

1 B

2 Example of an appropriate method

Length of segment AQ

$$(m \overline{AQ})^2 = (m \overline{AP})^2 + (m \overline{PQ})^2 - 2(m \overline{AP})(m \overline{PQ}) \cos (m \angle APQ)$$

$$(m \overline{AQ})^2 = 7^2 + 9^2 - 2(7)(9) \cos 116^\circ$$

$$m \overline{AQ} \approx 13.61 \text{ cm}$$

Measure of angle PAQ

$$\frac{\sin (m \angle PAQ)}{m \overline{PQ}} = \frac{\sin (m \angle APQ)}{m \overline{AQ}}$$

$$\frac{\sin (m \angle PAQ)}{9} = \frac{\sin 116^\circ}{13.61}$$

$$m \angle PAQ \approx 36.4665^\circ$$

Length of segment BC

$$\sin(m \angle BAC) = \frac{m \overline{BC}}{m \overline{AB}}$$

$$\sin 36.4665^\circ = \frac{m \overline{BC}}{m \overline{AP} + m \overline{PB}}$$

$$\sin 36.4665^\circ = \frac{m \overline{BC}}{22}$$

$$m \overline{BC} \approx 13.076 \text{ cm}$$

Answer: To the nearest centimetre, the length of segment BC is **13** cm.

Note: Do not penalize students who did not round off the final answer or who made a mistake in rounding it off.

Students who use an appropriate method in order to determine the length of segment AQ have shown that they have a partial understanding of the problem.

3 Example of an appropriate method

Length of segment BC

$$\sin 36^\circ = \frac{m\overline{QC}}{m\overline{BC}}$$

$$\sin 36^\circ = \frac{0.69}{m\overline{BC}}$$

$$m\overline{BC} \approx 1.174$$

Length of segment BQ

$$\tan 36^\circ = \frac{m\overline{QC}}{m\overline{BQ}}$$

$$\tan 36^\circ = \frac{0.69}{m\overline{BQ}}$$

$$m\overline{BQ} \approx 0.95$$

Length of segment CS

$$m\overline{SR} = m\overline{PQ} \approx 0.55 + 0.95 = 1.5 \text{ since PQRS is a rectangle.}$$

$$\sin 33^\circ = \frac{m\overline{SR}}{m\overline{CS}}$$

$$\sin 33^\circ \approx \frac{1.5}{m\overline{CS}}$$

$$m\overline{CS} \approx 2.754$$

Distance travelled by the ball

$$m\overline{BC} + m\overline{CS}$$

$$1.174 + 2.754$$

$$3.928$$

Answer: This ball travels a distance of **3.9** m, to the nearest tenth.

Notes: Do not penalize students who did not round off their final answer or who made a mistake in rounding it off.

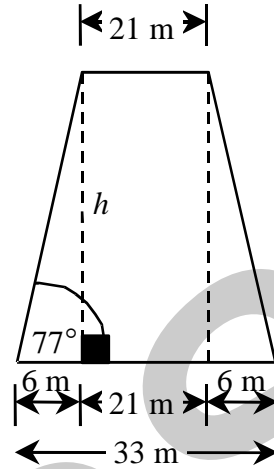
Students who use an appropriate method in order to determine the length of segment BC or the length of segment BQ have shown that they have a partial understanding of the problem.

4 Example of an appropriate solution

Height of the trapezoid

$$\tan 77^\circ = \frac{h}{6}$$

$$h \approx 25.989 \text{ m}$$



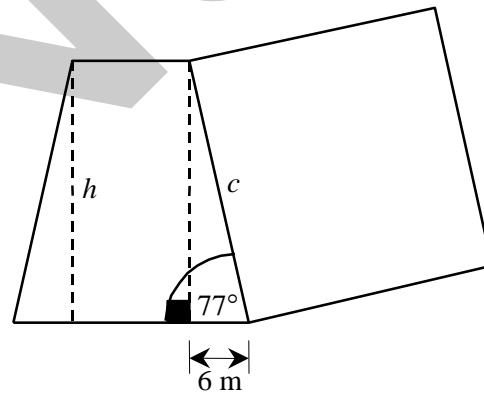
Area of the trapezoid

$$\frac{(21 + 33) \times 25.989}{2} \approx 701.703 \text{ m}^2$$

Length of the side of the square

$$\cos 77^\circ = \frac{6}{c}$$

$$c \approx 26.672 \text{ m}$$



Area of the square

$$26.672 \times 26.672 \approx 711.396 \text{ m}^2$$

Difference between the areas

$$711.396 - 701.703 \approx 9.693$$

Answer The difference between the areas of these two lots to the nearest square metre is 10 m^2 .

5 Example of an appropriate solution

Length of \overline{AB}

$$m \overline{AB} = d(A, B)$$

$$m \overline{AB} = \sqrt{(17 - 6)^2 + (9 - 20)^2}$$

$$m \overline{AB} = \sqrt{242}$$

$$m \overline{AB} \approx 15.5563$$

Length of \overline{BC}

$$m \overline{BC} = d(B, C)$$

$$m \overline{BC} = \sqrt{(17 - 14)^2 + (9 - 5)^2}$$

$$m \overline{BC} = \sqrt{25}$$

$$m \overline{BC} = 5$$

Length of \overline{AC}

$$m \overline{AC} = d(A, C)$$

$$m \overline{AC} = \sqrt{(14 - 6)^2 + (5 - 20)^2}$$

$$m \overline{AC} = \sqrt{289}$$

$$m \overline{AC} \approx 17$$

Measure of angle C

$$(m \overline{AB})^2 = (m \overline{BC})^2 + (m \overline{AC})^2 - 2 \times m \overline{BC} \times m \overline{AC} \times \cos C \text{ (Law of Cosines)}$$

$$242 = 25 + 289 - 2 \times 5 \times 17 \times \cos C$$

$$242 = 314 - 170 \times \cos C$$

$$-72 = -170 \times \cos C$$

$$0.4235 \approx \cos C$$

$$m \angle C \approx 64.94^\circ$$

Answer The measure of angle C is approximately 65° .

