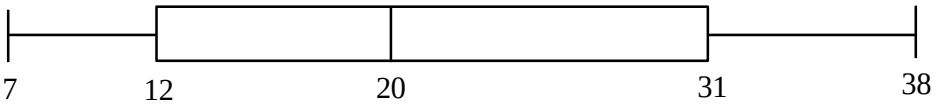
A) Alexander consulted a scientist in order to locate the nesting grounds of turtledoves.

B) Lisa visited some stores in her area in order to determine the cost of a bird feeder.

C) A manufacturer consulted some bird watchers in order to design a pair of binoculars that meets its needs.

D) Claudia questioned each student in her bird watching class in order to find out how many of them owned a pair of binoculars.

11 The following box-and-whisker plot is based on a statistical distribution consisting of 17 data values.



Below are 15 of these 17 values.

7 9 10 11 15 17 19 20 21 22 26 29 33 36 38

The mean of the two missing values is 25.

What are these two missing values?

Show all your work.

12 The scores of the 198 students who sat for a math test are given below in increasing order.

$$\underbrace{50, 50, 52, ..., 79, 79}_{163 \text{ scores}} \qquad \underbrace{80, 81, 81, ..., 99, 100}_{35 \text{ scores}}$$

Carl obtained a score of 80 in this test.

In what percentile is Carl's score?

13 In order to estimate the number of Secondary V students who have a part-time job, a school administration conducts a poll among 60 students.

The following table shows the distribution of the 750 students enrolled in this high school.

Level	Number of Students
Secondary III	300
Secondary IV	250
Secondary V	200

Which of the following random samples would be most representative of the population to be polled?

A) 20 students from each of the three levels

B) 60 students chosen from among the 750 students in this school

C) 60 students chosen from among the 200 Secondary V students

D) 24 Secondary III students, 20 Secondary IV students and 16 Secondary V students

14 Which of the following is an example of a poll?

A) Several tourists ask a guide about the history of a major tourist attraction.

B) A customs officer is asked how many trucks crossed the border yesterday.

C) A guide asks all the passengers on a tour bus to choose the site they would like to visit next.

D) An airline asks a number of passengers to indicate the extent to which they were satisfied with the service provided.

15 The 127 students in a school obtained the following marks on an exam. The marks are arranged in increasing order.

40 40 45 48 … 54 55 55 56 56 57 57 58 59 60 60

16 exam marks

61 … 61 62 63 63 64 64 64 65 66 … 66

12 exam marks 5 exam marks

67 … 68 68 69 69 69 70 70 70 71 71 71 72 72 73

8 exam marks

74 … 82 83 83 83 83 85 86 … 98 98 100

14 exam marks 20 exam marks

Which of the following four statements is true?

A) 65 is the lowest mark assigned a quintile rank of 3.

B) 55 is the highest mark assigned a quintile rank of 5.

C) A student with a mark of 60 is assigned a quintile rank of 2.

D) A student with a mark of 71 is assigned a quintile rank of 3.

16 The following table shows the distribution of 13 000 voters in a city.

Age	Women	Men	Total
[18, 30[	1200	1100	2300
[30, 45[	2600	2300	4900
[45, 60[	1800	2100	3900
[60, +∞[	1000	900	1900
Total	6600	6400	13 000

A sample of 390 people is required. This sample must be representative of the population distribution shown in the table.

How many women aged 30 to 45 should be in the sample?

17 The 26 students in a Math 436 class received the following marks:

49	54	57	58	58	60	61	61	63	66	69
70	71	75	79	79	82	85	86	87	88	91
91	93	94	99							

When Sarah and Carl wanted to know their marks, their teacher gave them the following information:

- Sarah's mark corresponds to one of the quartiles.
- No one else in the class received the same mark as Sarah.
- Carl's mark was assigned the same quintile rank as Sarah's.
- Carl's mark is an odd number.

What marks did Sarah and Carl receive?

Show all your work.

18 A journalist wants to publish the following article in a magazine.

Retired Quebecers Suffer from Loneliness.

This is the main finding of a survey of 50 residents of a Sherbrooke retirement home.

