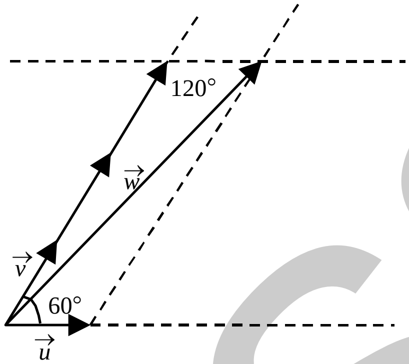
1

Example of an appropriate method

Draw the vector.



Since the adjacent angles in a parallelogram are supplementary,

180° – 60° = 120°

Therefore, using the Cosine Law

$$\left\|\vec{w}\right\|^2 = \left\|\vec{u} + 3\vec{v}\right\|^2$$

$$\left\|\vec{w}\right\|^2 = 1^2 + 3^2 - 2(1)(3)\cos 120°$$

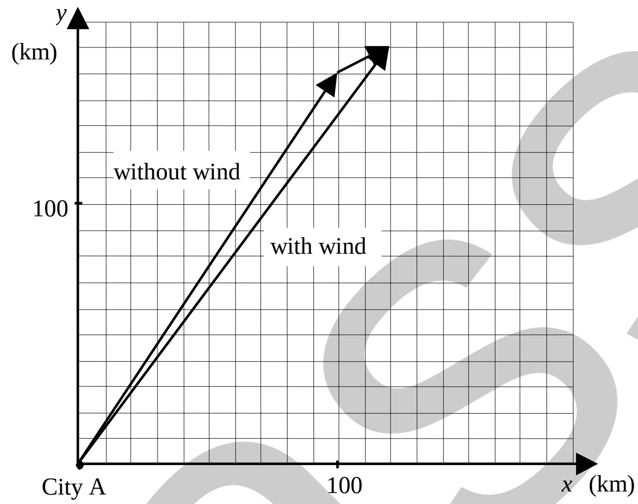
$$\left\|\vec{w}\right\|^2 = 13$$

$$\left\|\vec{w}\right\| = \sqrt{13} \approx 3.6$$

Answer The magnitude of the vector is 3.6 units.

2 Example of an appropriate method | /4

Let $(\overrightarrow{x,\ y})$ be the wind vector



$$(\overrightarrow{100,\ 150}) + (\overrightarrow{x,\ y}) = (\overrightarrow{120,\ 160})$$

$$(\overrightarrow{100 + x,\ 150 + y}) = (\overrightarrow{120,\ 160})$$

100 + x = 120 and x = 20

150 + y = 160 and y = 10

Therefore

$$(\overrightarrow{x,\ y}) = (\overrightarrow{20,\ 10})$$

The speed of the wind:

$$\left\|(\overrightarrow{20,\ 10})\right\| = \sqrt{20^2 + 10^2} \approx 22.36$$

Answer The wind speed is approximately 22.36 km/h.

3 D

4 B

5 The scalar product of vectors *u* and *v* is 16.

6 Example of an appropriate method

$$\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BN} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \frac{1}{2}\overrightarrow{AC}$$

$$\overrightarrow{PO} = \overrightarrow{PD} + \overrightarrow{DO} = \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} = \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{DC}\right) = \frac{1}{2}\overrightarrow{AC}$$

Vectors MN and PO are therefore parallel and of equal length.

Quadrilateral MNOP is therefore a parallelogram.

7 D

8

Example of an appropriate method

Given $\vec{u} = (5, 2)$ and $\vec{v} = (2, 3)$

Scalar product

$$\vec{u} \bullet \vec{v} = (5, 2) \bullet (2, 3)$$

$$\vec{u} \bullet \vec{v} = 5 \times 2 + 2 \times 3$$

$$\vec{u} \bullet \vec{v} = 16$$

Magnitude of the vectors

$$\|\vec{u}\| = \sqrt{5^2 + 2^2} = \sqrt{29} \approx 5.385$$

$$\|\vec{v}\| = \sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.606$$

Angle between $\vec{u}$ and $\vec{v}$

$$\vec{u} \bullet \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$\cos\theta = \frac{\vec{u} \bullet \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \frac{16}{\sqrt{29}\sqrt{13}}$$

$\cos\theta \approx 0.824\ 04$

$\theta \approx 34.51°$

Answer The angle between these two vectors measures 34.51°.

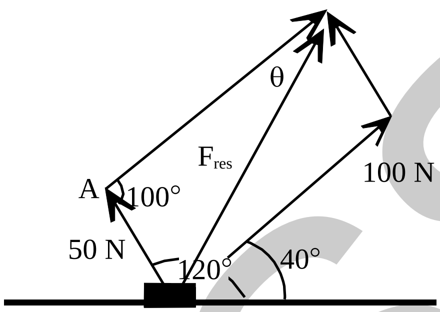
9

Example of an appropriate method

Measure of angle A

m ∠A = 180° – 80° = 100°

since two consecutive angle in a parallelogram are supplementary.