6 Example of an appropriate method

Measure of angle PRS

$$\frac{\sin \angle PRS}{20} = \frac{\sin 120^\circ}{27}$$

$$m \angle PRS \approx 39.9^\circ$$

Measure of angle RPS

$$m \angle RPS = 180^\circ - 120^\circ - m \angle PRS \approx 20.1^\circ$$

Length of segment PQ

$$\frac{m \overline{PQ}}{\sin 120^\circ} = \frac{6}{\sin \angle RPS}$$

$$m \overline{PQ} \approx 15.12 \text{ cm}$$

Answer The length of segment PQ to the nearest tenth is 15.1 cm.

Note Do not penalize students who did not round off their final answer or who made a mistake in rounding it off.

The following are the steps in another appropriate method:

- ▶ $m \overline{SR}$ using the law of cosines;
- ▶ $m \overline{PQ}$ using the ratio of similitude because triangles PQT and PRS are similar.

Students who correctly or incorrectly determine the measure of angle PRS **or** the length of segment SR have shown that they have a partial understanding of the problem.

7 C

8 Example of an appropriate solution

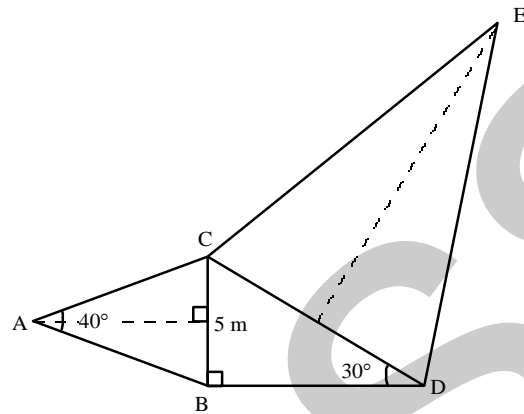
Measure of segment CD

$$m \overline{CD} = 2 \times m \overline{BC} = 10$$

Measure of angle CED

$$m \angle CED = m \angle BAC = 40^\circ$$

car $\triangle BAC \sim \triangle CED$



Measure of angle CEG

$$m \angle CEG = \frac{1}{2} m \angle CED = \frac{40^\circ}{2} = 20^\circ$$

Measure of segment CG

$$m \overline{CG} = \frac{1}{2} m \overline{CD} = \frac{10}{2} = 5$$

Measure of segment EG

$$\tan 20^\circ = \frac{5}{m \overline{EG}}$$

$$m \overline{EG} = \frac{5}{\tan 20^\circ} \approx 1374$$

Area of triangle ECD

$$\text{Area of } \triangle ECD \approx \frac{13.74 \times 10}{2} = 68.7$$

Final answer Area of triangle ECD is 68.7 m².

Accept any answer in the interval [67, 69].

9 Work : (example)

The measurement of \overline{BF} : $16.5 + 7.32 = 23.82$

The measurement of angle BAE :

$$\tan \angle BAE = \frac{m \overline{BE}}{m \overline{AB}} = \frac{16.5}{16.5} = 1$$

$$m \angle BAE = 45^\circ$$

or 45° since the
right triangle
is isosceles

The measurement of angle BAF :

$$\tan \angle BAF = \frac{m \overline{BF}}{m \overline{AB}} = \frac{23.82}{16.5}$$

$$m \angle BAF \approx 55.29^\circ$$

The measurement of angle EAF : $55.29^\circ - 45^\circ = 10.29^\circ$

Result The measure of angle FAE, (to the nearest degree) is 10 degrees.

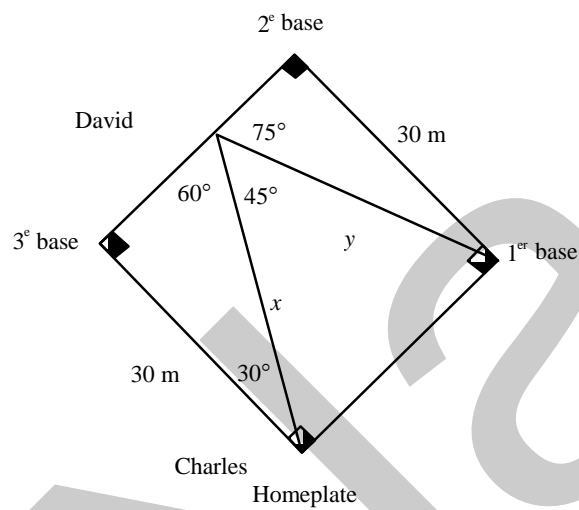
Any other complete and acceptable work with the correct result.

10 Work : (example)

Given x : the distance between Charles and David.

y : the distance between David and first base.

The distance between Charles and David.



$$\cos 30^\circ = \frac{30}{x}$$

$$x = \frac{30}{\cos 30^\circ}$$

$$x = \frac{30}{0.866}$$

$$x = 34.642$$

The distance between David and First base.

$$\sin 75^\circ = \frac{30}{y}$$

$$y = \frac{30}{\sin 75^\circ}$$

$$y = \frac{30}{0.966}$$

$$y = 31.066$$

The distance covered by the ball.

$$x + y = 34.642 + 31.066 = 65.708$$

Result To the nearest metre, the distance covered by the ball is 66 metres.

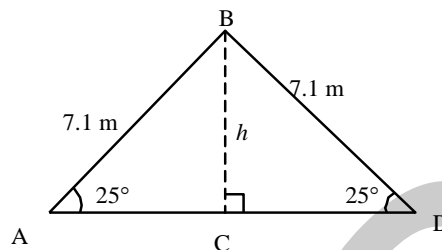
Any other complete and acceptable work with the correct result.