The people surveyed were between 75 and 89 years of age.

Do you feel lonely?		
	Women	*Men*
Yes	*82 %*	*80 %*
No	*18 %*	*20 %*

The editor of the magazine does not approve of the article because this survey contains several sources of bias.

The journalist wants to conduct another survey to ensure that the results are representative of the opinion of all retired people in Québec.

Identify 3 sources of bias in the first survey and indicate how you would solve the problem in each case.

Show all your work.

19 Annie, Claude, Kim and Sam were given their marks on the government history exam.

- Annie's mark was assigned a quintile rank of 5.
- Claude's mark lies in the 60^{th} percentile.
- Kim's mark lies between the first and second quartile.
- Sam's mark is equal to the third quartile.

Which one of these four people had the highest mark on this exam?

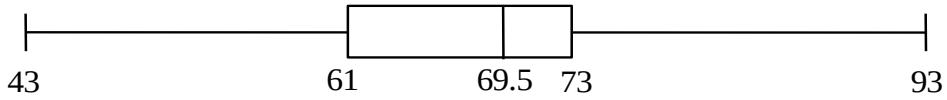
A) Annie

B) Claude

C) Kim

D) Sam

20 The marks on a history test for a class of 30 students are represented by the following box-and-whisker plot.



The following are the marks for 26 of the 30 students:

43	54	55	56	58	60	61	65	66	66
66	67	68	70	71	71	71	71	71	71
74	77	77	79	86	88				

Find the four missing marks.

Explain why you chose each mark.

You will be given 0 for each unexplained answer.

21 The data below shows the height, in centimetres, of 300 children.

$\underbrace{58, \ldots, 83, 84, 84}_{\text{90 children}}$ $\underbrace{85, 86, \ldots, 120}_{\text{210 children}}$

What percentile rank should be assigned to a child whose height is 85 cm?

22 Which of the following describes a census?

A) Alan visits a number of retailers in his area to determine the cost of a swimming pool.

B) A property assessor visits every home in a town to determine the number of residents who own a swimming pool.

C) A swimming pool dealer consults some chemists to find out how disinfectant pellets affect the pH of water.

D) A manufacturer consults some consumers so he can build the type of swimming pool they want.

23 The following table shows the distribution of the student population in a given school.

A sample of 180 students is required. This sample must be representative of the population distribution shown in table.

	Number of girls	Number of boys
First cycle	360	345
Second cycle	240	255

How many girls from the second cycle should be in the sample?

24

During a basketball free throw shooting contest, each participant was allowed to take 20 free throws. The results are recorded in the adjacent chart.

Number of Free Throws

Throws completed	Frequency
20	3
19	6
18	4
16	8
15	2
13	10
11	6
10	3
9	2
8	4
7	1
4	1

What is the percentile rank of 18 successful free throws?

25 On a physics examination, a group of 15 students received the following marks :

48, 53, 54, 58, 61, 63, 72, 73, 75, 78, 80, 82, 88, 90, 92

To which quintile rank does 80 belong?

A) 1^{st}

B) 2^{nd}

C) 4^{th}

D) 5^{th}

26 A school must assign a percentile rank for each Secondary I student's mathematics mark.

The results are shown in the following table.

Mark	Frequency
94	6
92	2
86	5
85	4
82	3
76	7
70	11
68	4
62	10
60	8
54	4
51	1

What is the percentile rank of a student whose mark is 70?

A) 71

B) 67

C) 59

D) 50

27 The following percentages are the marks obtained by two groups of students on a Mathematics exam. There are 20 students in each group.

Marks for students in group A

77 87 89 97 86 91 81 90 75 91

87 79 97 85 76 98 83 96 87 84

Marks for students in group B

76 95 75 84 96 88 79 94 77 87

98 85 75 85 88 78 77 78 75 89

George is in group A. He got 89 %.

Oliver is in group B. He got the same mark as George, 89 %.

The school's Parents Association gives a certain number of awards to deserving students.

From the strength of their position in the group, which of these two students has the better chance of receiving an award?

Show all the work needed to solve the problem.