Resultant force (strength)

$$\left\|\vec{F}_{res}\right\|^2 = 50^2 + 100^2 - 2(50)(100)\cos 100°$$

$$\left\|\vec{F}_{res}\right\| \approx 119.3\,N$$

Direction of resultant force

$$\frac{\sin 100°}{119.3} = \frac{\sin \theta}{50}$$

$\sin \theta \approx 0.412\ 74$

$\theta \approx 24.38°$

The direction is 24.38° + 40°, so about 64.38°.

Answer Tim must apply a force of 119.3 N with a direction of 64.38°.

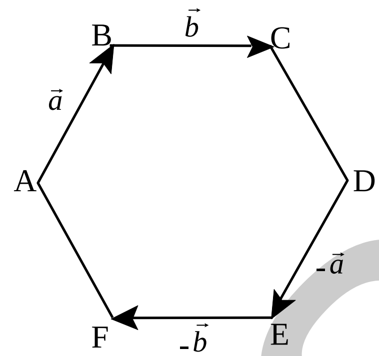
10 A

11 D

12

Example of an appropriate method

Hypothesis:



1. ABCDEF is a regular hexagon

2. $\overrightarrow{AB} = \vec{a}$

   $\overrightarrow{BC} = \vec{b}$

Conclusion : $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{DE} + \overrightarrow{EF} = \vec{a}$

		Proof			**Reasons**
1.	a)	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$	1.	a)	Chasles' relation
	b)	$\overrightarrow{AC} = \vec{a} + \vec{b}$		b)	By substitution
2.	a)	$\overrightarrow{DE} = -\overrightarrow{AB}$	2.	a)	$\overrightarrow{DE}$ and $\overrightarrow{AB}$ are non-collinear vectors by definition of a regular hexagon.
	b)	$\overrightarrow{DE} = -\vec{a}$		b)	By substitution
3.	a)	$\overrightarrow{EF} = -\overrightarrow{BC}$	3.	a)	$\overrightarrow{EF}$ and $\overrightarrow{BC}$ are non-collinear vectors by definition of a regular polygon.

b) $\overrightarrow{EF} = -\vec{b}$ — b) By substitution

4. $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{DE} + \overrightarrow{EF} =$ — 4. By vector addition

$\vec{a} + \vec{a} + \vec{b} + -\vec{a} + -\vec{b} = \vec{a}$

13 The scalar product of vectors *u* and *v* is -75.

14 B

15 D

16

Example of an appropriate solution

$\overrightarrow{CB} + \overrightarrow{AC} - \overrightarrow{FE} + \overrightarrow{GF} = \overrightarrow{GE}$ to be proved

1. $\overrightarrow{GA} + \overrightarrow{AC} - \overrightarrow{FE} + \overrightarrow{DE} = \overrightarrow{GE}$

   by substitution since $\overrightarrow{CB} = \overrightarrow{GA}$ and $\overrightarrow{GF} = \overrightarrow{DE}$

2. $\overrightarrow{GA} + \overrightarrow{AC} + \overrightarrow{EF} + \overrightarrow{DE} = \overrightarrow{GE}$

   because $\overrightarrow{EF}$ is the vector opposite to $\overrightarrow{FE}$

3. $\overrightarrow{GA} + \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{GE}$

   by substitution since $\overrightarrow{EF} = \overrightarrow{CD}$

4. $\overrightarrow{GC} + \overrightarrow{CE} = \overrightarrow{GE}$

   according to Chasles' Relation $\overrightarrow{GA} + \overrightarrow{AC} = \overrightarrow{GC}$ and $\overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{CE}$

5. $\overrightarrow{GE} = \overrightarrow{GE}$

   according to Chasles' Relation $\overrightarrow{GC} + \overrightarrow{CE} = \overrightarrow{GE}$

17

The components of vector *v* are (2, 4).