11

Work : (example)

Properties :

- 1) The sum of the interior angles of a triangle is 180° .
- 2) An isosceles triangle is also equiangular.



By drawing a height h from vertex B , 2 congruent right triangles are formed.

According to property 1, the measure of angle ABC is $180^\circ - (90^\circ + 25^\circ)$ or 65° .

The height h of the triangle can be found using the cosine.

$$\cos 65^\circ = \frac{h}{7.1 \text{ m}}$$

$$h = 7.1 \times \cos 65^\circ$$

$$h = 7.1 \times 0.4226$$

$$h = 3.00046 \text{ m}$$

The measure of base AD of triangle ABD can also be found using a trigonometric ratio.

Since $m \overline{AD} = m \overline{AC} + m \overline{CD} = 2 \times m \overline{AC}$ find the measure of segment \overline{AC} .

$$\sin 65^\circ = \frac{\text{m } \overline{AC}}{7.1 \text{ m}}$$

$$\text{m } \overline{AC} = 7.1 \text{ m} \times \sin 65^\circ$$

$$\text{m } \overline{AC} = 7.1 \text{ m} \times 0.9063$$

$$\text{m } \overline{AC} = 6.43473 \text{ m}$$

$$\text{m } \overline{AD} = 2 \times \text{m } \overline{AC} = 2 \times 6.43473 \text{ m} = 12.86946 \text{ m}$$

The measure of the area of this part of the roof can be found using the following formula.

$$\begin{aligned} \text{Area} &= \frac{\text{base} \times \text{height}}{2} \\ &= \frac{12.86946 \text{ m} \times 3.00046 \text{ m}}{2} \\ &= 19.30714998 \text{ m}^2 \approx 19 \text{ m}^2 \end{aligned}$$

Result The area of the part of the roof shown above, to the nearest square metre, is 19 m^2 .

12

Work : (example)

Measure of d_1

$$\frac{d_1}{30} = \tan 28^\circ$$

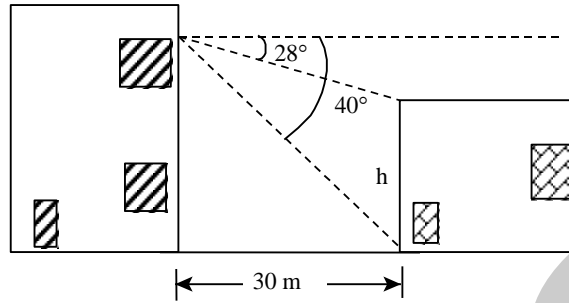
Where $d_1 = 30 \tan 28^\circ \approx 15.95$ mMeasure of d_2

$$\frac{d_2}{30} = \tan 40^\circ$$

Where $d_2 = 30 \tan 40^\circ \approx 25.17$ mHeight h of the house

$$h = d_2 - d_1 = 25.17 \text{ m} - 15.95 \text{ m} = 9.22 \text{ m}$$

Result The height of your neighbour's house is 9.22 m.



13 Work : (example)

In $\triangle ABC$

$$\sin 40^\circ = \frac{m \overline{BC}}{13}$$

$$\begin{aligned} m \overline{BC} &= 13 \sin 40^\circ \\ &\approx 8.36 \end{aligned}$$

In $\triangle BCD$

$$\sin 50^\circ = \frac{m \overline{BD}}{m \overline{BC}} \approx \frac{m \overline{BD}}{8.36}$$

$$\begin{aligned} m \overline{BD} &\approx 8.36 \times \sin 50^\circ \\ &\approx 6.4 \end{aligned}$$

Result The measure of segment BD is 6.4 m.

14 Work : (example)

$$\text{Height } h \text{ of the cone : } \tan 35^\circ = \frac{10}{h}$$

$$h \approx 14.281\ 48$$

Volume V of the pile of gravel :

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{\pi \times 10^2 \times 14.281\ 48}{3}$$

$$V \approx 1495.553\ 1\ \text{m}^3$$

Length L of the highway :

$$L = \frac{\text{volume}}{\text{width} \times \text{thickness}}$$

$$L = \frac{1495.553\ 1}{0.1 \times 8}$$

$$L \approx 1869.441\ 4$$

Result Rounded to the nearest whole number, the length of highway that can be covered is 1869 m.

NOTE : Accept any answer in the interval [1868, 1870].

15

Work : (example)

Measure of the sides of the base

$$c = \sqrt{A}$$

$$c = \sqrt{64} = 8 \text{ cm}$$

Measure of segment HF

$$m \overline{HF} = \frac{m \overline{AB}}{2} = 4 \text{ cm}$$

Measure of segment EF

- a) Segment EF, which is an altitude in triangle EBC, divides it into two congruent right-angled triangles.

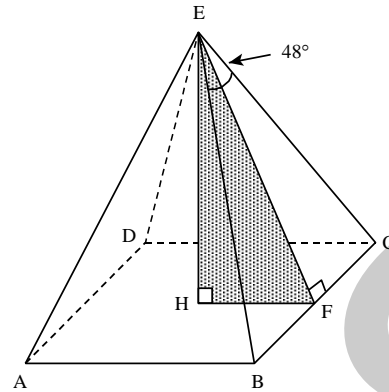
$$\text{Therefore, } m \overline{FC} = \frac{m \overline{BC}}{2} = 4 \text{ cm}$$

$$m \angle FEC = \frac{m \angle BEC}{2} = 24^\circ$$

$$\text{b) } \tan 24^\circ = \frac{m \overline{FC}}{m \overline{EF}}$$

$$\tan 24^\circ = \frac{4}{m \overline{EF}}$$

$$m \overline{EF} \approx 8.98 \text{ cm}$$



Measure of segment EH

$$(m \overline{EH})^2 = (m \overline{EF})^2 - (m \overline{HF})^2$$

$$(m \overline{EH})^2 \approx 8.98^2 - 4^2$$

$$m \overline{EH} \approx 8.04 \text{ cm}$$

c) Volume of the pyramid

$$V = \frac{\text{Area of base} \times \text{height}}{3}$$

$$V \approx \frac{64 \times 8.04}{3}$$

$$V \approx 171.6 \text{ cm}^3$$

Result The volume of the pyramid is approximately 171.6 cm^3 .

