2- Correction key

1 B

2 C

3 Example of an appropriate method

$$\frac{\text{cosec A} - \sin A}{\cos A} = \cot A$$

$$\frac{\text{cosec A} - \sin A}{\cos A} = \frac{\frac{1}{\sin A} - \sin A}{\cos A}$$

$$= \frac{\frac{1 - \sin^2 A}{\sin A}}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\sin A \times \cos A}$$

$$= \frac{\cos^2 A}{\sin A \times \cos A}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

4 Work : (example)

1. $\frac{2 - 2\sin A \cos A \times \tan A}{2} = \cos^2 A$
2. $\frac{2 - 2\sin^2 A}{2} = \cos^2 A$
3. $\frac{2(1 - \sin^2 A)}{2} = \cos^2 A$
4. $\frac{2 \times \cos^2 A}{2} = \cos^2 A$
5. $\cos^2 A = \cos^2 A$

5 Work : (example)

1. $\frac{2}{\text{cosec a}} + 2 \sin a \tan^2 a = \frac{2 \tan a}{\cos a}$

2. $2 \sin a + 2 \sin a \tan^2 a = \frac{2 \tan a}{\cos a}$

3. $2 \sin a (1 + \tan^2 a) = \frac{2 \tan a}{\cos a}$

4. $2 \sin a \sec^2 a = \frac{2 \tan a}{\cos a}$

5. $2 \sin a \times \frac{1}{\cos^2 a} = \frac{2 \tan a}{\cos a}$

6. $\frac{2 \sin a}{\cos a} \times \frac{1}{\cos a} = \frac{2 \tan a}{\cos a}$

7. $\frac{2 \tan a}{\cos a} = \frac{2 \tan a}{\cos a}$

6 Work : (example)

$$\frac{\sec\theta}{1-\cos\theta} - \frac{\sec\theta}{1+\cos\theta} = 2\operatorname{cosec}^2\theta$$

$$\frac{\sec\theta(1+\cos\theta) - \sec\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} = 2\operatorname{cosec}^2\theta$$

$$\frac{\sec\theta + \sec\theta\cos\theta - \sec\theta + \sec\theta\cos\theta}{(1-\cos\theta)(1+\cos\theta)} = 2\operatorname{cosec}^2\theta \quad \text{as } \sec\theta\cos\theta = 1$$

$$\frac{2}{(1-\cos\theta)(1+\cos\theta)} = 2\operatorname{cosec}^2\theta$$

$$\frac{2}{1-\cos^2\theta} = 2\operatorname{cosec}^2\theta$$

$$\frac{2}{\sin^2\theta} = 2\operatorname{cosec}^2\theta \quad \text{as } \sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

$$2\operatorname{cosec}^2\theta = 2\operatorname{cosec}^2\theta$$

7 Work : (example)

$$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

$$\frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}=2\sec\theta$$

$$\frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}=2\sec\theta$$

$$\frac{1+2\sin\theta+1}{\cos\theta(1+\sin\theta)}=2\sec\theta \qquad (\text{because } \sin^2\theta+\cos^2\theta=1)$$

$$\frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}=2\sec\theta$$

$$\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=2\sec\theta$$

$$\frac{2}{\cos\theta}=2\sec\theta \qquad \left(\text{because } \cos\theta=\frac{1}{\sec\theta}\right)$$

$$2\sec\theta=2\sec\theta$$

8 A

9 Example of an appropriate solution

1. $$\operatorname{cosec} A(\operatorname{cosec} A + \cot A) = \frac{1}{1-\cos A}$$

2. $$\frac{1}{\sin A}\left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right) = \frac{1}{1-\cos A}$$

3. $$\frac{1}{\sin A}\left(\frac{1+\cos A}{\sin A}\right) = \frac{1}{1-\cos A}$$

4. $$\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$$

5. $$\frac{1+\cos A}{1-\cos^2 A} = \frac{1}{1-\cos A}$$

6. $$\frac{1+\cos A}{(1+\cos A)(1-\cos A)} = \frac{1}{1-\cos A}$$

7. $$\frac{1}{1-\cos A} = \frac{1}{1-\cos A}$$

10 The exact values of x are $-\pi$, $\frac{-\pi}{3}$, $\frac{\pi}{3}$, π

11 Example of an appropriate method

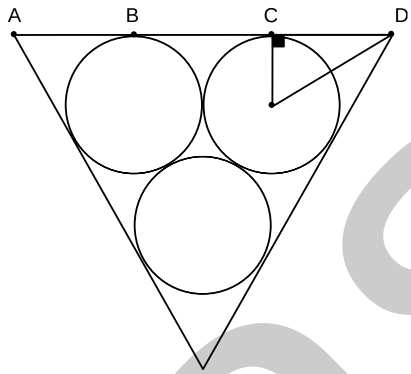
Equation of the circle in standard form

$$x^2 + 6x + y^2 - 2y = 26$$
$$(x-3)^2 + (y-1)^2 = 26 + 9 + 1$$
$$(x-3)^2 + (y-1)^2 = 36$$

Each radius measures 6 units

Since the three circles are congruent, the border forms an equilateral triangle.