18

Example of an appropriate method

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ Chasles Relation

$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$ Chasles Relation

Scalar product

$$\overrightarrow{AC} \bullet \overrightarrow{BD} = \left(\overrightarrow{AB} + \overrightarrow{BC}\right) \bullet \left(\overrightarrow{BC} + \overrightarrow{CD}\right)$$

$= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) \bullet \left(\overrightarrow{BC} - \overrightarrow{AB}\right)$ as $\overrightarrow{CD} = -\overrightarrow{AB}$ definition of a rhombus

$= \overrightarrow{AB} \bullet \overrightarrow{BC} - \overrightarrow{AB}^2 + \overrightarrow{BC}^2 - \overrightarrow{BC} \bullet \overrightarrow{AB}$ by distributivity

$= -\overrightarrow{AB}^2 + \overrightarrow{BC}^2$

$= -\left\|\overrightarrow{AB}\right\|^2 + \left\|\overrightarrow{BC}\right\|^2$ definition of scalar product

$= c^2 - c^2$ c = length of one side of the rhombus

$= 0$

Since $\overrightarrow{AC} \bullet \overrightarrow{BD} = 0$, $\overrightarrow{AC} \perp \overrightarrow{BD}$ Scalar product theorem

19 D

20

Example of an appropriate method

Components of vector AB

$\overrightarrow{AB}$ = (400 – 150, 200 – 125) = (250, 75)

Components of the unknown vector

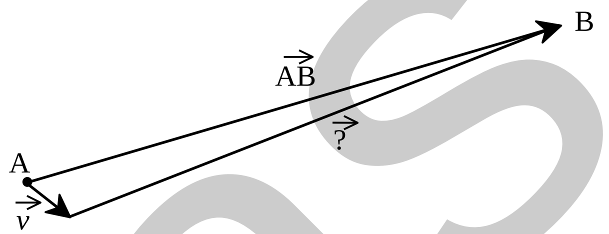
$\vec{v} + \vec{?} = \overrightarrow{AB}$

$\vec{?} = \overrightarrow{AB} - \vec{v}$

$\vec{?}$ = (250, 75) – (20, -15)

$\vec{?}$ = (250 – 20, 75 + 15)

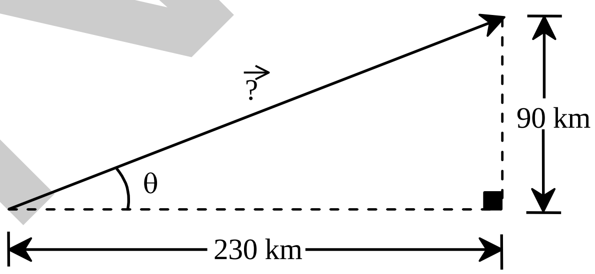
$\vec{?}$ = (230, 90)



Direction of the unknown vector

$\tan \theta = \frac{90}{230}$

$\theta \approx 21.37°$



Answer: To the nearest degree, the pilot should point the plane at an angle of **21**° relative to the east in order to reach airport B.

Note: Do not penalize students who did not round off their final answer or who made a mistake in rounding if off.

Students who used an appropriate method in order to determine the components of the unknown vector have shown that they have a partial understanding of the problem.

21 C

22 C

23 Rounded to the nearest tenth $\left\| \vec{u} + \vec{v} \right\|$ is 16.6.

24 A

25 A

26 To the nearest degree, the angle measure is **38°**.

Note: Do not penalize students who did not round their answer.

27 B

28

Example of an appropriate solution

$$\overrightarrow{AB} = (15 - 3, 8 - 4)$$
$$= (12, 4)$$

P is (x, 0) then $\overrightarrow{AP} = (x - 3, 0 - 4)$
$$= (x - 3, -4)$$

$$\overrightarrow{AB} \bullet \overrightarrow{AP} = (12, 4) \bullet (x - 3, -4)$$
$$= 12(x - 3) + 4(-4)$$
$$= 12x - 36 - 16$$
$$= 12x - 52$$

But $\overrightarrow{AB} \bullet \overrightarrow{AP} = -40$

$$12 \times -52 = -40$$
$$12x = -40 + 52$$
$$12x = 12$$
$$x = 1$$

P is (1, 0)

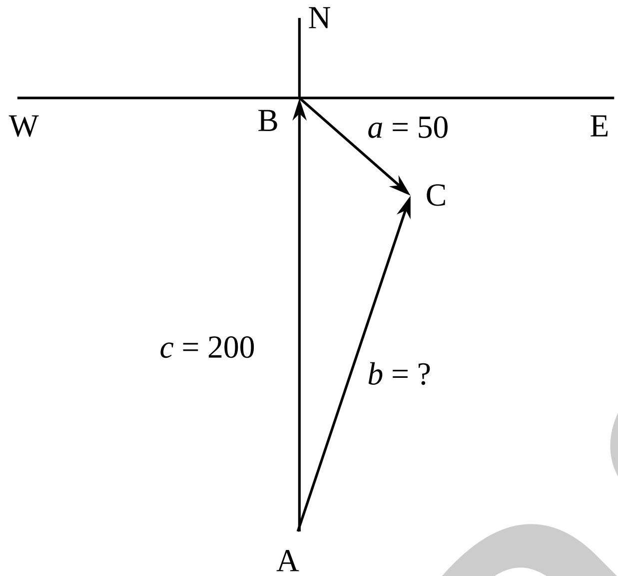
$$\left|\overrightarrow{AP}\right| = \sqrt{(1 - 3)^2 + (0 - 4)^2}$$
$$= \sqrt{(-2)^2 + (-4)^2}$$
$$= \sqrt{4 + 16}$$
$$= \sqrt{20}$$
$$= 4.4721\ldots$$

Answer: To the nearest tenth of a unit, the magnitude of $\overrightarrow{AP}$ is **4.5** units

Note: Students who have determined the value of *x* have shown they have a partial understanding of the problem.