16

Work : (example)

In triangle ABEMeasure of $\angle BAE$

$$\sin \angle BAE = \frac{4.8}{8}$$

$$m \angle BAE \approx 36.87^\circ$$

Measure of \overline{AE}

$$m \overline{AE} = \sqrt{8^2 - 4.8^2} = 6.4$$

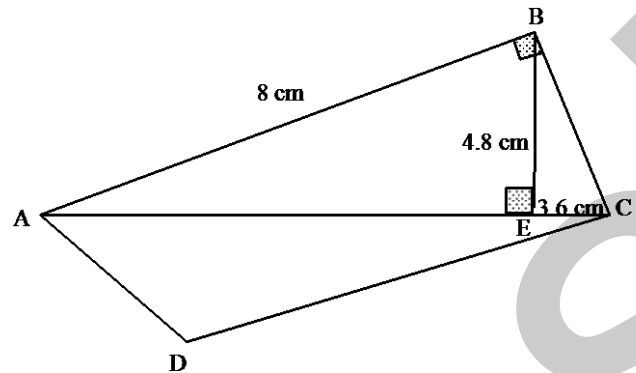
In triangle BECMeasure of \overline{BC}

$$(m \overline{BC})^2 = (m \overline{BE})^2 + (m \overline{EC})^2$$

$$m \overline{BC} = \sqrt{4.8^2 + 3.6^2} = 6$$

In triangle ADCMeasure of \overline{AC}

$$m \overline{AC} = 6.4 + 3.6 = 10$$



Measure of $\angle CAD$

$$m \angle CAD = 2 \times m \angle BAE$$

$$m \angle CAD \approx 2 \times 36.87^\circ = 73.74^\circ$$

Measure of \overline{AD}

$$m \overline{AD} = m \overline{BC} = 6$$

Using the law of cosines to find \overline{CD}

$$(m \overline{CD})^2 = (m \overline{AC})^2 + (m \overline{AD})^2 - 2(m \overline{AC})(m \overline{AD}) \cos \angle CAD$$

$$(m \overline{CD})^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \times \cos 73.74^\circ$$

$$m \overline{CD} \approx 10.12 \text{ cm}$$

Result : Segment DC is approximately 10.12 cm long.

17

B

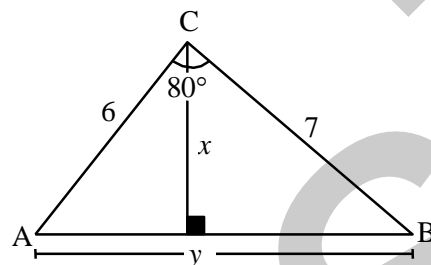
18

B

19

A

Mrs. Næss



Measure of \overline{AB} :

$$y^2 = 6^2 + 7^2 - 2(6)(7) \cos 80^\circ \quad (\text{cosine law})$$

$$y^2 = 36 + 49 - 84 \cos 80^\circ$$

$$y = 8.39$$

Measure of $\angle A$:

$$\frac{8.39}{\sin 80^\circ} = \frac{7}{\sin A} \quad (\text{sine law})$$

$$\sin A = \frac{7 \sin 80^\circ}{8.39} = 0.8216$$

$$m \angle A \approx 55.2^\circ$$

Measure of x :

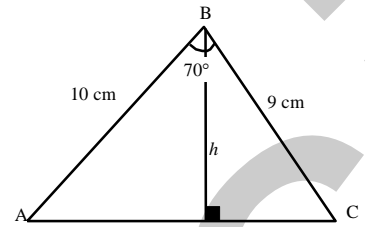
$$\sin 55.2^\circ = \frac{x}{6}$$

$$x = 6 \sin 55.2^\circ \approx 4.9292$$

Result : The value of x to the nearest hundredth is 4.93.

Mrs. Nassif

21 Work : (example)



Measure of \overline{AC} :

$$(\text{m } \overline{AC})^2 = 10^2 + 9^2 - 2(10)(9) \cos 70^\circ \quad (\text{cosine law})$$

$$(\text{m } \overline{AC})^2 = 100 + 81 - 180 \cos 70^\circ$$

$$\text{m } \overline{AC} \approx 10,92 \text{ cm}$$

Measure of $\angle A$:

$$\frac{10,92}{\sin 70^\circ} = \frac{9}{\sin A} \quad (\text{sine law})$$

$$\sin A = \frac{9 \sin 70^\circ}{10,92} \approx 0,7739$$

$$\text{m } \angle A \approx 50,7^\circ$$

Measure of the height :

$$\sin A = \frac{h}{10}$$

$$h = 10 \times \sin 50,7^\circ \approx 7,738 \text{ cm}$$

Result : The measure of height "h" to the nearest hundredth is 7.74 cm.