According to the diagram on the right,



Note: Adjustments will have to be made for different labelling.

Δ COD is a right triangle, $m\ \overline{OC} = 6$ units and $m\ \angle CDO = 30°$

$$\tan 30° = \frac{m\ \overline{OC}}{m\ \overline{CD}} = \frac{6}{m\ \overline{CD}}$$
$$m\ \overline{CD} = 10.39$$
$$m\ \overline{AD} = m\ \overline{AB} + m\ \overline{BC} + m\ \overline{CD}$$
$$= 10.39 + 12 + 10.39$$
$$32.78$$
$$\text{Perimeter: } P = 3 \bullet 32.78$$
$$= 98.34$$

Answer: To the nearest hundredth of a unit, the border measures **98.34**.

Note: Do not penalize students who did not round the answer.

Students who determined the radius of the circle have shown a partial understanding of the problem.

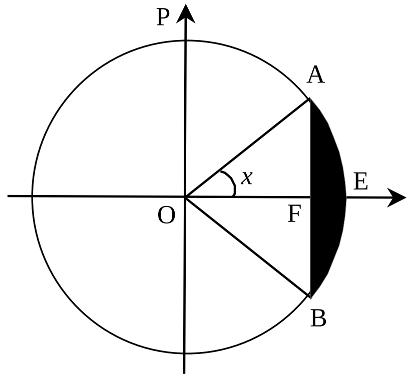
12 Example of an appropriate solution

Measure of angle x

$$\sin x = \frac{\sqrt{3}}{2}$$

$$m\ \angle x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$m\ \angle x = 60° \quad \text{or} \quad m\ \angle x = \frac{\pi}{3} \text{ radians}$$



Base AB of triangle AOB

$$m\ \overline{AB} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

Height FO of triangle AOB

$$m\ \overline{FO} = \cos\ 60° = \frac{1}{2}$$

Area of triangle AOB

$$\text{Area} = \left(\sqrt{3} \times \frac{1}{2}\right) \div 2 = \frac{\sqrt{3}}{4} \approx 0.43 \text{ square units}$$

Area of sector AOB

$$\text{Area of sector AOB} = \pi(1)^2 \times \frac{120°}{360°} \approx 1.05 \text{ square units}$$

Area of the shaded region $\approx 1.05 - 0.43 = 0.62$ square units

Answer: The area of the shaded region is 0.62 square units.

13 C

14 The exact values of x are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.

15 Example of an appropriate solution

$2\cos^2 x - 3\sin x - 3 = 0$

$2(1 - \sin^2 x) - 3\sin x - 3 = 0$

$2 - 2\sin^2 x - 3\sin x - 3 = 0$

$-2\sin^2 x - 3\sin x - 1 = 0$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0 \qquad \sin x + 1 = 0$

$2\sin x = -1 \qquad \sin x = -1$

$\sin x = \dfrac{-1}{2}$

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6} \qquad x = \dfrac{3\pi}{2}$

Answer The values of x in the domain are $\dfrac{11\pi}{6}$ and $\dfrac{3\pi}{2}$.

16 D

17 a) Work : (example)

$\sin^2 x + \cos x = 1$

$$1 - \cos^2 x + \cos x = 1$$
$$\cos^2 x - \cos x = 0$$
$$\cos x\,(\cos x - 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad \cos x = 1$$
$$x = \frac{\pi}{2} \ \text{ or } \ \frac{3\pi}{2} \ \text{ or } \ x = 0$$

Result : The solution set is $\left\{0, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$.

b) Work : (example)

$2\sin^2 x - \cos x - 2 = 0$

$$2(1 - \cos^2 x) - \cos x - 2 = 0$$
$$2 - 2\cos^2 x - \cos x - 2 = 0$$
$$2\cos^2 x + \cos x = 0$$
$$\cos x\,(2\cos x + 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad \cos x = \frac{-1}{2}$$
$$x = \frac{\pi}{2} \ \text{ or } \ \frac{3\pi}{2} \ \text{ or } \ x = \frac{2\pi}{3} \ \text{ or } \ \frac{4\pi}{3}$$

Result : The solution set is $\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$