29

Example of an acceptable solution



$\overrightarrow{AB} = c = 200$ km/h, North $\qquad \overrightarrow{BC} = a = 50$ km/h, from Northwest

$\overrightarrow{AC} = b = ?$

In Δ ABC

$b^2 = a^2 + c^2 - 2ac \cos(B)$

$b^2 = 50^2 + 200^2 - 2 \times 50 \times 200 \times \cos(45)$

$b = \sqrt{28\ 357.8644}$

$= 168.398$ km

Also

$$\frac{\sin(\text{A})}{a} = \frac{\sin(\text{B})}{b} = \frac{\sin(\text{C})}{c}$$

$$\frac{\sin(\text{A})}{50} = \frac{\sin(45)}{168.398}$$

$$\sin(\text{A}) = \frac{50 \times \sin(45)}{168.398} \rightarrow \text{A} = \sin^{-1}(0.2100)$$

$$\text{A} = 12.1195°$$

$$\text{A} = 12.1195° \text{ as shown or N } 12.1195° \text{ E}$$

Answer: The resulting speed of the airplane is **168.4** km/h in a direction of **12.12**° or **N 12.12° E**.

Name : ______________________________

Group : ______________________________

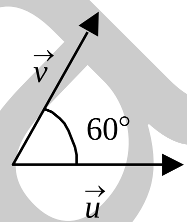
Date : ______________________________

568536 - Mathematics

Question Booklet

1

Two unit vectors, *u* and *v*, form a 60° angle as shown.



What is the magnitude of the vector *w* if

$\vec{w} = \vec{u} + 3\vec{v}$?

Show all your work.

2

A plane goes from city A to city B. In a Cartesian plane, city A is at the origin and city B has coordinates (100, 150). If there is no wind, the flight lasts one hour.

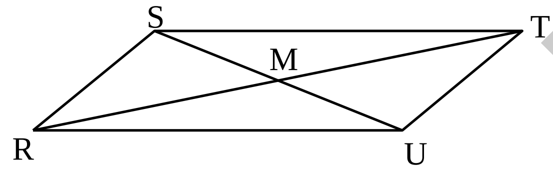
Unfortunately, there is a wind. If the pilot does not adjust his flight path, he will be at point (120, 160) after an hour.

What is the speed of the wind?

Show all your work.

3

Quadrilateral RSTU is a parallelogram and M is the point of intersection of its diagonals.



Antoine lists the following vector operation statements:

1) $\overrightarrow{ST} + \overrightarrow{SR} = 2\overrightarrow{MU}$

2) $\overrightarrow{UT} + \overrightarrow{UR} = 2\overrightarrow{SM}$

3) $\overrightarrow{RS} + \overrightarrow{RU} = \overrightarrow{RT}$

4) $\overrightarrow{MT} + \overrightarrow{MR} + \overrightarrow{MS} + \overrightarrow{MU} = \vec{0}$

5) $\overrightarrow{SR} - \overrightarrow{ST} = \overrightarrow{RT}$

Which of these statements are true?

A) 1, 2 and 3 only

B) 1, 2 and 5 only

C) 2, 4 and 5 only

D) 1, 3 and 4 only

4 Given the following information:

$\vec{a}$ and $\vec{b}$ are nonzero vectors in the plane

$\vec{a} \neq \vec{b}$

k is a scalar not equal to zero

$k \neq 1$

Which of the following statements is true?

A) $k(\vec{a} \bullet \vec{b}) = k\vec{a} \bullet k\vec{b}$

B) $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$

C) If $\vec{a} \bullet \vec{b} = 0$ then $\vec{a}$ and $\vec{b}$ are collinear

D) If $\vec{a} = k\vec{b}$ then $\vec{a}$ and $\vec{b}$ are noncollinear

5 Given vectors *u* and *v*.

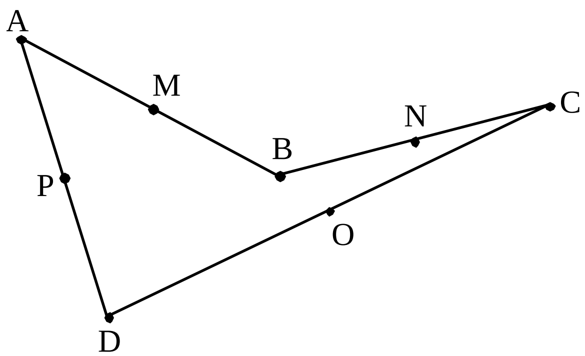
$\vec{u} = \overrightarrow{AB}$ where A(-2, 3) and B(6, 7)

$\vec{v} = (4, -4)$

What is the scalar product of vectors *u* and *v*?