Mrs. Nassif

22

Work : (example)

Measure of Angle CAB

$$20^2 = 6^2 + 25^2 - 2 \times 6 \times 25 \times \cos A$$

$$400 = 36 + 625 - 300 \times \cos A$$

$$400 = 661 - 300 \times \cos A$$

$$400 - 661 = -300 \times \cos A$$

$$\cos A = \frac{-261}{-300} = 0.87$$

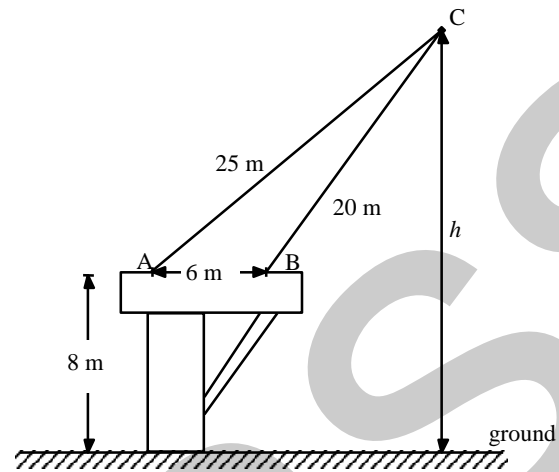
$$m \angle A \approx 29.54^\circ$$

Height (h_1)

$$\frac{h_1}{25} \approx \sin 29.54^\circ$$

$$h_1 \approx 25 \times \sin 29.54^\circ$$

$$h_1 \approx 12.33$$



Height (h)

$$h = h_1 + h_2$$

$$h \approx 12.33 + 8 = 20.33$$

Result : The end of the crane, point C, is 20.3 m above the ground.

Name : _____

Group : _____

Date : _____

568436 - Mathematics

Question Booklet

- 1 In the diagram on the right, triangles LMN and PLN are similar.

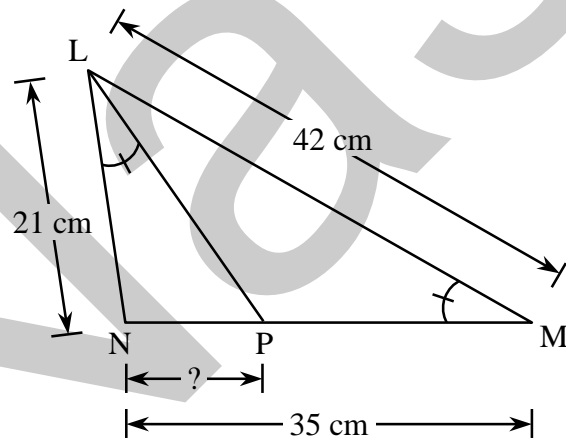
In addition:

$$m \overline{NL} = 21 \text{ cm}$$

$$m \overline{NM} = 35 \text{ cm}$$

$$m \overline{LM} = 42 \text{ cm}$$

$$m \angle NLP = m \angle NML$$



What is the length of segment NP to the nearest tenth of a centimetre?

A) 10.5 cm

C) 13.9 cm

B) 12.6 cm

D) 17.5 cm

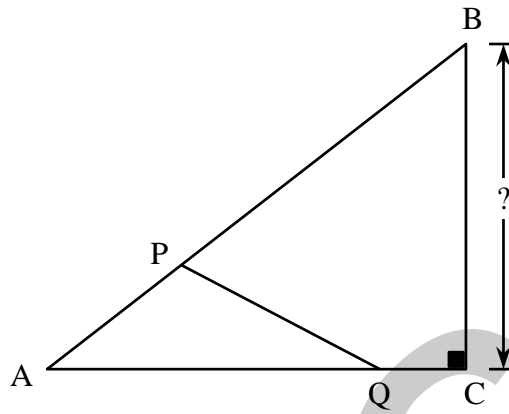
2 Line segment PQ was drawn in right triangle ACB given below.

$$m \angle APQ = 116^\circ$$

$$m \overline{PQ} = 9 \text{ cm}$$

$$m \overline{AP} = 7 \text{ cm}$$

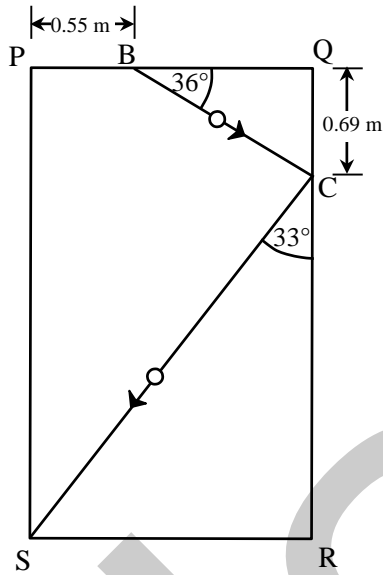
$$m \overline{PB} = 15 \text{ cm}$$



What is the length of segment BC to the nearest centimetre?

Show all your work.

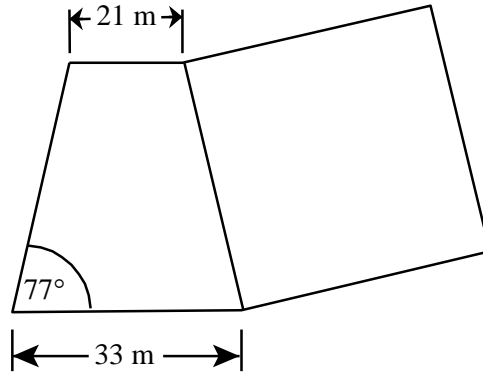
- 3 Rectangle PQRS below represents the surface of a pool table. A pool player makes a trick shot. Line segments BC and CS represent the path of one of the balls after the shot was made.



What distance does this ball travel to the nearest tenth of a metre?

Show all your work.

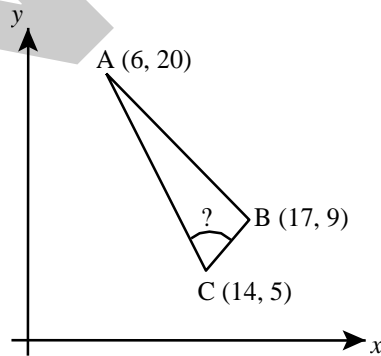
- 4 Two adjacent lots are represented by the isosceles trapezoid and square shown below.