18 Example of an appropriate method

$$\frac{1}{\sin x} - \sin x$$

$$\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$\frac{1 - \sin^2 x}{\sin x}$$

$$\frac{\cos^2 x}{\sin x}$$

$$\frac{\cos x}{\sin x} \bullet \frac{\cos x}{1}$$

$\cot x \bullet \cos x$

19 Example of an appropriate solution

$\sec t - \cos t\,(\sec^2 t - 1) = \cos t$

$$\frac{1}{\cos t} - \cos t\,(\tan^2 t) = \cos t$$

$$\frac{1}{\cos t} - \cos t\left(\frac{\sin^2 t}{\cos^2 t}\right) = \cos t$$

$$\frac{1}{\cos t} - \frac{\sin^2 t}{\cos t} = \cos t$$

$$\frac{1 - \sin^2 t}{\cos t} = \cos t$$

$$\frac{\cos^2 t}{\cos t} = \cos t$$

$\cos t = \cos t$

TRIGONOMETRIC FUNCTIONS/IDENTIES
SINE FUNCTION   COSINE FUNCTION

1 Which of the following is true for $f(x) = 2\cos\left(x + \frac{\pi}{3}\right) - 1$?

A) It increases on $\left[\frac{\pi}{6}, \frac{2\pi}{3}\right]$ and decreases on $\left[\frac{2\pi}{3}, \frac{7\pi}{6}\right]$

B) It decreases on $\left[\frac{\pi}{6}, \frac{2\pi}{3}\right]$ and increases on $\left[\frac{2\pi}{3}, \frac{7\pi}{6}\right]$

C) It increases on $\left[\frac{-\pi}{3}, \frac{\pi}{6}\right]$ and decreases on $\left[\frac{5\pi}{3}, 2\pi\right]$

D) It decreases on $\left[\frac{-\pi}{3}, \frac{\pi}{6}\right]$ and increases on $\left[\frac{5\pi}{3}, 2\pi\right]$

2 In which of the following intervals does the cosine function increase?

A) $[0, \pi]$

C) $[\pi, 2\pi]$

B) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

D) $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$

3 Prove the given trigonometric identity.

$$\frac{\text{cosec A} - \sin \text{A}}{\cos \text{A}} = \cot \text{A}$$

4 Suzanne is a sea captain. Her knowledge of trigonometry helps her calculate distances at sea. She must simplify a trigonometric expression to verify that her calculations are correct.

Show that Suzanne's calculations are correct by proving the following identity :

$$\frac{2 - 2 \sin \text{A} \cos \text{A} \times \tan \text{A}}{2} = \cos^2 \text{A}$$

5 Prove the following identity :

$$\frac{2}{\text{cosec a}} + 2 \sin \text{a} \tan^2 \text{a} = \frac{2 \tan \text{a}}{\cos \text{a}}$$

6 Prove the following identity :

$$\frac{\sec\theta}{1-\cos\theta}-\frac{\sec\theta}{1+\cos\theta}=2\,\text{cosec}^2\theta.$$

Show your work.

7 Prove the following identity :

$$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta.$$

Show your work.

8 The trigonometric expression $\dfrac{\text{cosec}\ x}{\tan x+\cot x}$ is equivalent to

A) $\cos x$

B) $\sin x$

C) $\sec x$

D) $\text{cosec}\ x$

9 Prove the following trigonometric identity :

$$\text{cosec A}\,(\text{cosec A}+\cot\text{A})=\frac{1}{1-\cos\text{A}}$$

Show your work.

10 Given the trigonometric equation:

$$2\cos^2 x+\cos x=1,\quad x\in[-\pi,\pi]$$

What are the exact values of x that satisfy this equation?

13 A kayaker is drifting on the Atlantic. The ocean is relatively calm and the movements of the waves can be represented by the equation below,

$$h(t)=2\sin\frac{2\pi}{9}\left(t-\frac{\pi}{3}\right)$$

where t represents the time in seconds and $h(t)$ represents the height in metres.

In one minute, how many times did the kayaker reach the top of a wave?

A) 14 times

B) 9 times

C) 7 times

D) 6 times

14 Given the following trigonometric equation:

$$2\sin^2 x + 5\sin x - 3 = 0,\ x \in [0, 2\pi]$$

What are the exact values of x that satisfy this equation?

15 Find all the solutions of the equation $2\cos^2 x - 3\sin x - 3 = 0$ such that $x \in \left[\frac{3\pi}{2}, 2\pi\right]$. Give your solution in radian measure.

16 What is the solution set of the equation $2\sin^2 x - 4\cos^2 x + 1 = 0$ where $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$?

A) $\left\{\frac{-\pi}{6}, \frac{\pi}{6}\right\}$

B) $\left\{\frac{-\pi}{3}, \frac{\pi}{3}\right\}$

C) $\left\{\frac{\pi}{4}\right\}$

D) $\left\{\frac{-\pi}{4}, \frac{\pi}{4}\right\}$

17 If $x \in [0, 2\pi[$, find the solution set of the following equations :

a) $\sin^2 x + \cos x = 1$

b) $2\sin^2 x - \cos x - 2 = 0$

18 Prove the following trigonometric identity.

$$\frac{1}{\sin x} - \sin x = \cot x \cos x$$

19 Prove the following trigonometric identity:

$$\sec t - \cos t\,(\sec^2 t - 1) = \cos t$$