6

In quadrilateral ABCD illustrated below, points M, N, O and P are the midpoints of segments AB, BC, CD and DA respectively.



Using the above figure, prove the following proposition: "The midpoints of the sides of any quadrilateral form the vertices of a parallelogram."

Show all your work.

7

Given $\vec{u}$ and $\vec{v}$ two vectors that are not opposite.

Which of the following is FALSE?

A) $\vec{u} \bullet \vec{v} = \vec{v} \bullet \vec{u}$

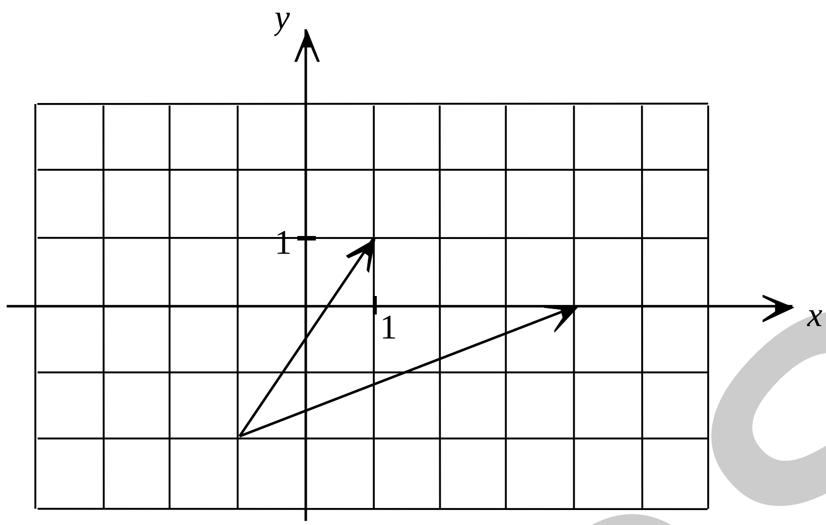
B) $2\vec{u} \bullet 3\vec{v} = 6\vec{v} \bullet \vec{u}$

C) $2(\vec{u} + \vec{v}) = 2\vec{u} + 2\vec{v}$

D) $2\vec{u} + 3\vec{v} = 3\vec{u} + 2\vec{v}$

8

Given $\vec{u}$ and $\vec{v}$ represented in the Cartesian plane below.

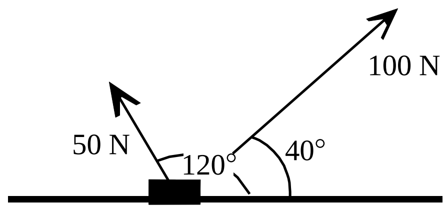


What is the measure of the angle between these vectors, rounded to the nearest hundredth?

Show all your work.

9

Peter and Marie are pulling on an object. The forces they applied are 100 N and 50 N respectively but in different directions: 40° and 120°. The situation is represented below.



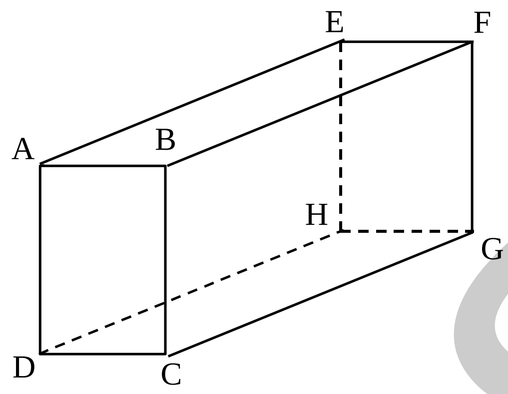
Tim is going to replace them.

What force must Tim apply to produce the same effect on the object (strength and direction)?

Show all your work.

10

Given the following prism having a rectangular base.



Which vector is equivalent to the resultant of the expression $\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{HE}} + \overrightarrow{\mathbf{AE}}$?