What is the difference between the areas of these two lots to the nearest square metre?

Show all your work.

- 5 Given triangle ABC shown on the right.



What is the measure of angle C to the nearest degree?

Show all your work.

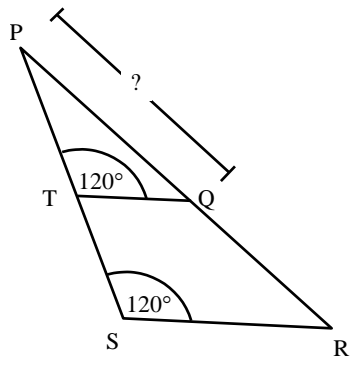
6

In the figure on the right,

$$m \overline{PS} = 20 \text{ cm,}$$

$$m \overline{PR} = 27 \text{ cm,}$$

$$m \overline{TQ} = 6 \text{ cm.}$$



What is the length of segment PQ to the nearest tenth?

Show all your work.

7

In a given figure,

- lines L_1 and L_2 are parallel;
- segment BD is perpendicular to line L_1 ;
- point B is on line L_1 and point D is on line L_2 ;
- C is the midpoint of segment BD;
- transversal AE passes through point C and forms an angle of 60° with segment BD;
- this transversal intersects line L_1 at A and line L_2 at E;
- segment BD measures 12 cm.

Rounded to the nearest tenth of a centimetre, what is the perimeter of triangle CDE?

A) 16.4 cm

C) 28.4 cm

B) 20.5 cm

D) 31.4 cm

8

In the adjacent diagram,

$\triangle ABC$ is similar to $\triangle ECD$,

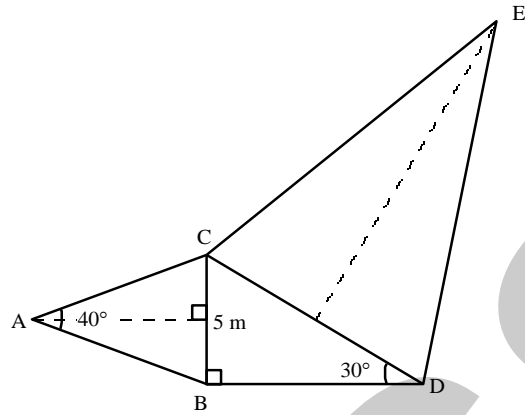
$\triangle ABC$ is isosceles and

$\triangle BCD$ is right angled at B

$m \overline{CB} = 5 \text{ m}$,

$m \angle CAB = 40^\circ$ and

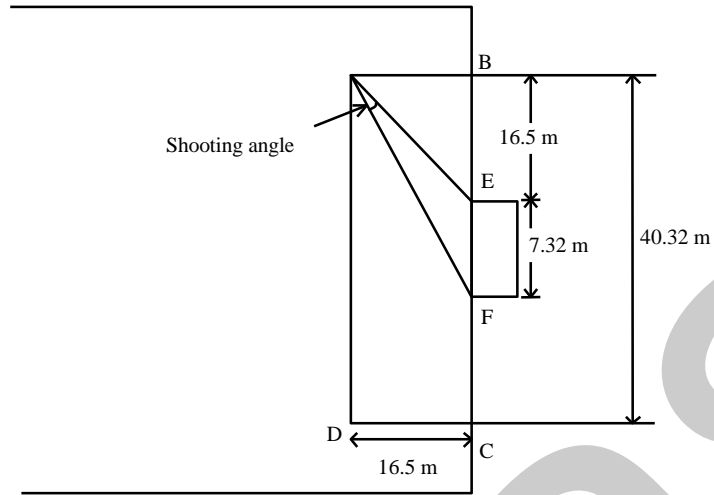
$m \angle CDB = 30^\circ$.



What is the area of triangle ECD?

Show all the work needed to solve the problem.

- 9 On a soccer field the region ABCD is marked by a white rectangle and EF represents the goal.

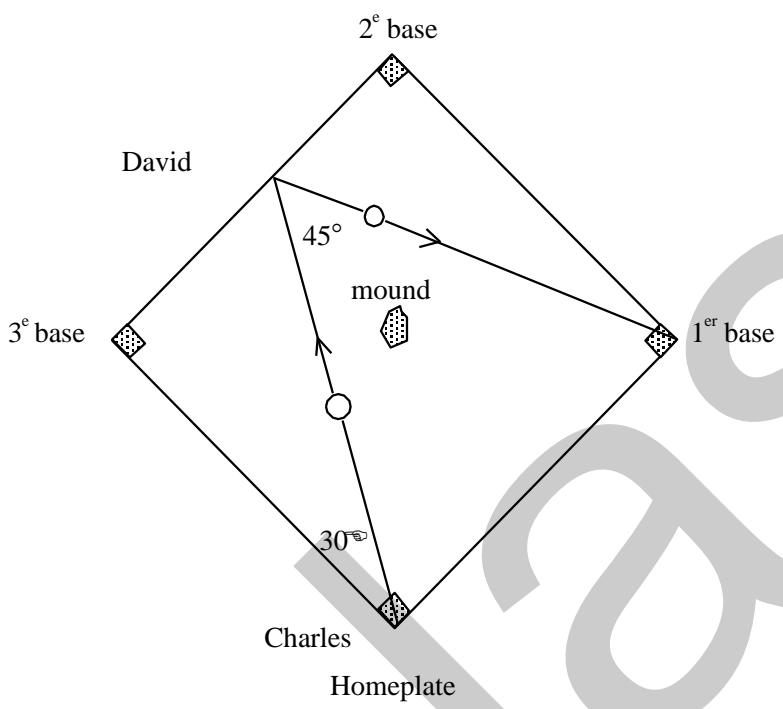


Using the given measurements, find the shooting angle FAE, (to the nearest degree) of a player at position A.