A) $\overrightarrow{DH}$

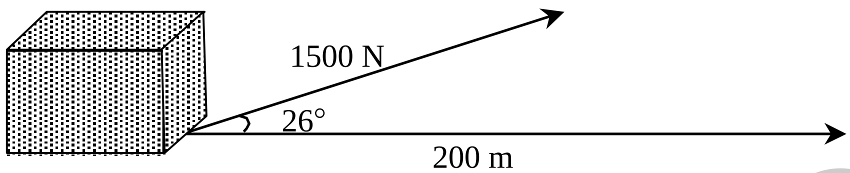
B) $\overrightarrow{BE}$

C) $\overrightarrow{FB}$

D) $\overrightarrow{BC}$

11

The Egyptians used an ingenious pulley system to move the blocks of stone used in the construction of pyramids. To minimize the work needed to displace the blocks, they applied a force oriented at 26°. (Work (Nm) is the scalar product of the force vector and the displacement vector.)



Rounded to the nearest Nm, what work is needed to displace a block of stone horizontally for a distance of 200 m, if the force applied to it is 1500 N oriented at 26°?

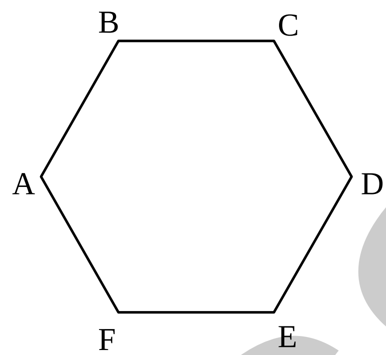
A) 131 511 Nm

B) 194 076 Nm

C) 228 768 Nm

D) 269 638 Nm

12 Given the regular hexagon on the right



where $\overrightarrow{AB} = \vec{a}$

and $\overrightarrow{BC} = \vec{b}$.

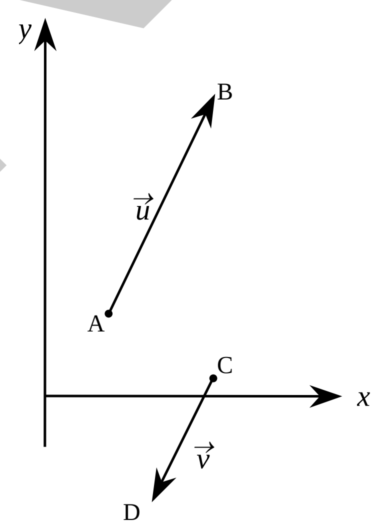
Prove the following identity: $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{DE} + \overrightarrow{EF} = \vec{a}$.

Show all your work.

13 Vectors *u* and *v* are represented in the Cartesian plane below.

$\vec{u} = \overrightarrow{AB}$ where A(3, 4) and B(8, 14)

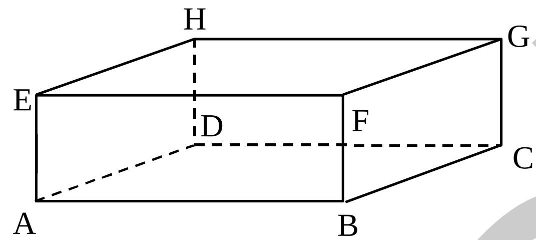
$\vec{v} = \overrightarrow{CD}$ where C(8, 1) and D(5, -5)



What is the scalar product of vectors *u* and *v*?

14

The following figure represents a right prism.



Which of these statements is FALSE?

A) $\overrightarrow{BC} + \overrightarrow{GF} = \vec{0}$

B) $\overrightarrow{AB} - \overrightarrow{FE} = \vec{0}$

C) $\overrightarrow{AB} \bullet \overrightarrow{AD} = 0$

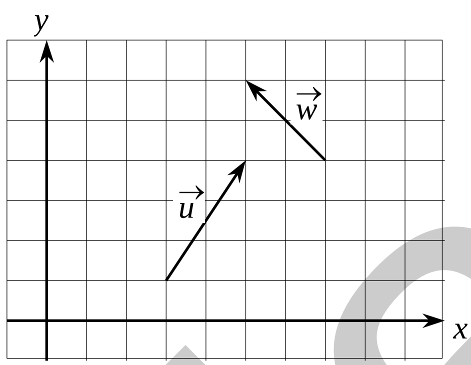
D) $\overrightarrow{EH} + \overrightarrow{HF} + \overrightarrow{FG} - \overrightarrow{EG} = \vec{0}$

15

Given the three vectors *u*, *v,* and *w*.

$\vec{v} = (-2, -3)$

$\vec{u}$ and $\vec{w}$ are represented in the Cartesian plane below:



Which of the following statements is TRUE?

A) $\vec{v}$ and $-\vec{u}$ are opposite.

B) $\vec{u}$ and $\vec{v}$ are equivalent.

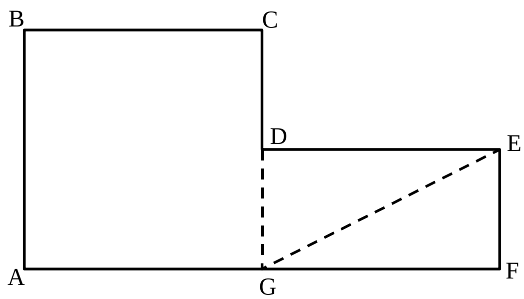
C) $\vec{w}$ and $(\vec{v} + \vec{w})$ are perpendicular.

D) $\vec{u}$ and $3\vec{v}$ are collinear.

16

In the polygon below, ABCG is a square.

D and G are the midpoints of sides CG and AF, respectively. Side AB is parallel to side EF.



Using the properties of vectors, show that $\overrightarrow{CB} + \overrightarrow{AC} - \overrightarrow{FE} + \overrightarrow{GF} = \overrightarrow{GE}$.

Show all your work.

17

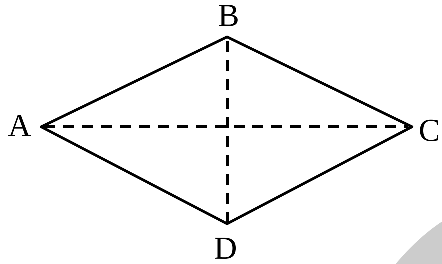
Given $\vec{u} = (9, 2)$ and $\vec{v} = (a, b)$, two vectors that form a basis.

Vector *w* = (24, 16) can be expressed by the following linear combination: $\vec{w} = 2\vec{u} + 3\vec{v}$.

What are the components of vector *v*?

18

Given the adjacent rhombus ABCD.



Use vectors to prove the following statement:

« The diagonals of the rhombus are perpendiculars. »

Show all your work.

19

Consider rectangle ABCD shown below.

Which of the following statements is true?

A) $\overrightarrow{DA}+\overrightarrow{AB}=\overrightarrow{AC}$

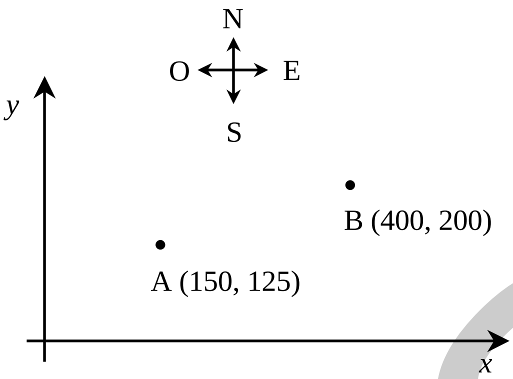
B) $\overrightarrow{AB}\bullet\overrightarrow{BC}=\overrightarrow{AC}$

C) $\overrightarrow{AB}\bullet\overrightarrow{BC}=\overrightarrow{DC}+\overrightarrow{CB}$

D) $\overrightarrow{AB}\bullet\overrightarrow{AD}=\overrightarrow{AB}\bullet\overrightarrow{BC}$

20

An airplane leaves airport A and must fly to airport B. In the Cartesian plane on the right, these airports are represented by points A and B respectively. The scale of the graph is in kilometres.



During the flight, the plane encounters a steady wind. This wind is represented by the vector $\vec{v}$ = (20, -15).

The pilot steers the plane so as to negate the effect of the wind.

To the nearest degree, at what angle relative to the east should the pilot point the plane in order to reach airport B?

Show all your work.

21

Given that *u* and *v* are vectors, which of the following is NOT a vector?

A) $\vec{u} + \vec{v}$

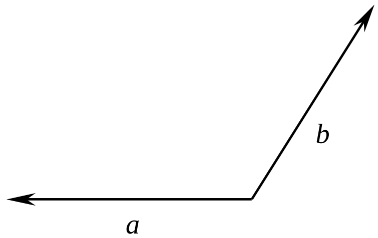
B) $\vec{u} - \vec{v}$

C) $\vec{u} \bullet \vec{v}$

D) $2(\vec{u} + \vec{v})$

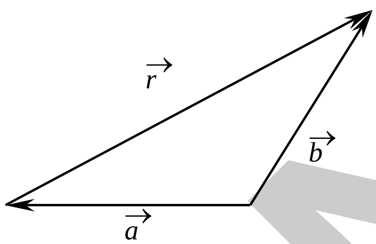
22

Given $\vec{a}$ and $\vec{b}$, two vectors illustrated below.