Show all your work.

10

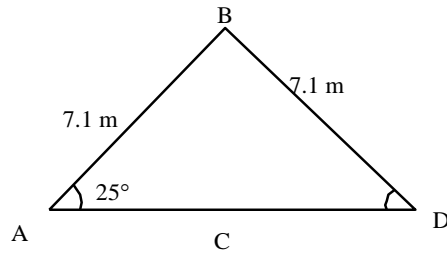
The bases of a baseball diamond form a square 30 m on each side. Charles comes to bat and hits a line drive which makes a 30° angle with the third base line. David catches the ball, pivots 45° and throws the ball to first base.



To the nearest metre, find the distance covered by the ball after it was hit.

Show all your work.

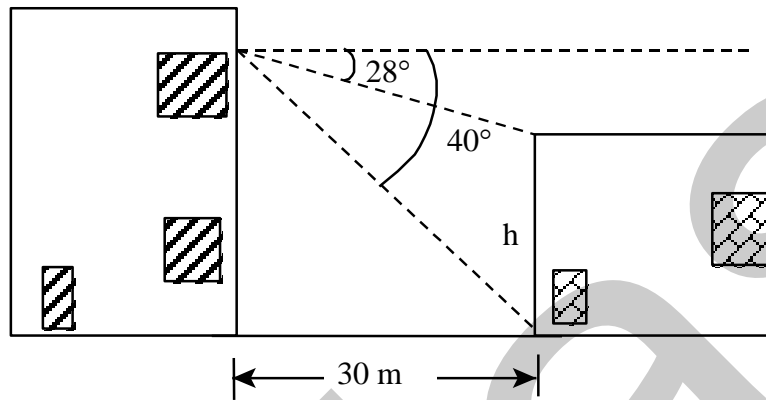
- 11 The angle of inclination of a roof is 25° . If each side of the roof measures 7.1 m, what is the area of the cross section of the roof shown below, to the nearest square metre?



Show your work.

- 12 You live on the second floor of a building. Your neighbour's house faces you on the opposite side of the street.

From your vantage point, if you look at the top of your neighbour's house, the angle of depression is 28° . If you look at the base of the house, the angle of depression is 40° .



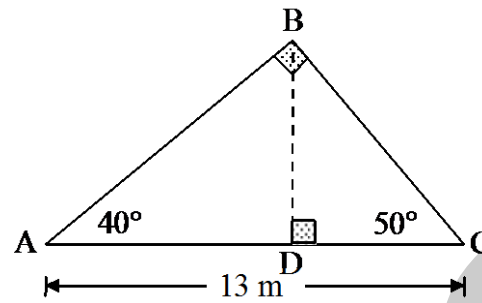
What is the height "h" of your neighbour's house if the width of the street is 30 metres?

Show your work.

- 13 Right triangle ABC is illustrated at the right.

Segment BD is the height.

What is the measure of segment BD to the nearest tenth of a metre?

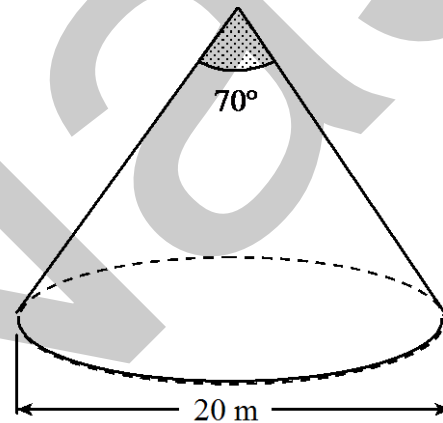


Show your work.

- 14 A pile of fine gravel is the shape of a cone. The diameter of the base measures 20 m. The angle at the vertex of the cone is 70° .

A layer of this gravel is spread along a highway which is 8 m wide. The layer is 10 cm thick.

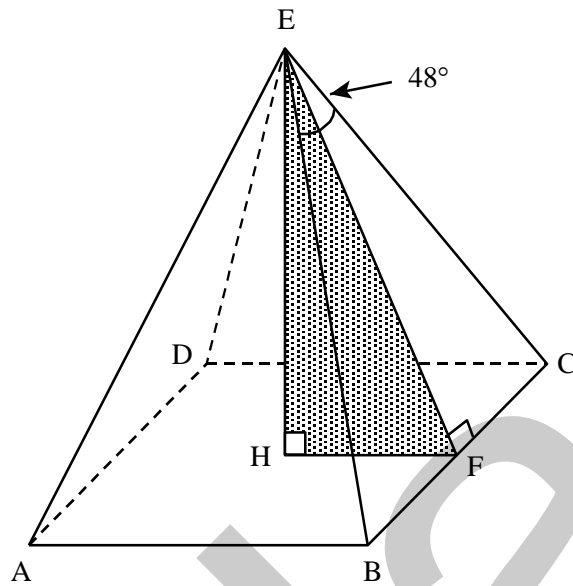
Rounded to the nearest whole number, what length of highway can be covered by this pile of gravel?



Show all your work

15

The base of the pyramid shown below is square; the area of the base is 64 cm^2 . The measure of angle BEC is 48° .

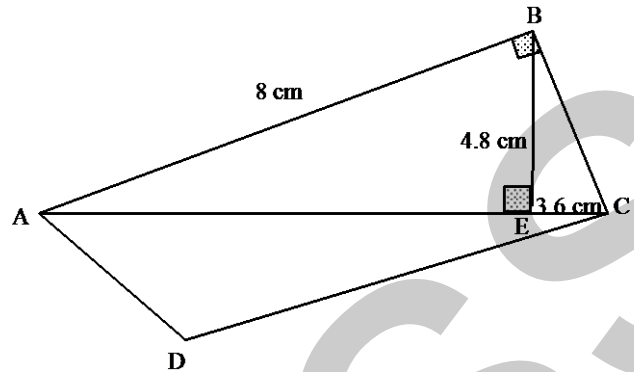


What is the volume of this pyramid?