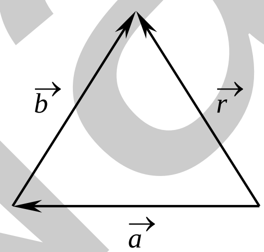


Which one of the following diagrams illustrates the relation between $\vec{a}$ and $\vec{b}$ and $\vec{r}$, the resultant vector?

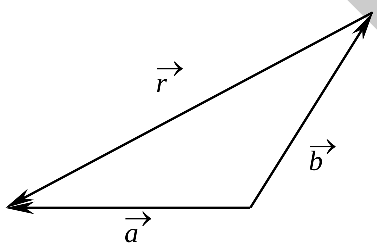
A)



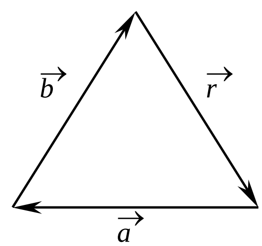
C)



B)



D)



23

Given vectors *u* and *v* where:

$\vec{u} = \overrightarrow{AB}$ with A(-5, 7) and B(3, -5)

$\vec{v} = (6, 3)$

Find $\left\| \vec{u} + \vec{v} \right\|$. Round your answer to the nearest tenth.

24

Given $\vec{u} = (3, 2)$, and $\vec{v} = (1, -4)$

What are the components of the resultant of the following vector operation?

$$\vec{u} - 2\vec{v}$$

A) (1, 10)

C) (2, 6)

B) (1, -6)

D) (5, -6)

25

An airplane flying East at 150 km/h encounters a 50 km/h wind blowing in a 30° East of North direction.

What will be the airplane's resultant velocity?

A) 180 km/h [E 14° N]

B) 195 km/h [E 7° N]

C) 200 km/h [N 30° E]

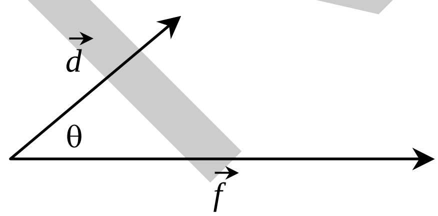
D) 132 km/h [E 19° S]

26

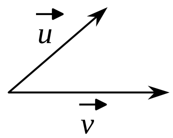
The scalar product of vectors *d* and *f* is 138. Their respective magnitudes are 7 and 25 units.

What is the measure of angle θ between vectors *d* and *f*?

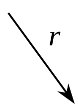
Round your answer to the nearest degree.

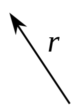


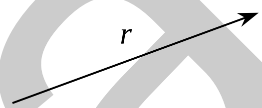
27 Given vectors $\vec{u}$ and $\vec{v}$ shown below.

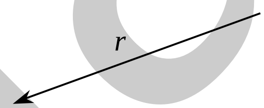


Which of the following vectors represents the resultant, r, of $\vec{u} - \vec{v}$?

A) 

B) 

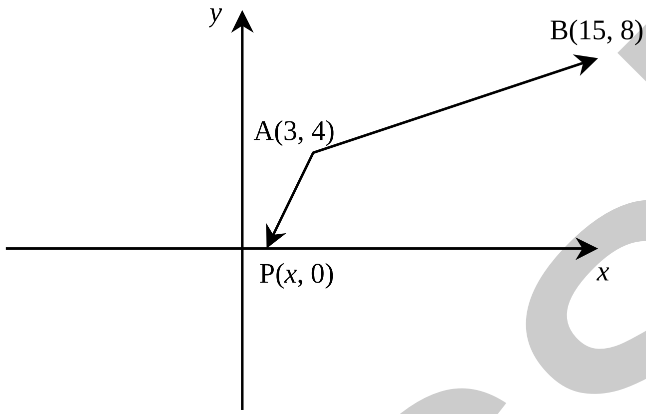
C) 

D) 

28

In the diagram on the right, the terminal point of $\overrightarrow{AP}$ lies on the *x* axis and

$\overrightarrow{AB} \bullet \overrightarrow{AP} = -40$.



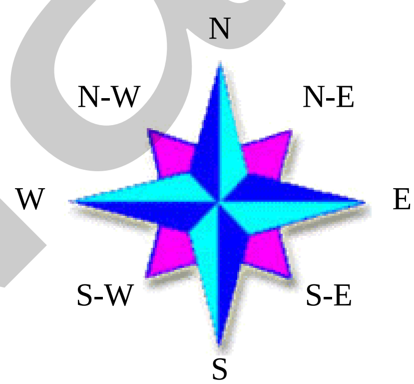
What is the magnitude of $\overrightarrow{AP}$, to the nearest tenth of a unit?

Show all your work.

29

An airplane is flying due north at 200 km/h.

Suddenly, the wind starts blowing from the northwest at 50 km/h.



What is the resulting speed and direction of the airplane?

Show all your work.