Show your work.

- 16 In quadrilateral ABCD on the right, angle ABC is 90° , \overline{AC} is a diagonal and \overline{BE} is a height.

- Sides \overline{BC} and \overline{AD} are congruent.
- The measure of angle CAD is double the measure of angle BAC.
- And, $m \overline{AB} = 8 \text{ cm}$
 $m \overline{BE} = 4.8 \text{ cm}$
 $m \overline{CE} = 3.6 \text{ cm}$



What is the length of side CD?

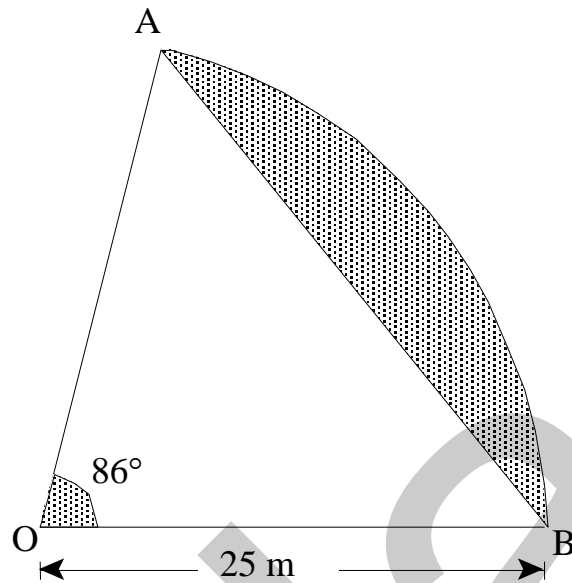
Show your work.

- 17 To construct a triangle ABC in which $a = 20 \text{ cm}$, $b = 30 \text{ cm}$ and $m \angle A = 25^\circ$, the measure(s) of c must be about

- | | |
|-------------------|-------------------|
| A) 12 cm | C) 42 cm |
| B) 12 cm or 42 cm | D) 15 cm or 50 cm |

18

One part of a secant-shaped garden is reserved for flowers and the other part, AOB, is covered in grass. The two straight sides AO and OB each measure 25 m. The angle between these two sides is 86° .



What is the area of the grass-covered part, to the nearest unit?

A) 271 m^2

C) 469 m^2

B) 312 m^2

D) 623 m^2

- 19 John lives in Town A and Eric lives in Town B, 4 km away. They both see the same airplane in the sky overhead between the two towns. John sees the airplane at an angle of elevation of 28° . At the same time, Eric sees the airplane at an angle of elevation of 40° .

Which of the following expressions could be used to find the altitude of the airplane, in kilometres?

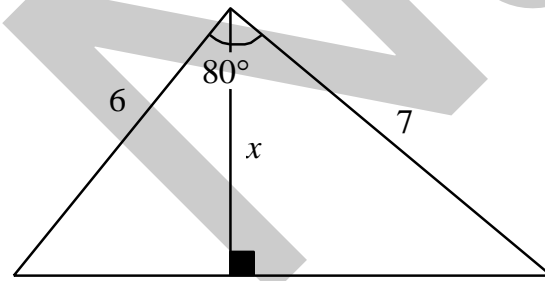
A) $\frac{4 \sin 28^\circ \sin 40^\circ}{\sin 112^\circ}$

C) $\frac{4 \sin 112^\circ}{\sin 28^\circ \sin 40^\circ}$

B) $\frac{4(\sin 28^\circ + \sin 40^\circ)}{\sin 112^\circ}$

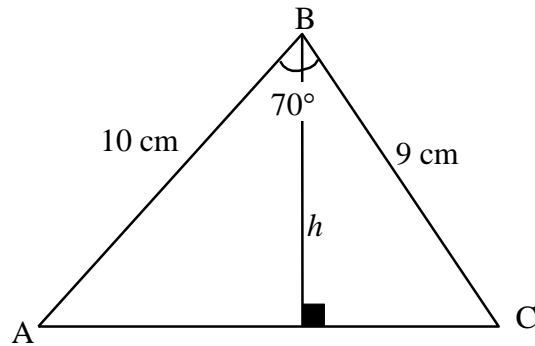
D) $\frac{4 \sin 112^\circ}{\sin 28^\circ + \sin 40^\circ}$

- 20 Find the value of x to the nearest hundredth given the data in the figure below.



Show your work.

21 In the triangle shown below, what is the measure of height h to the nearest hundredth?

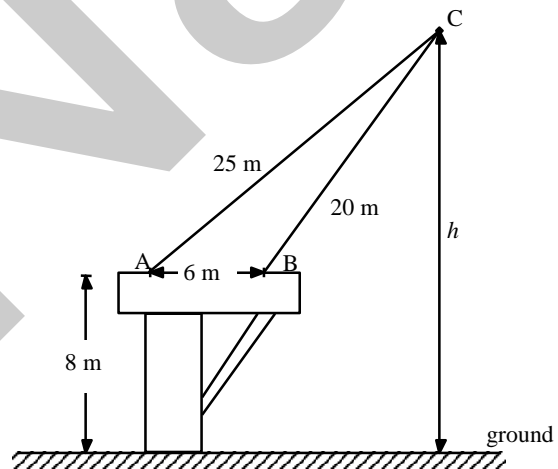


Show your work.

22 The Port of Montreal uses a special stationary crane to unload ships.

At the end of the day, the operator leaves the crane in the position illustrated by the adjacent diagram.

Rounded to the nearest tenth, how far above the ground is point C, the end of the crane?



Show your work.